



## Effect of basic Temperature Gradients on Electrothermal Convection in a Rotating Layer of Viscoelastic (Walter's B) Fluid Saturated Porous Medium Heated from Below

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**Abstract:-** Combined Effect of Thermal Modulation and Rotation on the Onset of Convection in Walter's B Viscoelastic Fluid Saturated Porous Medium has been investigated. The problem is numerically solved using the Galerkin method after applying linear stability analysis and normal modes. It is possible to derive the dispersion relation while accounting for the effects of wave number, electric Rayleigh number, Darcy number, Taylor number, and thermal electric Rayleigh number. Graphically, the effects of the electric Rayleigh number, Darcy number, Wave number, and Taylor number on the onset of stationary convection have been studied. Under the boundary conditions considered, The kinematic viscoelasticity accounting for rheology of the nanofluid has no effect on the stationary convection for Walters' (model B') nanofluids and behaves like an ordinary Newtonian nanofluid.

**Keywords:** Convection, Temperature Gradient, Porous Medium, Rotation, Walter's B Viscoelastic Fluid.

### 1. Introduction

Thermal convection in fluid-saturated porous media has piqued the interest of researchers in recent decades due to its importance in a variety of applications including geothermal energy utilisation, enhanced recovery of petroleum reservoirs, thermal insulation engineering, nuclear waste repository, grain storage, and mantle convection. The issue is also relevant in many engineering applications. Chandrasekhar [1] has provided a comprehensive account of Newtonian fluid thermal instability under various hydrodynamic and hydro magnetic assumptions. The Darcy model has been used to begin the investigation into porous media. In [2-4], a good account of convection problems in a porous medium is given. [5-7] has investigated the electrodynamics of continuous media and electrohydrodynamic convection in fluids. Electro hydrodynamics is a branch of fluid mechanics that studies fluid motion under the influence of electrical forces. It can also be thought of as the part of electrodynamics that deals with the effect of moving media on electric fields. The effect of fluid in motion is combined with the influence of the field in motion in electro hydrodynamics [8-9].



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Choi [10] was the first to use the term "nanofluid." The term "nanofluid" refers to a mixture of metallic nanoparticles suspended in base fluids like water, ethanol, or engine oils. The base fluids used in nanofluids can be oxide ceramics like  $\text{Al}_2\text{O}_3$  or  $\text{CuO}$ , nitride ceramics like  $\text{AlN}$  or  $\text{SiN}$ , or various metals like  $\text{Al}$  or  $\text{Cu}$ . Nanofluid has many uses in the automotive and energy-saving industries, among others. Additionally, nanoparticle suspensions are being developed for medical uses, such as cancer treatment. Numerous authors [12–22] have discussed the thorough investigation of thermal convection in a layer of nanofluid in a porous medium based on the Buongiorno [11] model.

In all the studies mentioned above, Newtonian nanofluids are involved. The study of non-Newtonian nanofluids, however, has garnered considerable interest due to the growing significance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology, and the petroleum industry. Walter's (Model B) [22] elastico-viscous fluid is one of these fluid types, and it has application in chemical technology and industry. Numerous significant polymers and practical goods are produced using Walter's (Model B) elastico-viscous fluid as a starting point. In [24-27], a thorough explanation of the issues with thermal instabilities in a Walter's (Model B) elastico-viscous fluid in a porous medium is provided.

The study of electrohydrodynamic thermal instability in viscous and viscoelastic fluid has attracted a lot of interest. Regarding the onset of convective instability in a dielectric fluid under the concurrent action of an AC electric field, Takashima [28], discussed the impact of uniform rotation. Researchers [29-36] investigated the onset of electrohydrodynamic instability in a horizontal layer of viscous and viscoelastic fluid.

Recently, The electrohydrodynamic thermal instability in a layer of porous medium saturated with a walter's (Model B) elastico-viscous nanofluid was recently studied by GianC.Rana[37]. Weakly nonlinear thermohaline rotating convection in a sparsely packed porous medium was studied by A. BenerjiBabu [38]. The combined effect of thermal modulation and an alternating current electric field on the beginning of electrothermoconvection in an anisotropic porous layer was studied by Mahantesh S. Swamy [41]. The combined effect of suspended particles and rotation on double-diffusive convection in a viscoelastic fluid saturated by a Darcy-Brinkman porous medium was studied by G. C. Rana [42]. M. S. Malashetty [43] investigated how rotation and thermal modulation interact to cause stationary convection to begin in a porous layer, Combining the Influence of Temperature Modulation and Magnetic Field on the Onset of Convection in an Electrically Conducting-Fluid Saturated Porous Medium by B. S. Bhadauria [44].

Examining the Combined Effect of Thermal Modulation and Rotation on the Onset of Convection in Walter's B Viscoelastic Fluid Saturated Porous Medium is the primary goal of the current study. For three different types of velocity boundary conditions, namely (i) rigid-rigid, (ii) free-free, and (iii) lower rigid and upper free, the system's stability characteristics are



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discussed. To our knowledge, no work has been started for such fluids in this direction. The boundary temperature modulation converts the fundamental temperature distribution from linear to nonlinear, which helps in effective control of convective instability.

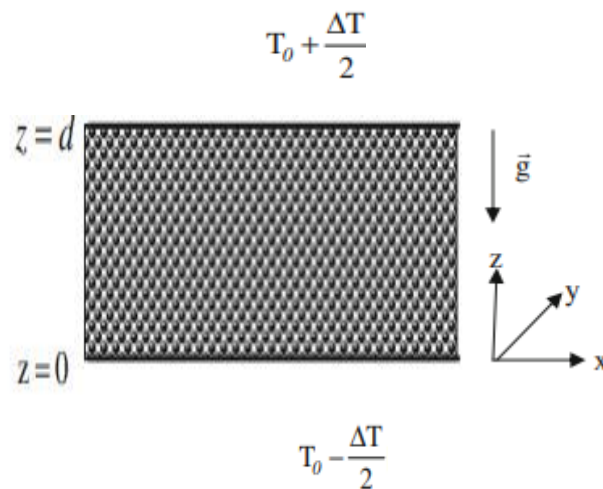
## 2. Objectives Mathematical Problem formulation

We consider a horizontal layer of Walter's B viscoelastic fluid-saturated porous medium of thickness  $d$  in the presence of gravity as shown in Fig. 1. The time-dependent temperature of lower and upper surfaces of the porous layer is externally imposed and is given by

$$T = T_0 + \frac{1}{2} \Delta T \quad \text{at} \quad z = 0 \quad (1)$$

$$T = T_0 - \frac{1}{2} \Delta T \quad \text{at} \quad z = d \quad (2)$$

$T_0$  is the reference temperature. The time-dependent parts denote the modulation imposed on the adverse thermal gradient caused by the temperatures  $T_0 + \Delta T/2$  and  $T_0 - \Delta T/2$  at the lower and upper surfaces, respectively. In addition, a vertical AC electric field is also imposed across the layer; the lower and upper surfaces are kept at an alternating potential  $V_0 + \Delta T/2$  and  $V_0 - \Delta T/2$ , respectively. A Cartesian coordinate system  $(x, y, z)$  is chosen such that the origin is at the bottom of the porous layer and  $z$ -axis is directed vertically upward.



**Figure. 1** Diagrammatic representation

The relevant basic equations are:

$$\Delta \cdot \vec{q} = 0 \quad (3)$$

$$\rho_0 \left[ \frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} - \frac{2}{\varepsilon} (\vec{q} \times \vec{\Omega}) \right] = -\nabla p + \rho \vec{g} - \frac{1}{k} \left( \mu - \mu_v \frac{\partial}{\partial t} \right) \vec{q} + f_e \quad (4)$$



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$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = K \nabla^2 T \quad (5)$$

$$f_e = \rho_e \vec{E} - \frac{1}{2} \vec{E} \cdot \vec{E} \nabla \varepsilon + \frac{1}{2} \nabla \left( \rho \frac{\partial \varepsilon}{\partial \rho} \vec{E} \cdot \vec{E} \right) \quad (6)$$

$$\rho = \rho_0 \{1 - \alpha(T - T_0)\} \quad (7)$$

where  $\vec{q}$  is the velocity,  $K$  is the permeability of the porous medium,  $k$  the effective thermal diffusivity,  $p$  the pressure,  $\vec{g}$  is acceleration due to gravity,  $T$  the temperature,  $\varepsilon$  the porosity of the medium,  $\alpha$  the volumetric expansion coefficient,  $\mu$  the viscosity,  $\mu_v$  the viscoelastic constant of Walters B liquid, and  $\rho_0$  is the reference density.  $\vec{E}$  is the electric field,  $\rho_e$  is the free charge density,  $\vec{\Omega} = (0, 0, \Omega)$  is the angular velocity,  $\rho$  is the density equal to at  $\rho_0$  the reference temperature. where the quantities have their pre-defined meaning. The coulomb force term  $\rho_e \vec{E}$ , is of negligible order compared with the dielectrophoretic force term for most dielectric fluids in a 60-Hz AC electric field. Therefore, the coulomb force term has been neglected in (6) and only the dielectrophoretic force term is retained.

The applicable Maxwell's equations are

$$\nabla \times \vec{E} = 0, \nabla \cdot (\varepsilon \vec{E}) = 0 \quad (8a, 8b)$$

$$\text{In view of (7), } \vec{E} \text{ defined as } \vec{E} = -\nabla V \quad (9)$$

Where the electric potential is  $V$ , the charge density is,  $\rho_e = \nabla \cdot (\varepsilon \vec{E})$  and the dielectric constant is  $\varepsilon$ .

The dielectric constant is assumed to be a linear function of temperature in the form

$$\varepsilon = \varepsilon_0 (1 - \gamma(T - T_0)) \quad (10)$$

Where the dielectric constant is ( $\gamma > 0$ ) & it is very small.

## 2.1 Basic State

The basic state is quiescent and is given by

$$\vec{q} = \vec{q}_b = 0, T = T_b(z), p = p_b(z), \vec{E} = E_b(z) \quad \vec{\rho} = \rho_b(z) \quad \varepsilon = \varepsilon_b(z) \quad v = v_b(z) \quad (11)$$



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Where  $b$  subscript denote the basic state.

$$\rho_b \vec{g} + \nabla p_b + f_e = 0 \quad (12)$$

$$\frac{dT_b}{dz} = - \frac{\Delta T}{d} f(z) \quad (13)$$

The expression for  $p_b$  and  $\rho_b$  is not given as they are not explicitly required in the subsequent analysis.

$$\rho_b = \rho_0 \{1 + \alpha \beta z\} \quad (14)$$

$$\nabla \times \vec{E}_b = 0, \quad \nabla \cdot (\epsilon \vec{E}_b) = 0 \quad (15)$$

It is found that

$$T_b - T_0 = -\beta z \quad (16)$$

$$\vec{E}_b(z) = \frac{E_0}{(1+\gamma\beta z)} \hat{k} \quad (17a)$$

And hence

$$V_b(z) = -\frac{E_0}{\gamma\beta} \log(1 + \gamma\beta z) \quad \text{where} \quad E_0 = -\frac{V_1 \gamma \beta}{\log(1 + \gamma\beta z)} \quad (17b)$$

Is the root mean square value of the electric field at  $z = 0$ .

$$\epsilon_b = \epsilon_0 (1 + \gamma\beta z) \quad (18)$$

$$\vec{E}_b = \nabla V_b \quad (19)$$

### 3. Analysis of linear stability

We give an infinitesimal disturbance to the basic state in the form

$$\vec{q} = \vec{q}', \quad p = p_b + p', \quad \vec{E} = \vec{E}_b + \vec{E}', \quad T = T_b + T', \quad \rho = \rho_b + \rho', \quad \epsilon = \epsilon_b + \epsilon' \quad (20)$$

where  $q', p', E', T', \rho'$  and  $\epsilon'$  represents the perturbs. Substituting Eq. (20) into Eq.(3)-(10), eliminating the pressure by operating curl twice, and retaining the vertical component, we get (after ignoring the primes).





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$$\left\{ \frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{\mu}{k\rho_0} \left( 1 - \frac{\mu_v}{\mu} \frac{\partial}{\partial t} \right) \right\} \nabla^2 w = \alpha g \nabla^2_h T + \frac{2\Omega}{\varepsilon} \frac{\partial \xi}{\partial z} + R_{ea} \nabla^2_h T - R_{ea} \nabla^2_h \frac{\partial v}{\partial z} \quad (21)$$

$$\left\{ \frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{\mu}{k\rho_0} \left( 1 - \frac{\mu_v}{\mu} \frac{\partial}{\partial t} \right) \right\} \xi = \frac{2\Omega}{\varepsilon} \frac{\partial w}{\partial z} \quad (22)$$

$$\left( M \frac{\partial}{\partial t} - \nabla^2 \right) T = -f(z)w \quad (23)$$

$$\nabla^2 V = \frac{\partial T}{\partial z} \quad (24)$$

Non-dimensionalizing the equations by setting

$$(x^*, y^*, z^*) = \left( \frac{x}{d}, \frac{y}{d}, \frac{z}{d} \right), \quad T^* = \frac{T}{\Delta T}, \quad w^* = \frac{w}{k/d}, \quad t^* = \frac{t}{d^2 \varepsilon / k}, \quad \vec{q}' = \frac{\vec{q}}{d}, \quad \xi' = \frac{\xi}{d^2},$$

$$V' = \gamma E_0 \Delta T d \quad (25)$$

and substituting Eq. (25) into Eq. (21) – (24), we obtain, respectively,

$$\left\{ \frac{1}{pr} \frac{\partial}{\partial t} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} \frac{\partial}{\partial t} \right) \right\} \nabla^2 w - R_t \nabla^2_h T + Ta^{1/2} \frac{\partial \xi}{\partial z} + R_{ea} \nabla^2_h T - R_{ea} \nabla^2_h \frac{\partial v}{\partial z} = 0 \quad (26)$$

$$\left\{ \frac{1}{pr} \frac{\partial}{\partial t} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} \frac{\partial}{\partial t} \right) \right\} \xi - Ta^{1/2} \frac{\partial w}{\partial z} \quad (27)$$

$$\left( M \frac{\partial}{\partial t} - \nabla^2 \right) T = -f(z)w \quad (28)$$

$$\nabla^2 V = \frac{\partial T}{\partial z} \quad (29)$$

Where  $R_t = \alpha g \Delta T d^3 / \nu k$  is the thermal Rayleigh number,  $R_{ea} = \gamma^2 \varepsilon_0 E^2_0 (\Delta T)^2 d^2 / \mu k$  is the AC electric Rayleigh number,  $Ta = 4\Omega^2 d^4 / \nu^2$  is the Taylor number,  $pr = \nu / k \varepsilon^2$  is the modified

Prandtl number,  $\Gamma p = \mu_v \varepsilon / \rho_0 d^2$  is the elastic parameter,  $M = A_h / \varepsilon$  is a modified thermal capacity ratio,  $Da = k / d^2$  is the Darcy number and  $\xi = \partial v / \partial x - \partial u / \partial y$  is the z-component of vorticity.



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The boundaries of the porous layer are assumed to be either free or rigid with fixed temperature and electric potential at the boundaries. Accordingly, the boundary conditions on the stress-free boundary are,

$$w = \frac{\partial^2 w}{\partial z^2} = T = \frac{\partial \xi}{\partial z} = \frac{\partial v}{\partial z} = 0 = DT \quad (30)$$

and on the rigid boundaries, the conditions are

$$w = \frac{\partial w}{\partial z} = T = \xi = V = 0 \quad (31)$$

It may be noted that only one type of boundary condition on  $V$  is considered on the rigid and stress-free boundaries in investigating the problem although either of the conditions can be imposed on these boundaries [8,11].

### 3.1 Normal Mode Study

To carry out the linear stability analysis, we employ the normal mode analysis procedure in which we look for the solution of the form

$$(w, T, V, \xi) = (W, \theta, \phi, Z) e^{(ilx + imy + \omega t)} \quad (32)$$

where ‘ $l$ ’ and ‘ $m$ ’ are the horizontal wave numbers in the  $x$  and  $y$  directions respectively and  $\omega = \omega_r + i\omega_i$  is the growth rate. Substituting Eq. (32) into Eq. (26)–(29), we obtain,

$$\left\{ \frac{w}{pr} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} w \right) \right\} (D^2 - a^2)W - R_t a^2 \theta + Ta^{1/2} DZ + R_{ea} a^2 \theta - R_{ea} a^2 D\phi = 0 \quad (33)$$

$$\left\{ \frac{w}{pr} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} w \right) \right\} Z - Ta^{1/2} DW = 0 \quad (34)$$

$$(Mw - (D^2 - a^2))\theta + f(z)w = 0 \quad (35)$$

$$(D^2 - a^2)\phi - D\theta = 0 \quad (36)$$

where  $D = d/dz$  and  $a = \sqrt{l^2 + m^2}$  is the dimensionless horizontal wave number,

The above equations are to be solved subject to appropriate boundary conditions.

At both the boundaries rigid,



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$$W = DW = \theta = Z = \phi = 0 \text{ at } z = 0, 1 \quad (37)$$

At the both the boundaries free,

$$W = D^2W = \theta = DZ = D\phi = 0 \text{ at } z = 0, 1 \quad (38)$$

and at the lower rigid surface, ( $z = 0$ )

$$W = DW = \theta = Z = \phi = 0 \quad (39a)$$

And at the upper free interface ( $z = 1$ )

$$W = D^2W = \theta = DZ = D\phi = 0 \quad (39b)$$

### 3.2 Numerical solution of the problem

It has been observed that oscillatory convection occurs only if the Prandtl number  $Pr$  is less than unity and the Taylor number exceeds a threshold. But for dielectric fluids, Prandtl number is much greater than unity (for example, for corn oil  $Pr = 480$ , silicone oil  $Pr = 100$  and for castor oil  $Pr = 10,000$ ) and hence the oscillatory convection is ruled out as the preferred mode of instability for dielectric fluids. Under the circumstances, we restrict ourselves to the case of steady onset and put  $\omega = 0$  in Eq. (26)–(29). As in the case of stress-free isothermal boundaries, an exact solution is not possible for the other types of velocity and temperature boundary conditions and then one has to resort to numerical methods to extract the critical stability parameters. For this, the Galerkin method is adopted to solve the resulting eigenvalue problem. Accordingly, the variables are written in a

$$W = \sum A_i W_i \quad \theta = \sum B_i \theta_i \quad Z = \sum C_i Z_i \quad \phi = \sum D_i \phi_i \quad (40)$$

where  $A_i, B_i, C_i$  and  $D_i$  are constants and the basis functions  $W_i, \theta_i, Z_i$  and  $\phi_i$  will be represented by the power series satisfying the respective boundary conditions. Substituting Eq. (40) into Eq. (26)–(29) (after noting  $\omega = 0$ ), multiplying the resulting momentum equation by  $W_j(z)$ , vorticity equation by  $Z_j(z)$ , energy equation by  $\theta_j(z)$ , electric potential equation by  $\phi_j(z)$ ; performing the integration by parts with respect to  $z$  between  $z = 0$  and  $z = 1$  and using the boundary conditions, leads to the following system of linear homogeneous algebraic equations:

$$A_i E_{ji} - B_i F_{ji} + C_i G_{ji} - D_i H_{ji} = 0 \quad (41)$$

$$-A_i I_{ji} + C_i J_{ji} = 0 \quad (42)$$

Where





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$$E_{ji} = \left\{ \frac{w}{pr} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} w \right) \right\} W_j (D^2 - a^2) W_i, F_{ji} = -(R_t a^2 W_j \theta_i + R_{ea} a^2 W_j \phi_i)$$

$$A_i K_{ji} + B_i L_{ji} = 0 \quad (43)$$

$$-B_i M_{ji} + D_i N_{ji} = 0 \quad (44)$$

Where

$$G_{ji} = T a^{1/2} W_j D Z_i$$

$$H_{ji} = R_{ea} a^2 W_j D \phi_i$$

$$I_{ji} = T a^{1/2} Z_j D W_i$$

$$J_{ji} = \left\{ \frac{w}{pr} + Da^{-1} \left( 1 - \frac{\Gamma p}{pr} w \right) \right\} Z_j Z_i$$

$$K_{ji} = \theta_j f(z) W_j$$

$$L_{ji} = M w \theta_j \theta_i - \theta_j (D^2 - a^2) \theta_i$$

$$M_{ji} = \phi_j D \theta_i$$

$$N_{ji} = \phi_j (D^2 - a^2) \phi_i$$

Where the inner product as  $(\dots) = \int_0^1 (\dots) dz$ . The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{bmatrix} E_{ji} & F_{ji} & G_{ij} & H_{ji} \\ I_{ji} & J_{ji} & 0 & 0 \\ K_{ji} & 0 & L_{ji} & 0 \\ 0 & M_{ji} & 0 & N_{ji} \end{bmatrix} = 0 \quad (45)$$

We select trial functions satisfying the appropriate boundary conditions as follows:

(i) Both the boundaries Rigid-rigid

$$W_i = (z^4 - 2z^3 + z^2) T_{i-1}^*, \quad \theta_i = (z - z^2) T_{i-1}^* = \phi_i, \quad Z_i = (z^4 - 5z^3/2 + 3z^2/2) T_{i-1}^* \quad (46a)$$



(ii) Both the boundaries Free-free

$$W_i = (z^4 - 2z^3 + z)T_{i-1}^*, \theta_i = (z - z^2)T_{i-1}^*, \phi_i = Z_i = (z^3 - 3z^2/2 + 3/2)T_{i-1}^* \quad (46b)$$

(iii) Lower rigid and upper free boundaries

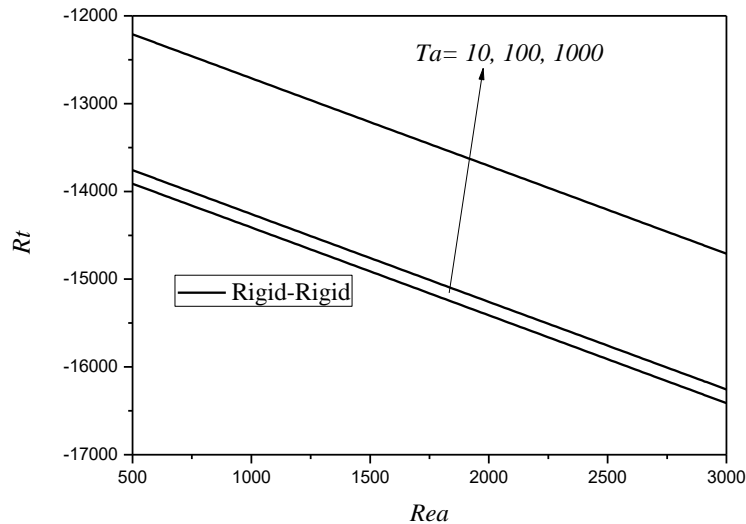
$$W_i = (z^4 - 5z^3/2 + 3z^2/2)T_{i-1}^*, \theta_i = (z - z^2)T_{i-1}^*, \phi_i = Z_i = (z^4 - 2z^3 + z^2)T_{i-1}^* \quad (46c)$$

$$\text{With temperature gradient } f(z) = 2z \text{ and } f(z) = z^2 \quad (46d)$$

where  $T_{i-1}^*$  ( $i = 1, 2, \dots, n$ ) is the modified Chebyshev polynomial of  $i$ th order. Substituting the above trial functions, depending on the boundary combinations considered, in Eq. (45) and expanding the determinant leads to the characteristic equation giving the thermal Rayleigh number  $R_t$  or the AC electric Rayleigh number  $R_{ea}$  as a function of the wave number  $a$  as well as other parameters  $R_{ea}$  or  $R_t$  as the case may be and the Taylor number  $Ta$ . The inner products involved in the determinant are evaluated analytically rather than numerically in order to avoid errors in the numerical integration. Numerical computations carried out reveal that the convergence in finding  $R_t$  or  $R_{ea}$  with respect to the wave number crucially depends on the value of  $Ta$ , and for higher values of  $Ta$  more number of terms are found to be required in the Galerkin expansion.

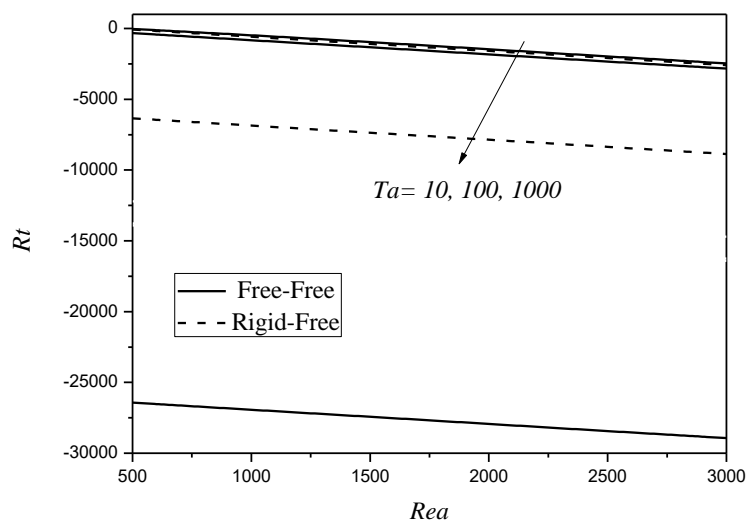
## 4. Results and discussion

Combined Effect of Thermal Modulation and Rotation on the Onset of Convection in Walter's B Viscoelastic Fluid Saturated Porous Medium is investigated. Since the viscoelastic parameter is not appearing in the expressions for  $R_t$  in the case of stationary convection there is no difference between the viscoelastic and viscous fluid results. Attention is focused on three kinds of velocity boundary conditions namely, free-free, rigid-rigid and lower rigid upper free which are considered to be either linear or parabolic temperature profiles. To solve the resulting eigenvalue problem, numerical techniques are used depending on the boundaries for the occurrence of stationary convection.



**Fig. 2(a):**  $R_t$  versus  $R_{ea}$  at  $Da^{-1} = 10$ ,  $Ta = 10, 100, 1000$  for Rigid-Rigid boundaries with linear temperature profiles.

The variations of  $R_t$  with  $R_{ea}$  for three different values of  $Ta = 10, 100$  and  $1000$  is plotted in Fig. 2(a), with fixed  $Da^{-1}$ . From figure 2(a) it is noted that the system become more stable with increase in the value of  $Ta$ .



**Fig. 2(b):**  $R_t$  versus  $R_{ea}$  at  $Da^{-1} = 10$ ,  $Ta = 10, 100, 1000$  for Rigid-Free, Free-Free boundaries with linear temperature profiles.

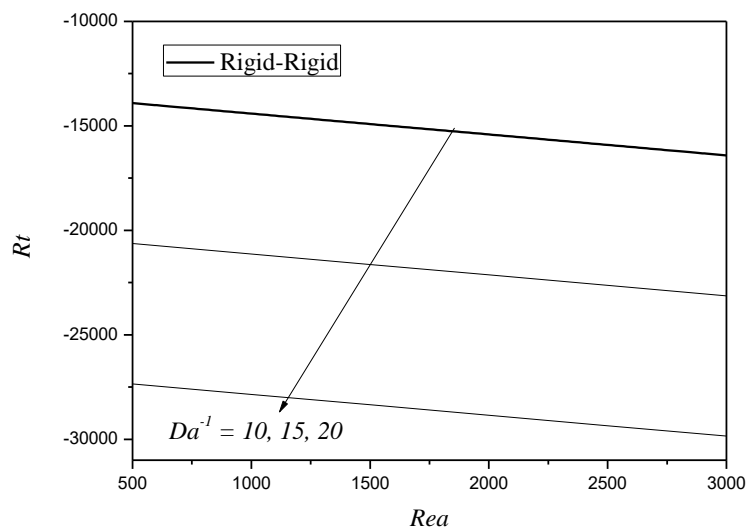


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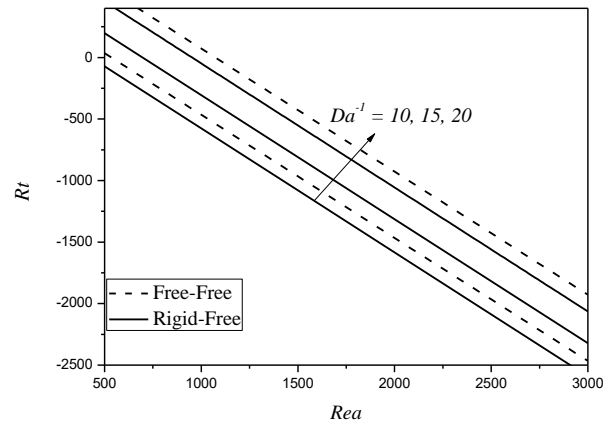
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The variations of  $R_t$  with  $R_{ea}$  for three different values of the  $Ta = 10, 100$  and  $1000$  is plotted in Fig.2(b), with fixed  $Da^{-1}$ . From the figure 2(b) it is found that  $R_t$  decreases with the increase in the value of  $R_{ea}$  and there by implying a destabilising effect and increase in  $Ta$  gives more stabilizing effect. And it is also noted that Free-Free boundaries have more non-destabilizing effect compared to Rigid- Free boundaries.



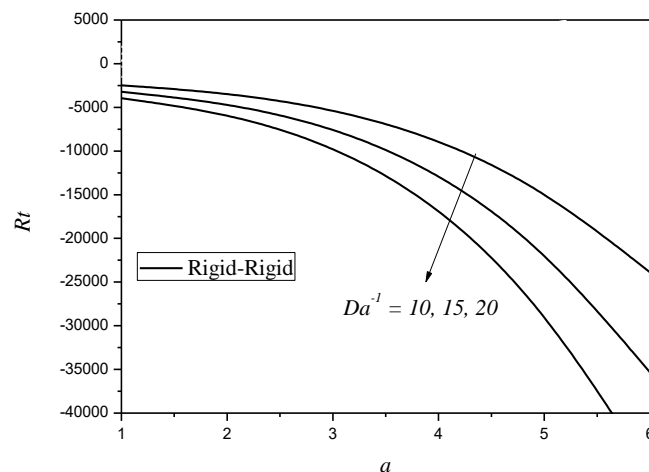
**Fig. 3(a):**  $R_t$  versus  $R_{ea}$  at  $Ta = 10$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Rigid, boundaries with linear temperature profiles.

The variations of  $R_t$  with the  $R_{ea}$  for three different values of the  $Da^{-1}$ , namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig.3(a), with fixed  $Ta$ . From the figure 3(a) it is evidenced that  $R_t$  decreases with the increase in the value of  $Da^{-1}$  and there by implying a destabilising effect.



**Fig: 3(b):**  $R_t$  versus  $R_{ea}$  at  $Ta = 10$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Free, Free-Free boundaries with linear temperature profiles.

The variations of  $R_t$  with  $R_{ea}$  for three different values of  $Da^{-1}$ , namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig.3(b), with fixed  $Ta$ . From the figure 3(b) it is found that  $R_t$  increases with the decrease in the value of  $R_{ea}$  and there by implying a destabilising effect. It is also noted that both the free boundaries have more non-destabilizing effect compared to Rigid- Free boundaries and has a non-destabilizing effect system in increase in the value of  $Da^{-1}$ .



**Fig: 4(a):**  $R_t$  versus  $a$  at  $Ta = 10$ ,  $R_{ea} = 1000$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Rigid boundaries with linear temperature profiles.



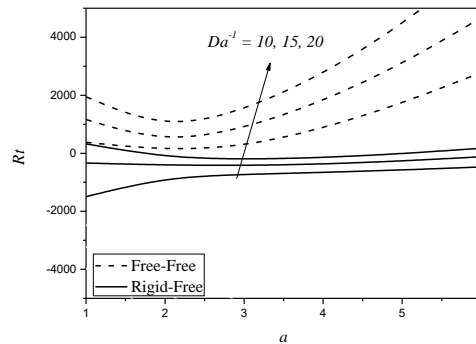


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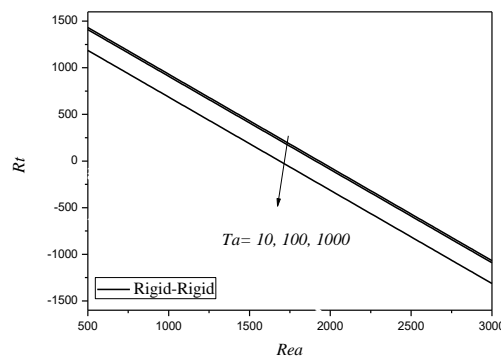
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The variations of  $R_t$  with  $a$  for three different values  $Da^{-1}$ , namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig:5(a), with fixed  $Ta$ . From the figure 4(a) shows the  $R_t$  decreases with an increase in the value of  $Da^{-1}$  and there by implying a destabilizing effect.



**Fig:4(b):**  $R_t$  versus  $a$  at  $Ta = 10$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Free, Free-Free boundaries with linear temperature profiles.

The variations of  $R_t$  with the  $a$  for three different values of the,  $Da^{-1}$ . namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig:4(b), with fixed  $Ta$ . Figure 4(b) demonstrates the neutral curves for Free-Free and Rigid-Free boundaries. The neutral curves exhibit single but different minimum w.r.to the wave number for various values of physical parameters and also for Rigid-Free and Free-Free boundaries. The region below each neutral curve corresponds to the stable state of the system. It is also observed that both the boundaries are free are more stable compared to Rigid-Free boundaries.



**Fig:6(a):**  $R_t$  versus  $Re_a$  at  $Da^{-1} = 10$ ,  $Ta = 10, 100, 1000$  for Rigid-Rigid boundaries with parabolic temperature profiles.

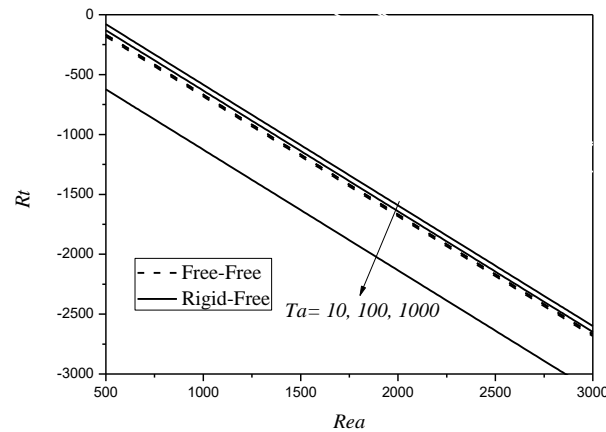


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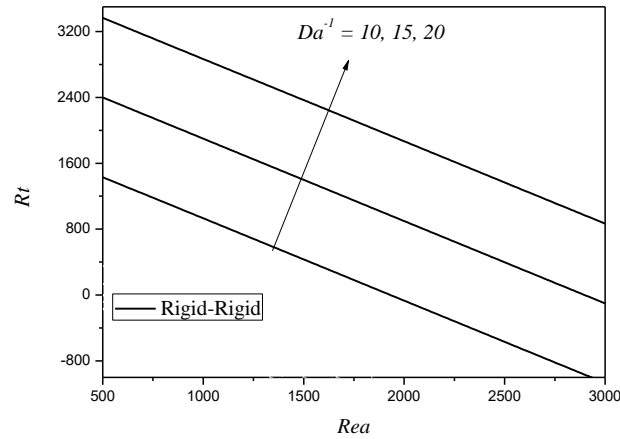
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The variations of  $R_t$  with the  $R_{ea}$  for three different values of the  $Ta$ , namely,  $Ta = 10, 100$  and  $1000$  is plotted in Fig:6(a), with fixed  $Da^{-1}$ . From the figure 6(a) it is observed that  $R_t$  decreases with the increase in the value of  $Ta$  and there by implying a destabilising effect.



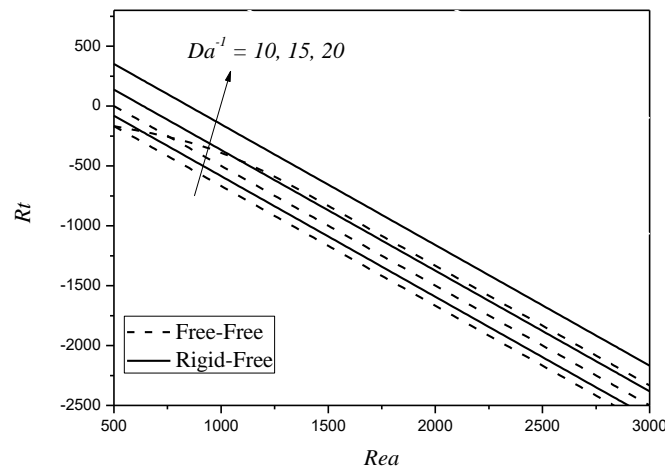
**Fig.6(b):**  $R_t$  versus  $R_{ea}$  at  $Da^{-1} = 10, Ta = 10, 100, 1000$  for Rigid-Rigid boundaries with parabolic temperature profiles

The variations of  $R_t$  with  $R_{ea}$  for three different values of  $Ta = 10, 100$  and  $1000$  is plotted in Fig.6(b), with fixed  $Da^{-1}$ . From the figure 6(b) it is found that  $R_t$  decreases with the increase in the value of  $R_{ea}$  and there by implying a destabilising effect and increase in  $Ta$  gives more stabilizing effect. And it is also noted that both the boundaries are free have more non-destabilizing effect compared to Rigid- Free boundaries.



**Fig:7(a):**  $R_t$  versus  $Re_a$  at  $Ta = 10$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Rigid boundaries with parabolic temperature profiles.

The variations of  $R_t$  with  $Re_a$  for three different values of the  $Da^{-1}$ , namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig:7(a), with fixed  $Ta$ . From the figure 7(a) it is observed that  $R_t$  increases with the increase in the value of  $Da^{-1}$  and there by implying a stabilizing effect.  $R_t$  decreases with the increase in the value of  $Re_a$  and there by implying a destabilizing effect.



**Fig.7(b):**  $R_t$  versus  $Re_a$  at  $Ta = 10$ ,  $Da^{-1} = 10, 15, 20$  for Rigid-Free and Free-Free parabolic temperature profiles

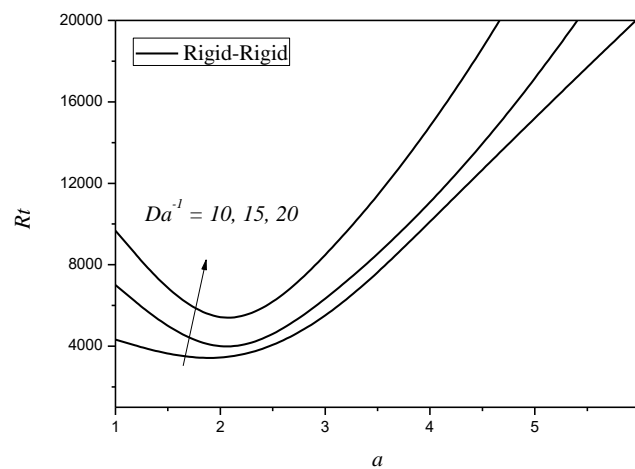


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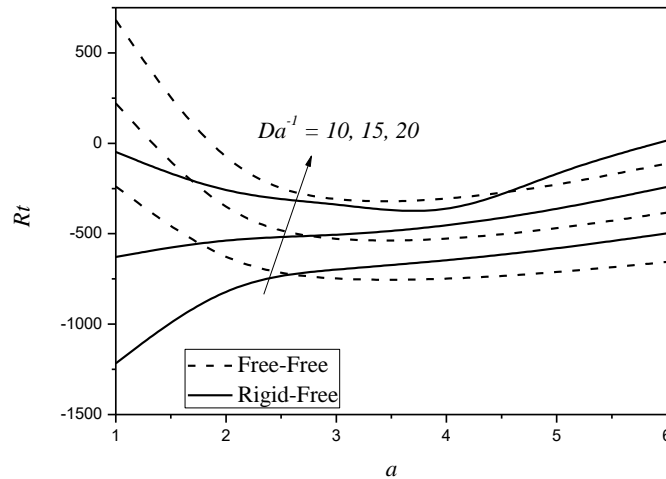
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The variations of  $R_t$  with  $R_{ea}$  for three different values of the  $Da^{-1}$ , namely,  $Da^{-1} = 10, 15$  and 20 is plotted in Fig:7(b), with fixed  $Ta$ . From the figure 7(b) it is absorbed that  $R_t$  decreases with the increase in the value of  $R_{ea}$  and there by implying a destabilising effect and increase in  $Da^{-1}$  gives more stabilizing effect. And it is also noted that rigid-free boundaries have more stabilizing effect compared to free- free boundaries.



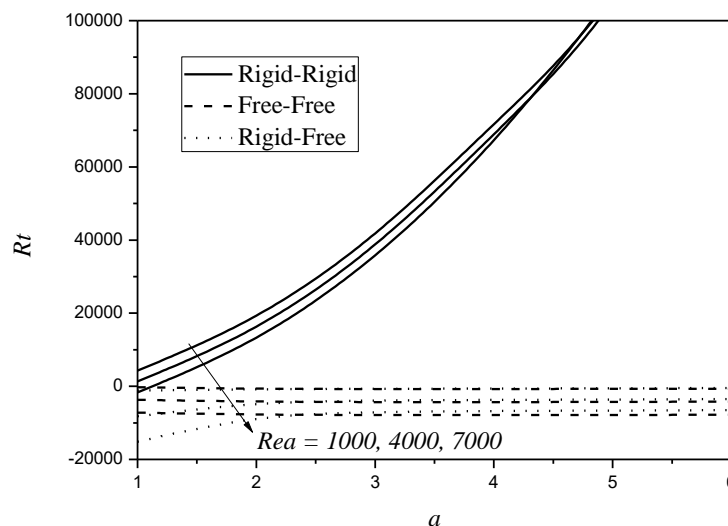
**Fig:8(a):**  $R_t$  versus  $a$  at  $Ta = 10, Da^{-1} = 10, 15, 20$  for Rigid-Rigid parabolic temperature profiles.

The variations of  $R_t$  with  $a$  for three different values of the,  $Da^{-1}$  namely,  $Da^{-1} = 10, 15$  and 20 is plotted in Fig:8(b), with fixed  $Ta$ . From the Figure 8(a) demonstrate the neutral cures for both the boundaries are rigid. The neutral cures exhibit single but different minimum with respect to the wave number for various values of physical parameters and also for rigid- rigid boundaries. The region below each neutral cures corresponds to the stable state of the system. It is also observed that both the boundaries are rigid are more stable with increase in value of  $Da^{-1}$ .



**Fig:8(b):**  $R_t$  versus  $a$  at  $Ta = 10, Da^{-1} = 10, 15, 20$  for Free-Free and Rigid-Free parabolic temperature profiles.

The variations of  $R_t$  with the  $a$  for three different values of the,  $Da^{-1}$  namely,  $Da^{-1} = 10, 15$  and  $20$  is plotted in Fig:8(b), with fixed  $Ta$ . System has a destabilizing effect with the increase in value of  $Da^{-1}$ .



**Fig:9:**  $R_t$  versus  $a$  at  $Da^{-1} = 10, Ta = 10, Re_a = 1000, 4000, 7000$  values of Rigid-Rigid, Free-Free and Rigid-Free parabolic temperature profiles.





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The variations of  $R_t$  with  $a$  for three different values of  $R_{ea} = 1000, 4000$  and  $7000$  is plotted in Fig:9, with fixed  $Da^{-1}$  &  $Ta$ . System has a destabilizing effect with the increase in value of  $R_{ea}$ .

## Conclusions

Effect of basic temperature gradients on electrothermal convection in a rotating layer of viscoelastic (Walter's B) fluid Saturated porous medium heated from below is investigated by using the Galerkin method. The major findings of this analysis are:

- (i) With respect to stationary convection,
  - (a) the Walter's -B viscoelastic fluid behaves like an ordinary fluid.
  - (b) On the onset of stationary convection has no effect of viscoelasticity.
- (ii) For the case of linear temperature gradient,
  - (a) The Taylor number has a stabilizing effect when both the boundaries are rigid.
  - (b) The free-free boundaries have more stabilizing effect compared to rigid- free boundaries.
  - (c) The  $Da^{-1}$  has a destabilizing effect when both the boundaries are rigid.
  - (d) The free-free & rigid-free boundaries have more stabilizing effect compared to rigid- free boundaries, while the case is reversed with respect to rigid -rigid boundary on non- oscillatory convection.
- (iii) For the case of parabolic temperature gradient,
  - (a) The Taylor number has a destabilizing effect when boundaries are rigid-rigid.
  - (b) the free-free boundaries have more non-destabilizing effect compared to rigid-free boundaries.
  - (c)  $Da^{-1}$  has a destabilizing effect when both the boundaries are rigid.
  - (d) the rigid-free boundaries have more non-destabilizing effect compared to both the boundaries are free.
  - (e) The  $R_{ea}$  has a non-stabilizing effect on stationary convection.
- (iv) By considering the boundary conditions, oscillatory convection has not happened.

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