



## Balancing Time and Quality Cost in Using Corner Cutters in Composite Roofs

Esmaeil Sadeghi pour Chahnasir<sup>1</sup>, \*Yosef Zandi<sup>2</sup>, Masood Paknahad<sup>3</sup>

<sup>1</sup>Department of Civil Engineering , Qeshm Branch , Islamic Azad University , Qeshm , Iran(  
[e.sadeghi.p@qeshm-it.ir](mailto:e.sadeghi.p@qeshm-it.ir))

<sup>\*2</sup>Department of Civil Engineering , Tabriz Branch , Islamic Azad University , Tabriz,  
Iran([zandi@iaut.ac.ir](mailto:zandi@iaut.ac.ir))

<sup>3</sup>Faculty of Engineering , Mahallat Institute of Higher Education , Mahallat, Iran  
([mpaknahad@mahallat.ac.ir](mailto:mpaknahad@mahallat.ac.ir))

### Abstract

Time, cost, and quality are the main goals of any project. Project managers have been focused on reducing time and cost in projects since the late 1950s, when the critical path was first expressed, and thus the issue of time-cost balance arose. In recent years, the attention of project managers to reducing project execution time while obtaining the desired quality for the project has increased. This has led researchers to develop time-cost balance models that include the quality factor. In this research, the issue of time, cost, and quality balance was studied, assuming that the quality of project activities depends on time and cost. For this purpose, a three-objective planning model was formulated, considering time, cost, and quality as objective functions and considering prerequisite relationships. A fuzzy decision-making approach was used to solve the problem. In order to implement this approach, the execution time and cost of compressing time and improving the quality of activities were described as fuzzy ideals so that a fuzzy decision-making approach could be used to balance them with quality. Combining membership functions related to the execution time of individual activities and membership functions of fuzzy ideals related to project execution time and compression cost, in line with the fuzzy decision approach, led to a one-objective planning model. To demonstrate the efficiency of the proposed model and approach to solving a real problem of time balance, cost, and quality related to time balance, the cost of using angles in composite ceilings was solved and the results were discussed. Finally, the validity of the models was evaluated using parametric analysis. To further adapt the proposed model and solution model to reality, nonlinear membership functions can be used instead of linear membership functions to describe quality dependence on time and even fuzzy ideals related to execution time and compression cost.

**Keywords:** Balance, Time, Quality, Cost, Corner Shear connector, Composite Roof



## 1- INTRODUCTION

In recent years, due to the complexity of project implementation, the competitive business environment, and the limited resources of organizations, the need to pay attention to project management in order to achieve project goals has received more attention. Therefore, they are considered in the implementation stages of projects. Therefore, project managers seek to achieve the above goals by allocating resources appropriately and choosing the best decision. Since most of the previous work has been done under uncertain and unrealistic conditions, there is a fundamental gap in the knowledge of managers in the field of project management. As a result, one of the most prominent features of this research is the consideration of uncertain conditions that are closer to the real world.

Projects are a collection of particular actions that are planned to accomplish a specific objective. The initial project schedule is established before project activities are anticipated for future implementation [1]. Since accurate future prediction is beyond human capacity, risks and uncertainties are bound to exist during the implementation of projects due to the ambiguity of the future. Due to the uncertainty of the plans made, projects are delayed and cost more money when they are planned based on the estimates that come from having little knowledge about their future status.

The time, cost, and quality of a project must be managed. The three most crucial objectives for any project are these three things. Time-cost balance has been the main topic of previous research. [2]. However, in recent times, the quality factor—which can be impacted by accelerating the project time—has been recognized as one of the fundamental requirements for project success. Each activity has a number of different modes of execution that can be used to balance time, cost, and quality; the best mode is chosen in order to reduce project time and cost while maximizing quality.

The theory of fuzzy logic as a new topic, combined with management science, is able to play a fundamental and useful role in risk management. Fuzzy management science is the application of classical management science in a fuzzy environment [3]. In classical management science, management issues are analyzed using rational and systematic methods, which help managers make decisions by providing quantitative models. Therefore, it is based on accurate and definite data, and therefore, in these methods, ambiguous and fuzzy data have no place in modeling [4].

Using fuzzy management science, classical management science methods are used in a fuzzy environment, and they can be used in various management tasks such as decision-making, policy-making, and planning. Fuzzy management science can design models that, like humans, have the ability to process qualitative information intelligently. Therefore, fuzzy management



science, while creating flexibility in the model, enters data such as knowledge, experience, and human judgments into the model and provides fully practical answers.

## 2- LITERATURE REVIEW

Information about the type of balance problem, its solution method, sample problem design method for evaluating algorithms and solution software for different articles are given in the Table 1.

Table1. Details of articles used in research

| Solution software | Solving method                                | Balance type                     | Year of publication | Reference |
|-------------------|-----------------------------------------------|----------------------------------|---------------------|-----------|
| MATLAB            | Multi-criterion decision-making               | Balance time, cost - and quality | 2015                | [5]       |
| GAMS              | Fuzzy AHP strategy and VIKOR method           | Balance time, cost - and quality | 2016                | [6]       |
| MATLAB            | Super Genetic Algorithm (SGA)                 | Balance time, cost - and quality | 2017                | [7]       |
| MATLAB            | Genetic Algorithm                             | Balance time and cost            | 2018                | [8]       |
| C++ programming   | A simulation-based Integer linear programming | Balance time, cost - and quality | 2019                | [9]       |
| MATLAB            | Genetic Algorithm                             | Balance time and cost            | 2019                | [10]      |



| Solution software          | Solving method                                                                                              | Balance type                     | Year of publication | Reference |
|----------------------------|-------------------------------------------------------------------------------------------------------------|----------------------------------|---------------------|-----------|
| MATLAB                     | Teaching- Learning Based Optimization (TLBO) incorporated with the Modified Adaptive Weight Approach (MAWA) | Balance time and cost            | 2019                | [11]      |
| MATLAB                     | Multiobjective teaching-learning-based optimization (TLBO)                                                  | Balance time and cost            | 2019                | [12]      |
| MATLAB                     | Noncritical Activity Rescheduling Method                                                                    | Balance time and cost            | 2020                | [13]      |
| Cplex solver in C language | A mixed integer linear programming (MILP)                                                                   | Balance time, cost - and quality | 2020                | [14]      |
| C++                        | An efficient optimization model where redundant paths are eliminated                                        | Balance time and cost            | 2020                | [15]      |
| MATLAB                     | exact solutions                                                                                             | Balance time, cost - and quality | 2021                | [16]      |
| MATLAB                     | multi-objective optimization of Non-Dominated Sorting Genetic Algorithms                                    | Balance time, cost - and quality | 2021                | [17]      |



| Solution software | Solving method                                                                                        | Balance type                     | Year of publication | Reference |
|-------------------|-------------------------------------------------------------------------------------------------------|----------------------------------|---------------------|-----------|
| MATLAB            | Opposition-based Multiple Objective Differential Evolution (OMODE)                                    | Balance time, cost - and quality | 2021                | [18]      |
| -                 | Data Envelopment Analysis (DEA) method                                                                | Balance time, cost - and quality | 2021                | [19]      |
| GAMS              | robust nonlinear programming (NLP)                                                                    | Balance time, cost - and quality | 2022                | [20]      |
| MATLAB            | Interval-valued fuzzy sets method                                                                     | Balance time, cost - and quality | 2022                | [21]      |
| LINGO             | exact solution based on the network reduction approach                                                | Balance time and cost            | 2022                | [22]      |
| MATLAB            | A population-based meta-heuristics approach the nondominated sorting genetic algorithm III (NSGA III) | Balance time, cost - and quality | 2022                | [23]      |
| MATLAB            | AHP and NSGA-II                                                                                       | Balance time, cost - and quality | 2022                | [24]      |
| MATLAB            | Slime Mold Algorithm (SMA)                                                                            | Balance time, cost - and quality | 2022                | [25]      |



| Solution software | Solving method                                                                                    | Balance type                     | Year of publication | Reference |
|-------------------|---------------------------------------------------------------------------------------------------|----------------------------------|---------------------|-----------|
| MATLAB            | Fuzzy Logic and Genetic Algorithm                                                                 | Balance time, cost - and quality | 2022                | [26]      |
| MATLAB            | Nondominated Sorting Genetic (NSGA-II) and Multi-objective Particle Swarm Optimization algorithms | Balance time, cost - and quality | 2023                | [27]      |
| MATLAB            | A hybrid multi-verse optimizer model (hDMVO)                                                      | Balance time, cost - and quality | 2023                | [28]      |
| GAMS              | A novel fuzzy Bayesian network-based                                                              | Balance time, cost - and quality | 2023                | [29]      |

### 3- METHODS AND MATERIALS

In this section, first the problem of time, cost, and quality balance in the use of corner shear connectors in composite ceilings is defined with a new approach, and then its mathematical model is formulated. In this regard, an attempt has been made to highlight the critical relationship between time and project activity quality.

#### 3-1- PROBLEM DEFINITION

A project with  $n$  activities is considered, each of which has a normal execution time and an intensive execution time. According to these times, a normal cost and a compression cost are predicted for each activity, respectively. The purpose of solving the time-cost balance problem is to carry out the project on a predetermined date so that the costs of compressing the time to carry out activities are minimized. It is clear that performing an activity in a shorter time than normal reduces the quality of doing it, so it is necessary to consider quality as an effective parameter in the process of time-cost balance.



In general, quality is defined as the degree to which the qualitative characteristics of an activity conform to the standard levels defined for that activity, usually expressed as a percentage [30]. In this study, it is assumed that the quality of activities depends on the time of their performance. To show this dependence, time is described as a fuzzy number, the degree of membership of which indicates the quality of the activity performed. Thus, if the activity is performed at normal time, the quality of its implementation will be at the highest possible level (membership level one); on the other hand, performing the activity in the shortest possible time reduces the quality of the activity to the lowest possible level (membership rate less than one). In addition to the negative impact that shortening the execution time of an activity has on its execution quality, the activities may be designed in such a way that it is possible to avoid the reduction in execution quality by incurring additional costs.

For example, it is possible to use advanced equipment in the execution of an activity, which improves the quality of execution. The use of such advanced equipment will impose new costs on the project, which we call "quality creation costs." As a result, the final quality is determined by the execution time and the cost of creating the quality, such that the shorter the execution time, the lower the level of quality of the activity; conversely, as the cost of creating the quality increases, the higher the level of quality of the activity.

However, if the normal and compact times of activity  $i$  are equal to  $nt_i$  and  $ct_i$ , respectively, the execution time of this activity can be described by a fuzzy number  $x_i$ , the degree of membership of which corresponds to the quality of execution [31]. Thus, if  $x_i$  is a fuzzy variable related to the fuzzy number  $x_i$ , the membership function of this number is defined as follows:

$$q_i^T(x_i) = \begin{cases} 0 & x_i < ct_i \\ lq_i + \frac{(x_i - ct_i)(1 - lq_i)}{nt_i - ct_i} & ct_i < x_i < nt_i \\ 1 & x_i > nt_i \end{cases} \quad (1)$$

Where  $x_i$  the variable corresponding to the execution time of the activity is,  $lq_i$  is the lowest level of the execution quality of the activity (in full compression mode) and  $q_i^T(x_i)$  is the execution quality of the activity  $i$  with the time  $x_i$ . It is clear that if the activity is performed at its normal time (not compressed), the level of performance quality will not decrease and the activity will have its normal quality.



The following diagram (Fig. 1) shows the fuzzy number membership function that describes the runtime and the corresponding quality. It should be noted that in real-world problems, the relationship between the quality of execution and the time of execution of an activity is nonlinear, but since the compression time of each activity is short compared to the execution time of the project, with a slight negation of the linear approximation, the description of the relationship between the compression time of the activity and the quality of it is used. Based on this, the proposed membership function is defined by the linear equation (1).

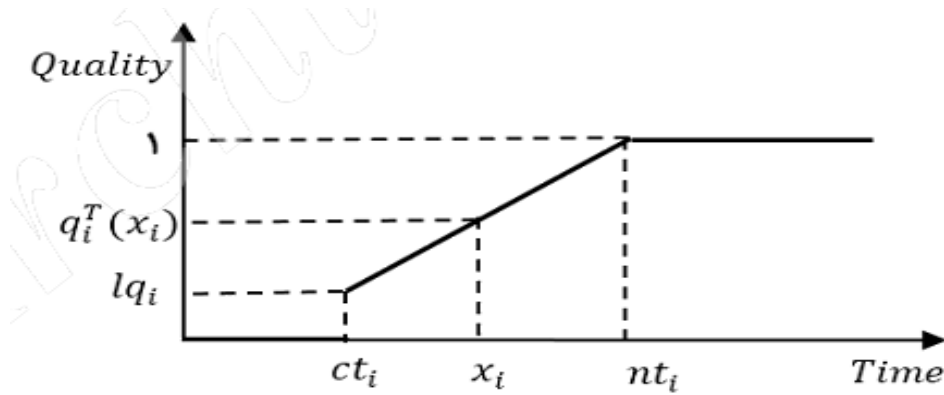


Fig. 1 Performance quality as a function of time-dependent activity

By reducing the execution time of the activity, the quality of performing the activity decreases and reaches the level of  $q_i^T(x_i)$ , in which case the quality level resulting from the compression of the execution time (ie  $q_i^T(x_i)$ ) can be applied with higher costs. For this purpose, if it indicates the cost required to increase the quality level to the highest possible value (ie one) and the  $c_i$  variable is related to the cost required to improve the quality, then the cost-time-dependent quality function can be defined as follows [31].

$$q_i(x_i, c_i) = \begin{cases} 0 & c_i < 0, x_i < ct_i \\ q_i^T(x_i) + \frac{c_i(1 - q_i^T(x_i))}{c_i^u} & 0 < c_i < c_i^u, ct_i < x_i < nt_i \\ 1 & c_i > c_i^u \text{ or } x_i > nt_i \end{cases} \quad (2)$$

If the execution time (i.e.  $x_i$ ) is a definite and constant value, then  $q_i^T(x_i)$  is constant and the quality function will depend only on the cost, in which case the diagram below describes the quality function  $q_i$ .

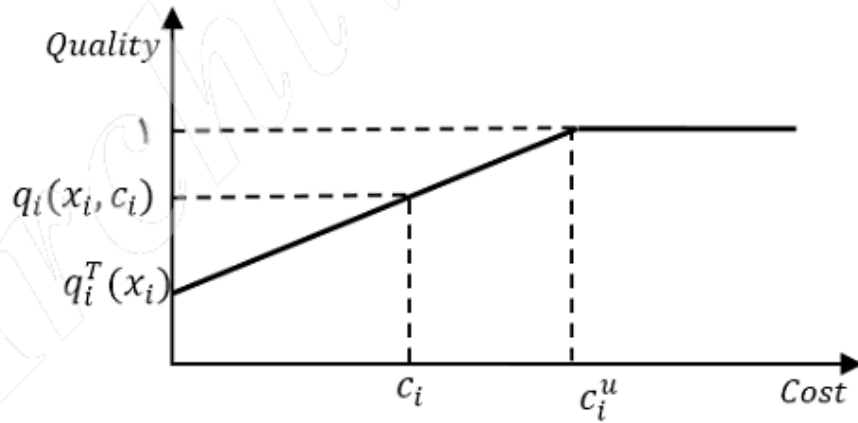


Fig. 2 Quality function of activity  $i$  related to the cost of quality improvement, assuming that  $x_i$  is constant

But the fact is that  $x_i$  are variable and therefore  $q_i(x_i, c_i)$  as a bivariate function can be represented in the following diagram.

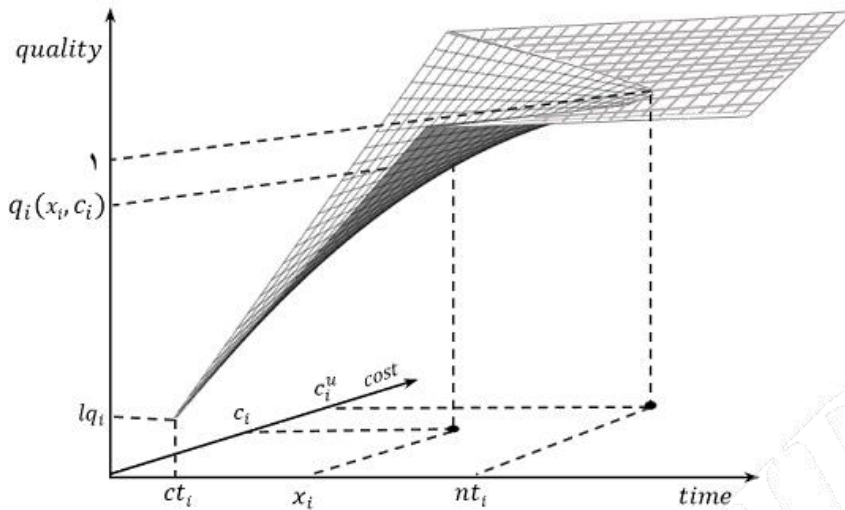


Fig. 3 Quality function of the activity depends on the time of execution and the cost of quality improvement

As can be seen in the diagram above, the quality of the activity, as defined in the previous relation, depends on the execution time  $x_i$  and the amount of quality creation cost  $c_i$ , and this dependence is due to the presence of the expression  $c_i(1 - q_i^T(x_i))$  is nonlinear. The symbols used in the proposed model are described below [31].



## Sets and parameters:

|                                                                                                                                                                                |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $V$ : A set of all activities (nodes)                                                                                                                                          |
| $E$ : A set of all arcs to consider prerequisite relationships.                                                                                                                |
| $E_{FS}$ : A set of arcs such as $(i,j) \in E$ in which it is necessary to start the $j$ th activity with a certain minimum delay after the end of the $i$ th activity.        |
| $E_{SS}$ : A set of arcs such as $(i,j) \in E$ in which $j$ th activity is required to start with a specified minimum delay after the start of $i$ -th activity.               |
| $E_{SF}$ : A set of arcs such as $(i,j) \in E$ in which $j$ -th activity is required to be completed with a specified minimum delay after the start of $i$ -th activity.       |
| $E_{FF}$ : A set of arcs such as $(i,j) \in E$ in which the $j$ -th activity is required to be completed with a specified minimum delay after the end of the $i$ -th activity. |
| $FS_{ij}$ : The minimum time required from the end of the activity to the beginning of the activity.                                                                           |
| $SS_{ij}$ : The minimum time required from the start of the activity to the beginning of the activity.                                                                         |
| $SF_{ij}$ : The minimum time required from the beginning of the activity to the end of the activity.                                                                           |
| $FF_{ij}$ : The minimum time required from the end of the activity to the end of the activity.                                                                                 |
| $nt_i$ : Normal time of $i \in V$ activity.                                                                                                                                    |
| $ct_i$ : Compressed $i \in V$ activity time.                                                                                                                                   |
| $nc_i$ : The cost of doing $i \in V$ activity in normal time.                                                                                                                  |
| $cc_i$ : The cost of performing $i \in V$ activity in intensive time.                                                                                                          |
| $CS_i^T = \frac{cc_i - nc_i}{nt_i - ct_i}$ Compression-dependent cost factor (rate of increase per unit decrease in time).                                                     |
| $lq_i$ : The lowest level of quality of $i \in V$ activity execution when the activity execution time is fully compressed.                                                     |



$c_i^u$  Cost in excess of  $cc_i$  paid to improve the quality of  $i\epsilon V$  activity to level one.

$CS_i^Q = \frac{c_i^u}{(1 - |q_i|)}$  Quality-dependent coefficient of quality upgrade for  $i\epsilon V$  activity (rate of increase per one percent increase in quality).

$T_P$  The predetermined time for completion of the project is determined by the decision maker.

### Decision variables:

$x_i$  Activity time  $i\epsilon V$ .

$s_i$  Activity start time  $i\epsilon V$ .

$c_i$  The cost of improving the quality of  $i\epsilon V$  activity.

$q_i^T(x_i)$  The quality of  $i\epsilon V$  activity in terms of time, ie  $x_i$  (without paying for quality upgrades).

$q_i(x_i, c_i)$  Final quality of  $i\epsilon V$  activity in terms of time and cost of quality upgrade.

$C$  Total cost of compressing execution time and improving the quality of project activities.

$T$  Total project execution time.

### 3-2- MATHEMATICAL PROPOSED MODEL

Suppose that for a project network, a loop-oriented directional graph is defined as  $G(V, E)$ , in which  $V$  is a set of nodes (including  $n$  nodes for the main activity and a virtual end node) and  $E$  is an arc set. Describes the interactivity prerequisite relationships. The mathematical model related to the problem of balancing time, cost, and quality is formulated as a program with the following three objectives:

|                                                                                                  |                                 |
|--------------------------------------------------------------------------------------------------|---------------------------------|
| $\min T = S_{n+1}$                                                                               | (a)                             |
| $\min C = \sum_{i \in V} CS_i^T(nt_i - x_i) + \sum_{i \in V} CS_i^Q[q_i(x_i, c_i) - q_i^T(x_i)]$ | (b)                             |
| $\max Q = \min\{q_i(x_i, c_i)\}$                                                                 | (c)                             |
| $S_j + SS_{ij} \leq S_i$                                                                         | $\forall (i, j) \in E_{SS}$ (d) |
| $S_i + SF_{ij} - x_i \leq S_j$                                                                   | $\forall (i, j) \in E_{SF}$ (e) |
| $S_i + x_i + FS_{ij} \leq S_j$                                                                   | $\forall (i, j) \in E_{FS}$ (f) |
| $S_i + x_i + FF_{ij} - x_j \leq S_j$                                                             | $\forall (i, j) \in E_{FF}$ (g) |
| $ct_i \leq x_i \leq nt_i$                                                                        | $\forall i \in V$ (h)           |
| $S_{n+1} \leq T_P$                                                                               | (i)                             |
| $x_i \geq 0, S_i \geq 0$                                                                         | $\forall i \in V$ (j)           |



In the above model, (a) the objective function corresponds to the project execution time, which we seek to minimize; (b) the cost objective function, which consists of two separate parts, The first part is related to the cost of compression during the execution of activities, which is defined as the total cost of compression for each activity, and the second part is related to the cost of improving the quality of activities after compression. It should be noted that the expression  $q_i(x_i, c_i) - q_i^T(x_i)$  in the second part indicates the rate of improvement of the quality of  $i$ -th activity (in terms of compression) after compression, which is a cost-dependent coefficient of quality upgrade ( $Ie^{CS_i^q}$ ) is multiplied. (c) The objective function is related to the quality of project implementation. In this article, the overall quality of project implementation is considered equal to the minimum quality of project activities, which we seek to maximize. Limits (d) to (g) are related to the observance of public prerequisite relations. The constraint (h) indicates that the execution time of the  $i$  activity must be between normal time and compressed time. Constraint (i) guarantees the completion of the project before the time set by the decision maker ( $T_P$ ).

### 3-3- ANALYTICAL METHOD

In this section, the proposed model of time, cost, and quality balance (Model 3) is solved with a fuzzy decision-making approach. For this purpose, first a review of fuzzy decision-making approaches is provided, and then the objective functions of time and cost are described as fuzzy ideals. Then, the research model is formulated using a fuzzy decision-making approach as a solvable one-objective programming model.

When the decision-making process takes place in a fuzzy environment, compatibility between ideals and constraints is a very important feature. This compatibility eliminates the differences between them and provides a relatively simple way to express the concept of decision based on the goals, ideals, and limitations of a decision-making process. Based on this property of adaptability, Belman and Zadeh [32] proposed that a fuzzy decision be defined as a fuzzy set of choices resulting from the sharing of goals (ideals) and constraints. To this end, they introduced three main concepts: fuzzy ideal or goal, fuzzy constraint, and fuzzy decision, then presented the application of these concepts in decision-making processes under fuzzy conditions in a fuzzy environment.

$X$  is assumed to be a set of different options for a decision problem. A fuzzy ideal  $G$  is a fuzzy set on  $X$  with a membership function  $\mu_G$  and a fuzzy constraint or constraint  $C$  is a fuzzy set on  $X$  with a membership function  $\mu_C$ . It is clear that both fuzzy ideals and fuzzy constraints tend to be at their best at the same time. In fact, fuzzy set  $D$  is specified as  $D = G \cap C$  with the following membership function [32].



$$\mu_D(x) = \min\{\mu_G(x), \mu_C(x)\} \quad (3)$$

Where  $x \in X$  is a specific option. Accordingly, the optimal option is the option whose membership in set D has the highest possible value. In mathematical terms,  $x^*$  is the best option based on fuzzy decision D if:

$$\mu_D(x^*) = \max \mu_D(x) = \max - \min\{\mu_G(x), \mu_C(x)\} \quad (4)$$

In the following, the problem of time, cost, and quality balance is solved using the fuzzy decision approach. For this purpose, time and cost objectives are first described in the form of fuzzy ideals, and according to the fuzzy structure previously defined for quality, the Belman-Zadeh decision criterion is used to find the optimal solution to the problem.

To describe the objective function of minimizing project execution time as a fuzzy ideal, the lower and upper limits allowed for project execution time are used. The upper limit for project execution time is the normal project execution time ( $T_{max}$ ) and the lower limit can be considered equal to the project execution time at full compression time of execution activities ( $T_{min}$ ). Therefore, the membership function related to the fuzzy ideal of project execution time is described in the following relation and diagram.

$$\mu(T) = \begin{cases} 1 & T < T_{min} \\ \frac{T_{max} - T}{T_{max} - T_{min}} & T_{min} \leq T \leq T_{max} \\ 0 & T > T_{max} \end{cases} \quad (5)$$

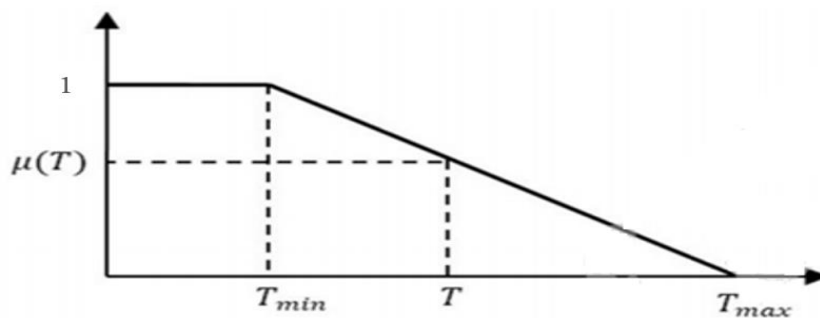


Fig. 4 Membership function related to project execution time



To describe the objective function of minimizing the cost of project implementation, as a fuzzy ideal, we use the costs of compressing time and improving the quality of project activities. In this case, the lower cost limit is related to the situation where no compression is performed and no cost is paid. The maximum cost of compression is when the project is completed in the shortest possible time and we have to upgrade the quality of all activities, which have been reduced due to compression, to level one; therefore, the membership function related to the fuzzy ideal of project compression cost is described in the following relation and diagram.

$$\pi(C) = \begin{cases} 1 & C < C_{\min} \\ \frac{C_{\max} - C}{C_{\max} - C_{\min}} & C_{\min} \leq C \leq C_{\max} \\ 0 & C > C_{\max} \end{cases} \quad (6)$$

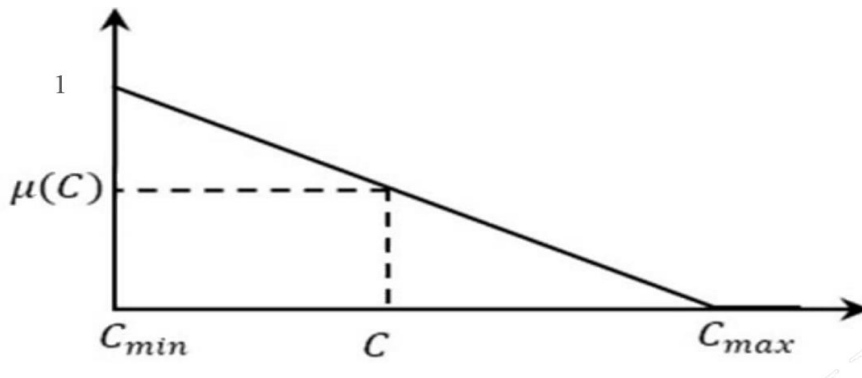


Fig. 5 Membership function Compression cost of project implementation

Given the foregoing and the primary goal of balancing time, cost, and quality, the implementation of project activities in the shortest possible time at the lowest possible cost with the highest possible quality can be described as follows using the Belman-Zadeh decision criterion.

$$\max -\min\{\mu(T), \pi(C), q_i(x_i, c_i)\} \quad (7)$$

Assuming  $\alpha = \min\{\mu(T), \pi(C), q_i(x_i, c_i)\}$  and according to the objectives and limitations of the problem, the above model can be rewritten as follows.



|                                                                                                                                     |                            |
|-------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| $\max \alpha$                                                                                                                       |                            |
| $(T_{\max} - T_{\min})\alpha + S_{n+1} \leq T_{\max}$                                                                               |                            |
| $(C_{\max} - C_{\min})\alpha + \sum_{i \in V} CS_i^T(nt_i - x_i) + \sum_{i \in V} CS_i^Q[q_i(x_i, c_i) - q_i^T(x_i)] \leq C_{\max}$ |                            |
| $\alpha \leq q_i(x_i, c_i) = q_i^T(x_i) + \frac{C_i(1 - q_i^T(x_i))}{c_i^U}$                                                        |                            |
| $S_i + x_i + FS_{ij} \leq S_j$                                                                                                      | $\forall (i,j) \in E_{FS}$ |
| $S_i + SS_{ij} \leq S_j$                                                                                                            | $\forall (i,j) \in E$      |
| $S_i + SF_{ij} - x_j \leq S_j$                                                                                                      | $\forall (i,j) \in E_{SF}$ |
| $S_i + x_i + FF_{ij} - x_j \leq S_j$                                                                                                | $\forall (i,j) \in E_{FF}$ |
| $ct_i \leq x_i \leq nt_i$                                                                                                           | $\forall i \in V$          |
| $S_{n+1} \leq T_p$                                                                                                                  |                            |
| $x_i \geq 0, S_i \geq 0$                                                                                                            | $\forall i \in V$          |

The first three constraints correspond to the fuzzy ideals of project execution time, the cost of compressing project execution time, and the quality of project activities, respectively, which are the simplified forms  $\alpha \leq q_i^T(x_i)$ ,  $\alpha \leq q_i(x_i, c_i)$ , and  $\alpha \leq q_i^T(x_i) + \frac{C_i(1 - q_i^T(x_i))}{c_i^U}$ , respectively. The other limitations are the same as the limitations of the main research model. According to the formulas related to  $q_i^T(x_i)$  and  $q_i(x_i, c_i)$ , the above model can be rewritten as follows.

|                                                                                                                                                                     |                            |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| $\max \alpha$                                                                                                                                                       |                            |
| $(l_{\max} - l_{\min})\alpha + S_{n+1} \leq l_{\max}$                                                                                                               |                            |
| $(C_{\max} - C_{\min})\alpha + \sum_{i \in V} CS_i^T(nt_i - x_i) + \sum_{i \in V} CS_i^Q[\frac{c_i}{c_i^U}(1 - q_i)(\frac{nt_i - x_i}{nt_i - ct_i})] \leq C_{\max}$ |                            |
| $\alpha \leq q_i(x_i, c_i) = q_i^T(x_i) + \frac{C_i(1 - q_i^T(x_i))}{c_i^U}$                                                                                        |                            |
| $S_i + x_i + FS_{ij} \leq S_j$                                                                                                                                      | $\forall (i,j) \in E_{FS}$ |
| $S_i + SS_{ij} \leq S_j$                                                                                                                                            | $\forall (i,j) \in E$      |
| $S_i + SF_{ij} - x_j \leq S_j$                                                                                                                                      | $\forall (i,j) \in E_{SF}$ |
| $S_i + x_i + FF_{ij} - x_j \leq S_j$                                                                                                                                | $\forall (i,j) \in E_{FF}$ |
| $ct_i \leq x_i \leq nt_i$                                                                                                                                           | $\forall i \in V$          |
| $S_{n+1} \leq T_p$                                                                                                                                                  |                            |
| $x_i \geq 0, S_i \geq 0$                                                                                                                                            | $\forall i \in V$          |



The above model is a comprehensive model for balancing time, cost and quality in continuous mode, which in addition to observing the general prerequisite constraints and compression constraints of project execution time, shows the dependence of the quality of project activities on compression time and quality creation cost. It should be noted that this model is a nonlinear

model due to the presence of the expression  $\frac{c_i}{c_i^u}(nt_i - x_i)$  in the second and third constraints, which we use in section to solve a real problem. It is very clear that the answer obtained from solving the above model includes the time of activities, project execution time, compression costs, quality creation costs and the final quality of activities. This answer strikes a logical balance between time, cost and quality.

In the following, by applying simpler assumptions to the problem, a special case of the model is presented.

In the final model, an attempt is made to achieve a reasonable balance between time, cost, and quality of project activities. Sometimes it may be necessary to look for an answer that minimizes the time it takes to complete a project so that the degree to which cost and quality goals are met has a certain minimum level, for example. With such an assumption, the following model can be proposed, in which the objective function is to minimize the project execution time; in comparison with the final model, the limitation related to the membership function of the execution time and the predetermined time limit is eliminated, and the minimum degree of achievement is limited. Cost and quality have been added.

|                                                                                                                                                                                                    |                            |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| $\min \quad s_{n+1}$                                                                                                                                                                               |                            |
| $\alpha \geq \lambda$                                                                                                                                                                              |                            |
| $(C_{\max} - C_{\min})\alpha + \sum_{i \in V} CS_i^l (nt_i - x_i) + \sum_{i \in V} CS_i^q \left[ \frac{c_i}{c_i^u} (1 - lq_i) \left( \frac{nt_i - x_i}{nt_i - ct_i} \right) \right] \leq C_{\max}$ |                            |
| $(nt_i - ct_i)\alpha \leq lq_i (nt_i - ct_i) + (1 - lq_i) \left[ x_i - ct_i + \frac{c_i}{c_i^u} (nt_i - x_i) \right]$                                                                              |                            |
| $S_i + x_i + FS_{ij} \leq S_j$                                                                                                                                                                     | $\forall (i,j) \in E_{FS}$ |
| $S_i + SS_{ij} \leq S_j$                                                                                                                                                                           | $\forall (i,j) \in E$      |
| $S_i + SF_{ij} - x_i \leq S_j$                                                                                                                                                                     | $\forall (i,j) \in E_{SF}$ |
| $S_i + x_i + FF_{ij} - x_j \leq S_j$                                                                                                                                                               | $\forall (i,j) \in E_{FF}$ |
| $ct_i \leq x_i \leq nt_i$                                                                                                                                                                          | $\forall i \in V$          |
| $x_i \geq 0, S_i \geq 0$                                                                                                                                                                           | $\forall i \in V$          |



Proposing the appropriate value of the parameter  $\lambda$  can be done by the decision maker according to the priority of the objectives. Also, by changing the parameter  $\lambda$ , in the range of zero to one, a set of answers can be obtained by proposing them, the decision maker can choose the desired option.

In the following, the application of these models and the proposed solution approach to the problem of balancing time, cost, and quality with real data in the use of corner shear connectors in composite roofs are presented.

### 3-4 BALANCING THE USE OF CORNER SHEAR CONNECTOR IN COMPOSITE CEILINGS

The discussion of time, cost, and quality balance in the implementation of construction projects has become important and is a significant part of planning. In this section, the issue of time, cost, and quality balance in the project with corner shear connectors in composite ceilings is investigated using the proposed models and solution approach. This project has 4 activities; the corresponding node network is shown in the figure below, and its data is shown in the corresponding table.

It is necessary to explain that the quality of each activity is defined based on the degree of conformity of its characteristics with the standard characteristics and is expressed as a percentage (sometimes a number between zero and one, which is zero to zero percent and one to 100 percent). In this study, the consensus of experts (consulting engineers, supervisors, and contractors of construction projects) has been used to estimate the parameters of the problem. For this purpose, first a questionnaire to estimate the parameters was provided to the experts, then the final value of the quality parameters was obtained by calculating the geometric mean of the experts' opinions and the final parameters of time and cost by rounding the arithmetic mean of the proposals.

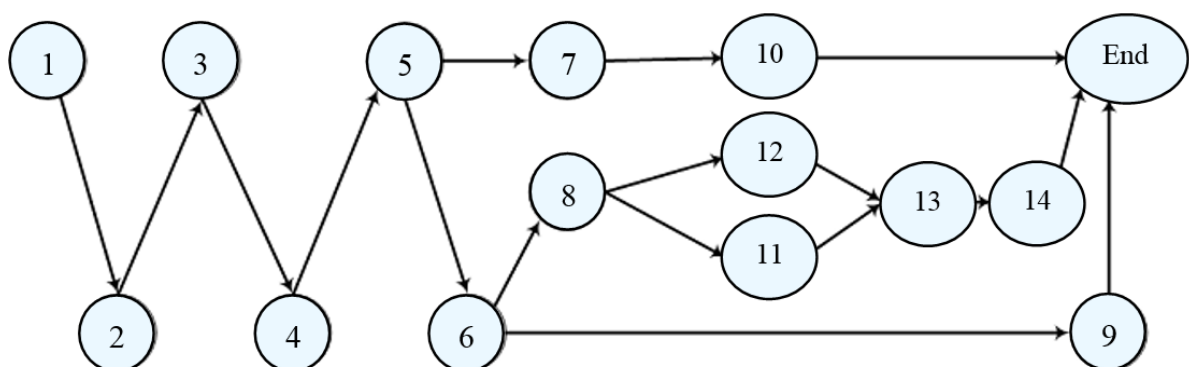


Fig.6 Project related network



Table2 .Problem data

|    | lqi | Million<br>tomans | Million<br>tomans | Day | Day | Relationship<br>type /<br>proposal | Description of<br>activity                                      | activity |
|----|-----|-------------------|-------------------|-----|-----|------------------------------------|-----------------------------------------------------------------|----------|
| 50 | 0.7 | 600               | 300               | 15  | 30  | -                                  | Excavation and<br>concrete<br>foundations of                    | 1        |
| 40 | 0.6 | 1500              | 1000              | 25  | 40  | 1 / FS (5)                         | Execution of the<br>foundation                                  | 2        |
| 70 | 0.5 | 2500              | 2000              | 20  | 40  | 2 / FS (10)                        | Skeleton<br>installation                                        | 3        |
| 20 | 0.8 | 800               | 600               | 8   | 15  | 3 / FS (0)                         | Implementation<br>of ceiling<br>composite<br>sectioning corners | 4        |
| 10 | 0.6 | 400               | 300               | 20  | 30  | 4 / SS (5)                         | Porcelain wall,<br>chair and<br>sandstone                       | 5        |
| 40 | 0.3 | 600               | 500               | 10  | 20  | 5 / SS (5)                         | Execution of basic<br>facilities                                | 6        |
| 50 | 0.4 | 550               | 450               | 25  | 40  | 5 / FS (0)                         | Namasazi                                                        | 7        |
| 30 | 0.5 | 500               | 400               | 18  | 30  | 6 / FS (0)                         | Interior stonework                                              | 8        |
| 50 | 0.6 | 330               | 280               | 8   | 15  | 6 / SS (10)                        | Landscaping                                                     | 9        |
| 10 | 0.8 | 270               | 150               | 8   | 15  | 7 / FF (5)                         | Install doors and<br>windows                                    | 10       |
| 30 | 0.5 | 350               | 250               | 25  | 40  | 8 / FS (0)                         | False ceilings and<br>plastering                                | 11       |
| 30 | 0.6 | 400               | 300               | 15  | 25  | 8 / FS (0)                         | Flooring                                                        | 12       |



|    |     |     |     |    |    |                                 |                                  |    |
|----|-----|-----|-----|----|----|---------------------------------|----------------------------------|----|
| 20 | 0.4 | 250 | 180 | 20 | 40 | 11 / FS<br>(10) 12 /<br>FS (10) | painting                         | 13 |
| 40 | 0.8 | 200 | 150 | 10 | 20 | 13 / FS<br>(10)                 | Execution of final<br>facilities | 14 |

## 5- RESULTS

Before solving the problem for the project in question, the values of  $T_{min}$  (the fastest possible time to complete the project (this value is obtained when the execution time of all activities is in full compression mode)),  $T_{max}$  (normal time to complete the project without any compression) are required. ),  $T_P$  (predetermined time to complete the project),  $C_{min}$  (corresponding to the state in which no compression is performed, so no cost is paid) and  $C_{max}$  (corresponding to the state in which the project is completed in the shortest possible time) And we have to set the quality of all activities, which have been reduced due to compression, to the desired level (one).

The values  $T_{min}=180$ ,  $T_{max}=305$ ,  $C_{min}=0$  and  $C_{max}=2880$  were obtained for the problem and the predetermined time for completion of the project was  $T_P=260$ . It is necessary to explain that all models of this research have been solved in Lingo software. The results of solving the model by applying the above values are summarized in the table below. The minimum degree of achievement of the objectives of the research model is and at this level, the values of the objective function of time, cost and total quality of the project are 221 days, 831.17 million Tomans and 71%, respectively.

Table3. Results of model solution for real research data

| Cost of improving<br>the quality of<br>activity i (million<br>tomans | Activity compression<br>cost i (million<br>Tomans | $q_i^*(x_i^*, c_i^*)$ | $q_i^T(x_i^*)$ | $x_i^*$ | activity |
|----------------------------------------------------------------------|---------------------------------------------------|-----------------------|----------------|---------|----------|
| 1.9                                                                  | 300                                               | 0.71                  | 0.7            | 15      | 1        |
| 0                                                                    | 0                                                 | 1                     | 1              | 40      | 2        |
| 0                                                                    | 50                                                | 0.95                  | 0.95           | 38      | 3        |



|       |     |      |     |    |    |
|-------|-----|------|-----|----|----|
| 0     | 0   | 1    | 1   | 15 | 4  |
| 0     | 0   | 1    | 1   | 30 | 5  |
| 23.51 | 100 | 0.71 | 0.3 | 10 | 6  |
| 0     | 0   | 1    | 1   | 40 | 7  |
| 12.68 | 100 | 0.71 | 0.5 | 18 | 8  |
| 0     | 0   | 1    | 1   | 15 | 9  |
| 0     | 0   | 1    | 1   | 15 | 10 |
| 12.68 | 100 | 0.71 | 0.5 | 25 | 11 |
| 0     | 0   | 1    | 1   | 25 | 12 |
| 10.38 | 70  | 0.71 | 0.4 | 20 | 13 |
| 0     | 50  | 0.8  | 0.8 | 10 | 14 |

As expected, the cost of compression and the cost of creating quality are zero for activities that are performed in their normal time because performing the activity in its normal time does not reduce the quality of the activity and consequently the cost of creating quality. On the other hand, the execution times of the first, third, and fourteenth activities have been compressed, but no cost has been incurred to improve their quality because the quality reduction due to the compression obtained by the model was not so great that it needed to be upgraded. The results obtained for activities six, eight, eleven, and thirteen show that for these activities, both the cost of compression and the cost of quality improvement are paid.

Now, assuming that a minimum amount is considered necessary to achieve the cost and quality objectives of the activities, it is possible to obtain the minimum possible time for the project by solving the model. In order for the minimum degree of realization of cost and quality goals to be achieved, solving the model leads to the results presented in the table below. The time taken to implement the project in this situation is 245 days, and the cost of compression and quality improvement of activities is 420.4 million Tomans, of which 350 million Tomans are related to the compression of activities and 70.4 million Tomans are related to improving the quality of activities to a minimum level of 85 percent.



Table.4 The results of model solving for real research data with the assumption

| Cost of improving the quality of activity i ((million tomans | Activity compression cost i (million (Tomans | $q_i^*(x_i^*, c_i^*)$ | $q_i^+(x_i^*)$ | $x_i^*$ | activity |
|--------------------------------------------------------------|----------------------------------------------|-----------------------|----------------|---------|----------|
| 0                                                            | 0                                            | 1                     | 1              | 30      | 1        |
| 0                                                            | 0                                            | 1                     | 1              | 40      | 2        |
| 0                                                            | 0                                            | 1                     | 1              | 40      | 3        |
| 0                                                            | 0                                            | 1                     | 1              | 15      | 4        |
| 0                                                            | 0                                            | 1                     | 1              | 30      | 5        |
| 3.42                                                         | 30                                           | 0.85                  | 0.79           | 17      | 6        |
| 0                                                            | 0                                            | 1                     | 1              | 40      | 7        |
| 21                                                           | 100                                          | 0.85                  | 0.5            | 18      | 8        |
| 0                                                            | 0                                            | 1                     | 1              | 15      | 9        |
| 0                                                            | 0                                            | 1                     | 1              | 15      | 10       |
| 21                                                           | 100                                          | 0.85                  | 0.5            | 25      | 11       |
| 0                                                            | 0                                            | 1                     | 1              | 25      | 12       |
| 15                                                           | 70                                           | 0.85                  | 0.4            | 20      | 13       |
| 10                                                           | 50                                           | 0.85                  | 0.8            | 10      | 14       |

It is clear that the level of  $\alpha$  is determined by the decision maker depending on the importance of achieving the goals. If we want to make a comparison between the results of the first model and the second model, it should be noted that the first model maximizes the minimum degree of achievement of time, cost and quality goals, while the second model minimizes project execution time in a way that the degree of achievement of cost and quality goals is above a predetermined level  $\alpha$ . Accordingly, the first model has set the minimum degree of achievement of goals by 71% with a time of implementation of 221 days, and the second model has increased



the project implementation time to 245 days in order to comply with the 85% limit of realization of cost and quality goals.

In order to validate the research models, the status of their answers to systematic changes in key parameters is examined to see if the model's behavior in relation to the changes is logical or not. To evaluate the first model, since the execution time was assumed to be 221 days with the assumption of  $T_p = 260$ , the minimum degree of achievement of goals, as well as execution time, cost, and quality, can be estimated by systematically changing the predetermined time to values less than 221. The projects are calculable, and the results are presented in the table.

As can be seen from the results of this table, with the reduction of the predetermined time for the completion of the project, the minimum degree of achievement of the goals and the quality of project implementation have decreased, and the costs of compression and quality improvement have increased. These changes seem logical because shortening the project run time reduces quality and increases the cost of compression.

Table.5 Results of solving the first model for different parameter values

| quality | Cost (million tomans) | execution time | Minimum degree of achievement ( ) of goals | $T_p$ |
|---------|-----------------------|----------------|--------------------------------------------|-------|
| 0.704   | 853.26                | 220            | 0.704                                      | 220   |
| 0.664   | 969.17                | 215            | 0.664                                      | 215   |
| 0.623   | 1085.66               | 120            | 0.623                                      | 210   |
| 0.581   | 1206.21               | 205            | 0.581                                      | 205   |
| 0.533   | 1346.18               | 200            | 0.533                                      | 200   |
| 0.48    | 1499.56               | 195            | 0.48                                       | 195   |
| 0.423   | 1661.15               | 190            | 0.423                                      | 190   |

In order to evaluate the performance of the second model by changing the parameter , which indicates the minimum degree of achievement of cost and quality objectives, the following table summarizes the results of time, cost, and quality of project implementation.



Table.6 The results of solving the second model for different parameter values

| quality | Cost (million tomans) | execution time |     |
|---------|-----------------------|----------------|-----|
| 1       | 0                     | 305            | 1   |
| 0.9     | 286.67                | 260            | 0.9 |
| 0.8     | 571.24                | 235            | 0.8 |
| 0.7     | 851.86                | 220            | 0.7 |
| 0.6     | 1147.48               | 208            | 0.6 |
| 0.5     | 1434.76               | 197            | 0.5 |
| 0.4     | 1725.78               | 188            | 0.4 |
| 0.4     | 1725.78               | 188            | 0.3 |

As we know, by reducing the parameter, the minimum level expected to achieve the cost and quality goals decreases, and the project execution time, which is the main goal of the second model, is also reduced. In other words, by reducing the zoning parameter of the second model, it expands, and thus the optimal value of the objective function is improved.

## 6- CONCLUSION

In this research, the issue of time, cost, and quality balance was studied, assuming that the quality of project activities depends on time and cost. For this purpose, a three-objective planning model was formulated, considering time, cost, and quality as objective functions and considering prerequisite relationships. A fuzzy decision-making approach was used to solve the problem. In order to implement this approach, the execution time and cost of compressing time and improving the quality of activities were described as fuzzy ideals so that a fuzzy decision-making approach could be used to balance them with quality.

Combining membership functions related to the execution time of individual activities and membership functions of fuzzy ideals related to project execution time and compression cost, in line with the fuzzy decision approach, led to a one-objective planning model. To demonstrate the efficiency of the proposed model and approach to solving a real problem of time balance, cost, and quality related to time balance, the cost of using angles in composite ceilings was solved and the results were discussed. Finally, the validity of the models was evaluated using parametric analysis. To further adapt the proposed model and solution model to reality, nonlinear membership functions can be used instead of linear membership functions to describe quality dependence on time and even fuzzy ideals related to execution time and compression cost.



## References

1. Kai Guo, Limao Zhang, Multi-objective optimization for improved project management: Current status and future directions, *Automation in Construction*, Volume 139, 2022, 104256, ISSN 0926-5805, <https://doi.org/10.1016/j.autcon.2022.104256>.
2. Teresa Beste, Ole Jonny Klakegg, Strategic change towards cost-efficient public construction projects, *International Journal of Project Management*, Volume 40, Issue 4, 2022, Pages 372-384, ISSN 0263-7863, <https://doi.org/10.1016/j.ijproman.2022.04.006>.
3. Gul, M., Yucesan, M. & Ak, M.F. Control measure prioritization in Fine – Kinney-based risk assessment: a Bayesian BWM-Fuzzy VIKOR combined approach in an oil station. *Environ Sci Pollut Res* 29, 59385–59402 (2022). <https://doi.org/10.1007/s11356-022-19454-x>
4. da Silva EM, Ramos MO, Alexander A, Jabbour CJ. A systematic review of empirical and normative decision analysis of sustainability-related supplier risk management. *Journal of Cleaner Production*. 2020 Jan 20;244:118808.
5. Hajiagha SH, Akrami H, Hashemi SS, Mahdiraji HA. An integer grey goal programming for project time, cost and quality trade-off. *Engineering Economics*. 2015 Feb 26;26(1):93-100.
6. Eydi A, Farughi H, Abdi F. A hybrid method based on fuzzy AHP and VIKOR for the discrete time-cost-quality trade-off problem. *Journal of Optimization in Industrial Engineering*. 2016 Nov 1;19(2):105-16.
7. Hosseini-Nasab H, Pourkheradmand M, Shahsavaripour N. Solving multi-mode time-cost-quality trade-off problem in uncertainty condition using a novel genetic algorithm. *International journal of management and fuzzy systems*. 2017;3(3):32-40.
8. Agdas D, Warne DJ, Osio-Norgaard J, Masters FJ. Utility of genetic algorithms for solving large-scale construction time-cost trade-off problems. *Journal of Computing in Civil Engineering*. 2018 Jan 1;32(1):04017072.
9. Mrad M, Al-Gahtani KS, Hulchafo R, Souayah N, Bamatraf K. Risk assessment for discrete stochastic time-cost-quality trade-off problem using simulation-based integer linear programming approach. *IEEE access*. 2019 Mar 7;7:32453-63.
10. Ballesteros-Pérez P, Elamrousy KM, González-Cruz MC. Non-linear time-cost trade-off models of activity crashing: Application to construction scheduling and project compression with fast-tracking. *Automation in Construction*. 2019 Jan 1;97:229-40.
11. Toğan V, Eirgash MA. Time-cost trade-off optimization of construction projects using teaching learning based optimization. *KSCE Journal of Civil Engineering*. 2019 Jan;23:10-20.



12. Eirgash MA, Toğan V, Dede T. A multi-objective decision making model based on TLBO for the time-cost trade-off problems. *Structural Engineering and Mechanics*. 2019 Jan 1;71(2):139-51.
13. Sonmez R, Aminbakhsh S, Atan T. Activity uncrashing heuristic with noncritical activity rescheduling method for the discrete time-cost trade-off problem. *Journal of Construction Engineering and Management*. 2020 Aug 1;146(8):04020084.
14. Jeunet J, Orm MB. Optimizing temporary work and overtime in the Time Cost Quality Trade-off Problem. *European Journal of Operational Research*. 2020 Jul 16;284(2):743-61.
15. Ammar MA. Efficient modeling of time-cost trade-off problem by eliminating redundant paths. *International Journal of Construction Management*. 2020 Sep 21;20(7):812-21.
16. Hamta N, Ehsanifar M, Sarikhani J. Presenting a goal programming model in the time-cost-quality trade-off. *International Journal of Construction Management*. 2021 Jan 2;21(1):1-1.
17. Wang T, Abdallah M, Clevenger C, Monghasemi S. Time–cost–quality trade-off analysis for planning construction projects. *Engineering, Construction and Architectural Management*. 2021 Feb 3;28(1):82-100.
18. Luong DL, Tran DH, Nguyen PT. Optimizing multi-mode time-cost-quality trade-off of construction project using opposition multiple objective difference evolution. *International Journal of Construction Management*. 2021 Mar 4;21(3):271-83.
19. Banihashemi SA, Khalilzadeh M. Time-cost-quality-environmental impact trade-off resource-constrained project scheduling problem with DEA approach. *Engineering, Construction and Architectural Management*. 2021 Jul 16;28(7):1979-2004.
20. Lotfi R, Yadegari Z, Hosseini S, Khameneh A, Tirkolaee E, Weber GE. A robust time-cost-quality-energy-environment trade-off with resource-constrained in project management: A case study for a bridge construction project. *Journal of Industrial and Management Optimization*. 2022;18(1).
21. Mahdavi-Roshan P, Mousavi SM. A new interval-valued fuzzy multi-objective approach for project time–cost–quality trade-off problem with activity crashing and overlapping under uncertainty. *Kybernetes*. 2022 Jul 6.
22. Van Eynde R, Vanhoucke M. A reduction tree approach for the discrete time/cost trade-off problem. *Computers & Operations Research*. 2022 Jul 1;143:105750.
23. Sharma K, Trivedi MK. Latin hypercube sampling-based NSGA-III optimization model for multimode resource constrained time–cost–quality–safety trade-off in construction projects. *International Journal of Construction Management*. 2022 Dec 10;22(16):3158-68.



24. Sharma K, Trivedi MK. AHP and NSGA-II-Based time–cost–quality trade-off optimization model for construction projects. In Artificial Intelligence and Sustainable Computing: Proceedings of ICSISCET 2020 2022 (pp. 45-63). Springer Singapore.
25. Son PV, Khoi LN. Utilizing artificial intelligence to solving time–cost–quality trade-off problem. Scientific Reports. 2022 Nov 22;12(1):20112.
26. Banihashemi SA, Mohammad M. Time-Cost-Quality-Risk Trade-off Project Scheduling Problem in Oil and Gas Construction Projects: Fuzzy Logic and Genetic Algorithm. Jordan Journal of Civil Engineering. 2022;16(2).
27. Sadeghi R, Heidari A, Zahedi F, Khordehbinan MW, Khalilzadeh M. Application of NSGA-II and fuzzy TOPSIS to time–cost–quality trade-off resource leveling for scheduling an agricultural water supply project. International Journal of Environmental Science and Technology. 2023 Jan 23:1-28.
28. Son PV, Nguyen Dang NT. Solving large-scale discrete time–cost trade-off problem using hybrid multi-verse optimizer model. Scientific Reports. 2023 Feb 3;13(1):1987.
29. Haghighi M, Ashrafi M, Nazerfard E. A novel fuzzy Bayesian network-based approach for solving the project time-cost-quality trade-off problem. AUT Journal of Modeling and Simulation. 2023 Jan 14.
30. Johnson JL, Adkins D, Chauvin S. A review of the quality indicators of rigor in qualitative research. American journal of pharmaceutical education. 2020 Jan 1;84(1).
31. Dang-Trinh Nguyen, Long Le-Hoai, Putri Basenda Tarigan, Duc-Hoc Tran, Tradeoff time cost quality in repetitive construction project using fuzzy logic approach and symbiotic organism search algorithm, Alexandria Engineering Journal, Volume 61, Issue 2, 2022, Pages 1499-1518, ISSN 1110-0168, <https://doi.org/10.1016/j.aej.2021.06.058>.
32. Reda Boukezzoula, Didier Coquin, A decision-making computational methodology for a class of type-2 fuzzy intervals: An interval-based approach, Information Sciences, Volume 510, 2020, Pages 256-282, ISSN 0020-0255, <https://doi.org/10.1016/j.ins.2019.09.020>.