



Numerical Study of Delamination of a DCB Beam Under Dynamic Loading by Mixed Finite Element

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Abstract:- The main objective of the present study is the calculation of the Energy Release Rate (ERR) in the case of delamination of composite structures subjected to dynamic load. A new numerical model based on the Mixed Finite Element RMQ-7 (Reissner Modified Quadrilateral with 7-nodes) and associated to the Virtual Crack Extension Method (VCE) has been developed to this purpose. The energy balance principal has made it possible to introduce Kinetic Energy into computation of ERR. Numerical analysis of delamination in composite laminates was investigated to validate the model. It was carried on a Double Cantilever Beam (DCB) sample. The numerical results obtained for the DCB delamination under dynamic loading in Mode-I was in good agreement with those found experimentally by other authors. These results confirming the accuracy and efficiency of the proposed model.

Keywords: *Composite, Energy, Dynamic, Fracture, Delamination.*

1. Introduction

Currently, the use of composite materials is now steadily rising due to its beneficial mechanical qualities, which enable the creation of high-strength structures. Aerospace, civil engineering, and the marine sector are just a few of the domain disciplines that employ composite materials. Since to their composite nature, the stacking of these layers on top of one another may produce delamination and a loss of bond between these layers [1]–[6].

Delamination is one of the three most critical failure modes in composite layered materials; especially when loaded dynamically. It can cause a considerable loss of strength and the in-plane stiffness [7], [8]. The delamination can propagate when subjected to various loadings such as low- and high-velocity impacts, static loads and dynamic loads acting on the structure [9].



To fully understand this failure mechanism, it is necessary to determine the energy release rate (ERR) G . To compute it, several analytical and numerical methods have been proposed [6], [10], [11], including the Virtual Crack Extension Approach (VCE) proposed by authors in [12], [13], [14]. This approach relies on the correlation between the stress intensity factor and the energy released rate (ERR) in a virtual quantity of crack formation, which was based on a hypothesis forth by Rice and the J-integral method created by the authors in [15], [16]. It also involves determining concentrated strain fields near of cracks.

The relation between the energy release rate (ERR) and interlaminar fracture toughness of composite laminates in Mode-I Fracture (Opening Mode) under dynamic loading has been proven by several authors, such as those in [17], [18], who presented an experimental and analytical solution of dynamic characterization of interlaminar fracture in Mode-I for two different material systems under both quasi-static and dynamic loading rates. The results indicated a positive relation between dynamic loading rates and interlaminar fracture toughness.

Author in [19], studied experimentally how the dynamic loading rate affects the interlaminar fracture toughness in composite laminates.

Experimental literature by authors in, [20], [21], has showed that the energy release rate (ERR) G , based on the symmetric loading of the Double Cantilever Beam (DCB) specimen arms, can be solved using the Euler-Bernoulli beam theory.

In this paper, a numerical method using the mixed finite element RMQ-7 (Reissner Modified Quadrilateral with 7-nodes) designed by [22], was employed to solve fracture mechanics problems. This element was coupled with the virtual crack extension method (VCE) to evaluate the energy release rate (ERR).

The objective of this study is to develop a numerical model to calculate the value of (ERR) in the dynamic situation for composite laminates that are subject to a speed opening force in Mode-I Fracture (Opening Mode) and comparing the obtained values of (ERR) with previous experimental results.

2. Fracture Mechanic Theorie

According to linear-elastic fracture mechanics, the energy dW_p required to propagate the crack of a length da and subsequently produce a new surface dA is defined as the Griffith energy release rate (ERR) G [23] expressed as follows:

$$G = \lim_{\Delta A \rightarrow 0} \frac{\Delta W_p}{\Delta A} = \frac{dW_p}{dA} \quad (1)$$

The surface that the cracks expansion produced is :



$$dA = bda \quad (2)$$

Where W_p is the work done by the external forces, a and b are the crack length and width respectively.

Energy conservation can be used to determine the energy variation within the model and the stress distribution around the fracture point (The virtual crack extension approach) [24]. Mott modifies the Griffith energy balance in the scenario of a central crack in a plate with infinite kinetic energy for the fracture event. The modified form of the equation of Griffith is used to express of energy release rate (ERR) for linear elastic solids [25], [26], as seen in equation (3).

$$\frac{1}{b} \frac{d}{da} (W_{ext} - U_s - U_{kin}) = G_{Ista} - \frac{1}{b} \frac{d}{da} U_{kin} \quad (3)$$

Where W_{ext} is the external work, U_{kin} is the total kinetic energy, G_{Ista} is the static energy release rate in Mode-I and U_s is the elastic total strain energy.

Fundamentally, the fracture propagation phase is a dynamic process requiring kinetic energy. Additionally, kinetic energy in the substrates is mentioned in relation to dynamic loading. Both of these phenomena can be taken into account. In this case, the dynamic energy release rate (ERR) is described as follows [27]:

$$G_{Idyn} = G_{Ista} - \frac{1}{b} \frac{d}{da} U_{kin} \quad (4)$$

Where G_{Idyn} is the dynamic energy release rate in Mode-I.

3. Governing Equations

More clear approach to evaluating the kinetic term was provided by authors in [25], [28]. For this, the "Berry Method" has been used to estimate the deflection u along the beam. A vertical displacement profile both ends of beam is needed. According to the Euler-Bernoulli beam theory, the crack opening displacement rate $y(x)$, prior to crack propagation is given by Smiley [29]:

$$y(x) = \frac{Pa^3}{3EI_z} \left(\frac{3ax^2 - x^3}{2a^3} \right) \quad (5)$$

Where P is applied load, E is Young's modulus of the material, I_z the moment of inertia corresponding to bending around the z-axis and a is initial crack length.

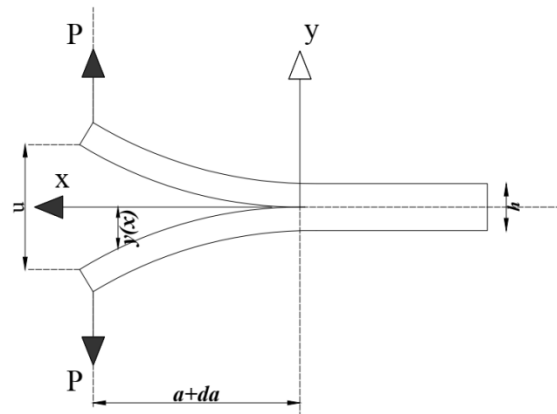


Fig. 1 Delamination of double cantilever beam (DCB).

When the two arms experience a displacement that is symmetrical with respect to the crack's plane, the arm movement's maximum velocity may reach: $\frac{\dot{u}}{2}$

Where \dot{u} is an opening speed for both ends of beam.

The displacement speed in the (y) direction prior to the crack propagating was detailed by the author in [30]. Equation (6) depicts the relationship between the crack length a , the vertical displacement speed in y direction $\dot{y}(x)$ and the opening speed for both ends of beam \dot{u} . For a (DCB) specimen.

$$\dot{y}(x) = \frac{\dot{u}(3ax^2 - x^3)}{2a^3} \quad (6)$$

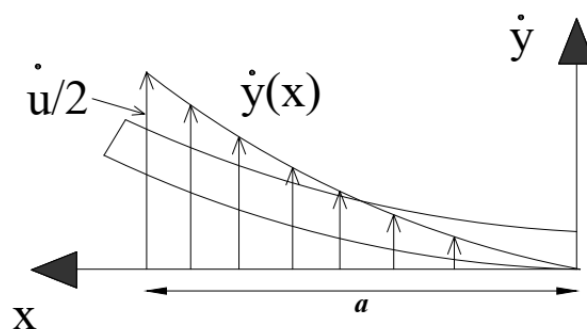


Fig. 2 The vertical displacement for of an arm of beam.



In order to obtain the contribution of the kinetic energy to the energy release rate (ERR) in Mode-I at the beginning of the crack growth, by integrating the displacement speed $\dot{y}(x)$, it is possible to compute energy of an arm during an opening movement, the expression obtained is the derived the vertical displacement speed for an arm of the beam with respect to the crack length a :

$$U_{kin} = \int_0^a \frac{1}{2} (\dot{y}(x))^2 dm \quad (7)$$

Where m is arm mass.

If ρ is the mass density of material and bh is the arm's section, equation (7) may be expressed as follows.

$$U_{kin} = \frac{1}{2} \rho b h \int_0^a (\dot{y}(x))^2 dx \quad (8)$$

Author in [30]. Show that the expression of kinetic energy takes into account volumetric mass:

$$\frac{1}{b} \frac{dU_{kin}}{da} = \frac{33}{560} \rho h \dot{u}^2 \quad (9)$$

Where ρ , h and \dot{u} are the mass density of material, the sample thickness and the speed for arm successively. In the dynamical situation of the Mode-I, the energy release rate (ERR) under dynamic loading may therefore be expressed as follows:

$$G_{Idyn} = G_{Ista} - \frac{33}{560} \rho h \dot{u}^2 \quad (10)$$

4. Implementation Of The Model

In order to create a numerical model that conforms with the mechanical hypotheses and experimental parameters, we examine a test with a double cantilever beam (DCB). The end of the beam is fixed and cannot move or rotate. At each the beam' free ends, a concentrated load is placed. The bending moments and shear forces produced by this load have an impact on the structural integrity and possible fracture propagation. This sample is widely used in fracture toughness research on the Mode-I of composite materials [31].

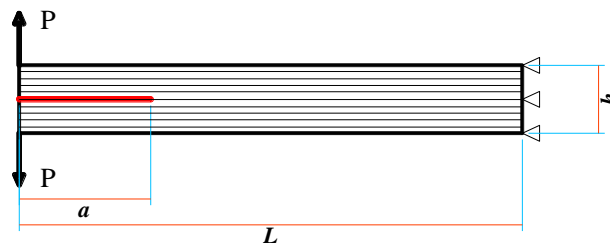


Fig. 3 Schematic drawing of a symmetrically loaded DCB specimen

Two samples with distinct geometric and mechanical characteristics were used for this test. A bi-material beam was used for the first test and a mono-material beam for the second. This test was performed at two different opening speeds, 3 m/s in the first sample and 1 m/s in the second, with a constant force of 60 N for each. The mechanical and geometrical properties are summarized in Tab. 1 and in Tab.2 [30], [32].

Tab. 1 Geometrical properties

Test:	L L(mm)	Width b(mm)	Thickness h(mm)	Crack length a(mm)
01	200	25	4.7	50
02	80	1	4	60

Tab. 2 Mechanical properties

Test:	E_1 (GPa)	E_2 (GPa)	ν_{12}	ρ (Kg/m ³)
01	32.5	10.23	0.27	1570
02	10	-	0.3	1000

5. Development Of The Element Used In Numerical Analysis (RMQ7)

In this study, a specific mixed finite element (RMQ-7) is used. It is an improved version of the parent (Reissner Modified Quadrilateral) Reissner element with four points and five degrees of freedom in each. The process of formation of this element has passed at several stages:

Firstly, a displacement node was added to four-pointed Reissner element to create RMQ5, resulting in an element with five nodes and 22 degrees of freedom (Fig. 4).

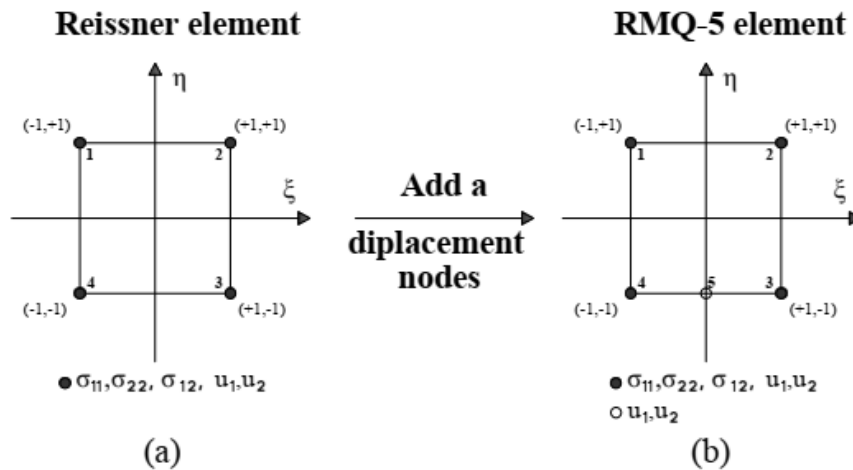


Fig. 4 Element formation RMQ5

Secondly, the five-node reference element RMQ5 were used to create the RMQ11 itself, by shifting a few variables and the static nodal from the corners to the side, It element with 11 nodes and 2 degrees of freedom at each (Fig. 5).

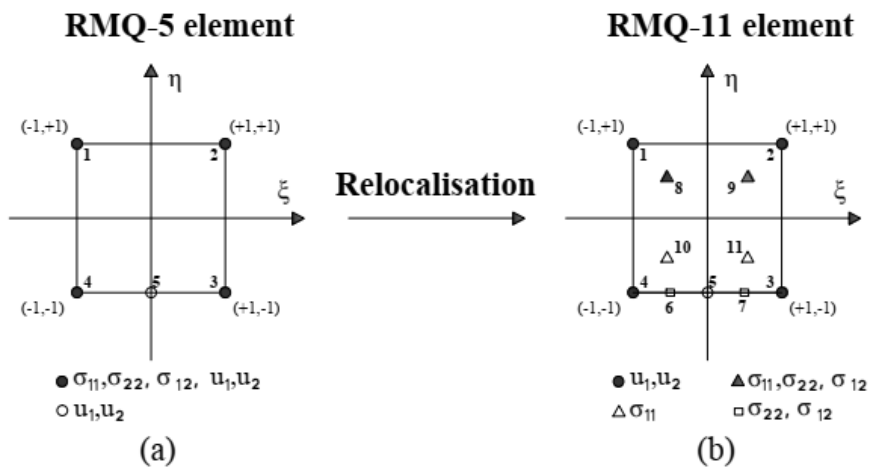


Fig. 5 Element formation RMQ11

Finally, bouzerd [33] proposed the final element RMQ7 (Fig. 6. b) for the modeling of cracked interfaces. Which based on RMQ11 element, and that using a static condensation process, which reduces the size of equations with the removal of a certain number of variables, therefore merging the internal degrees of freedom, and it has the shape of a quadrilateral with seven nodes and fourteen degrees of freedom, with an arrangement in an actual (x, y) plane, as shown in (Fig. 6). Five of its nodes are a displacement (u_1, u_2) nodes, and the remaining two are stress $(\sigma_{12}, \sigma_{22})$ nodes. The corners of the quadrilateral include four displacement nodes, while the center of the side with the specific interface contains the



fifth displacement node, it's mediating stress nodes. In the modeling of cracked structures, this node corresponds to the crack's starting location [34], [35]. Bouziane [36], regenerate the element starting in a natural (ξ, η) plane leading to an isoparametric formulation [37]

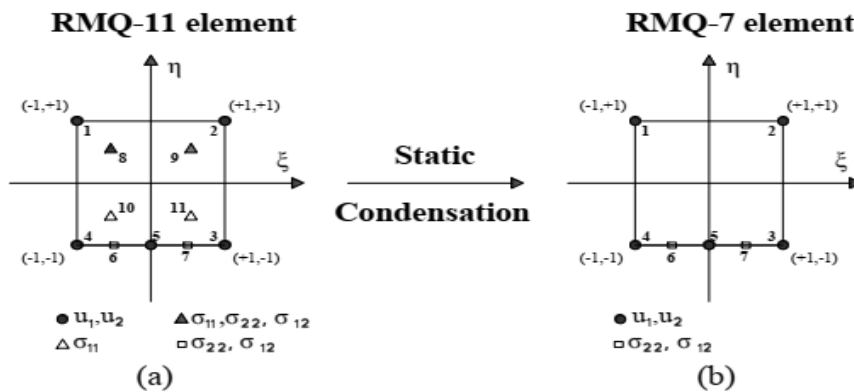


Fig. 6 Element formation RMQ7

6. Numerical Validation

To compute the energy release rate (ERR) values, we have created a program that includes equation (10). This program appropriate for computing the (ERR) values in a double cantilever beam (DCB) analysis in the dynamic state. Where the current mixed finite element (RMQ-7) described previously is the basis for this program.

A modeling of the DCB beam was done in this program, with 740 elements formed and 2013 nodes. To effectively depict stress concentration effects, we create a finite element mesh with an adequate density, particularly close to the fracture tip. Fig. 7 is a partial representation of the DCB beam by the developed program. It also shows the position of the (RMQ-7) elements around the crack length.

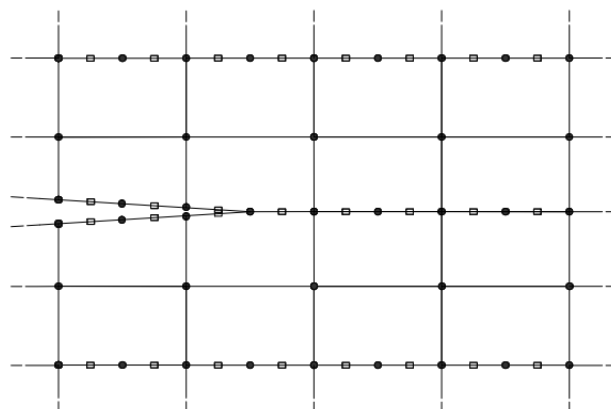


Fig. 7 Example of Mesh around tip crack by RMQ-7of (DCB) beam



7. Results And Discussion

The previous experimental results are taken into consideration as the basis for comparing the obtained outcomes. As a consequence of these findings, as shown in Tab. 3, we observe the following:

At an opening speed of 3 m/s in the first test, the value of the energy release rate determined by new program is 195 J/m². The inaccuracy is around 12.82 % when compared to the typical experimental G_I value of 170 J/m² found for the same plate by Author in [30]. For the second test, the energy release rate (ERR) is 471 J/m², at an opening speed of 1 m/s. When compared to the results of the [32], where the energy release rate (ERR) value was about 450 J/m², the error value is 4.45 %.

Tab. 3 Values the energy release rate (ERR)

Test	\dot{u} (m/s)	Force P (N)	energy release rate (J/m ²)		
			[30]	Chen <i>et al.</i> , 2020	simulation results\
01	3	60	170	-	195
02	1	60	-	450	471

This results show a small variation between the numerical simulation and the earlier experimental studies.

8. Conclusions

Considering most previous studies, whether experimental or numerical, focused on evaluating the energy release rate (ERR) in the static state, it is of great importance in this work to evaluate the energy release rate (ERR) in the dynamic state.

This paper focuses on the study and implementation of a simulation model of layered dynamic delamination for a sample representative of composite material. The RMQ-7 element, the virtual crack extension method, and the dynamic Euler-Bernoulli beam theory are the basis of this model for the dynamic evaluation of the energy release rate (ERR), which represents the delamination in Mode-I (Opening Mode).

Firstly, the vertical displacement velocity of the arm was derived relative to the crack length a . This allowed us to obtain an expression of kinetic energy, which then permitted us to calculate the dynamic energy release rate (ERR).

Secondly, The static model was transformed to a dynamic model. To do this, a subroutine has been created to generate initial program and to make it conform to the dynamic state.



Finally, to verify the effectiveness and feasibility of the current model, a double cantilever beam (DCB) was used as a representative application of the delamination. The effect of loading-rate on the delamination of composite materials was studied. The results obtained showed that the Mode-I energy release rate (ERR) obtained from this model was in good agreement with those found experimentally and in literature. This study also showed that the opening speed in the first mode (opening mode) has a significant impact on the energy release rate (ERR).

Furthermore, this finding opens the door for further research into the potential that the energy release rate (ERR) is influenced by variables such as volumetric mass and crack length. To further understand these impacts and their influence on the energy release rate (ERR) in the dynamic state, more research may need to be done using systematic experiments or simulations.

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