



Designing and Solving two Mathematical Models for the Optimal Distribution of Urban Water Resources

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Abstract

The lack of optimal distribution of water resources in cities with a lack of quality water resources can be a threat to the sustainability of these cities.

The present study seeks to design a method for urban water distribution among urban districts using two optimization mathematical models applied in Qom. Responding to the water needs of the consumption districts was to be carried out with the least distribution costs, towards which end the first mathematical model was designed. Thus, by solving the first model, it will be clear how much water is needed to be transferred from each surface and ground resources that, while considering resources' limitation of allocation to consumption districts, the needs of the districts are satisfied and the costs of the water company kept at the minimum.

The scarcity of high-quality water resources in Qom has rendered water allocation from these resources to all consumption districts in the city impossible. On the other hand, urban consumption districts demand the highest-quality water given their predominant fabric (residential, commercial, or industrial). Thus, distribution management must take measures to allocate high-quality water to each district depending on its fabric, to which end the second mathematical model has been designed. Hence, solving the second model would answer the question of whether, by using the mathematical model of water distribution from various resources with different qualities, it would be possible to satisfy citizens' needs in terms of water quality (so that the quality of the allocated water is higher than the quality demanded by the district) and ensure that optimized resource allocation would take place at minimum costs for the company.



The results of the two designed models can help planning managers make decisions regarding the implementation of the first model given its cost-effectiveness or the second model given that it satisfies the demands of all consumption districts in terms of access to high-quality water.

Keywords: Optimize distribution of water resources, Mathematical modeling, Ground water resources, Surface water resources

1. Introduction

A sustainable city is a city in which the needs of residents are adequately and fairly met; Without jeopardizing the interests of future generations and the city and urban activities have the least adverse impact on the environment. In the discussion of the sustainability of cities, water is one of the most fundamental issues. Since water plays an important role in the United Nations Sustainable Development Goals (SDGs) and is directly or indirectly linked to almost all SDGs, its success is an important factor for other SDGs (Koop and et al.2022). The role of water resources management should be such that it ultimately leads to the optimal use of resources and also guarantees the demands of society (Yilmaz Salman & Hasar, 2023).

Over the past few years, due to increasing population, limited water resources, and other pressures on the urban water systems (UWS), urban water management has become a major concern for urban policymakers to develop efficient management solutions (Nezami et al., 2022). Because water is a vital issue for sustainable cities (Gao et al., 2023) .

The accelerating growth in the urban population of the world has intensified the significance of fair water resource distribution among the stakeholders. Previous research suggests that around two-thirds of the world's population will be living in cities by 2050, which will lead to a 55% increase in water consumption by urban users (Arfanuzzaman& Rahman, 2017).

Still, limited access to water resources alongside economic issues have forced planning managers to optimize the use of the available resources (Magri&Berezowska-Azzag, 2018). On the other hand, different water uses do not necessarily require the same water quality (Yuan et al., 2019). This limited access to water resources to respond to the needs of urban districts indicates the significance of urban water management (Darbandsari et al, 2020). Considering issues such as the accelerating increase in urban population and environmental problems, management of harvesting finite water resources by urban, agricultural, and industrial stakeholders is among the challenging issues water managers face (Ni et al, 2014. Zhang et al, 2014. Mankad, 2012).

Hence, urban water resource management requires adequate knowledge of the economic and social conditions of the studied region Therefore, the management of urban water resources is an issue that includes several objectives that examine the supply chain and distribution of



urban water. (Ramírez-Agudelo, 2021. Mittal et al, 2022. Özerol et al 2020. Sivagurunathan et al, 2022).

Therefore, it is necessary to include diverse ways to achieve different goals for sustainable development to be presented in different cities and to reduce the consumption of non-renewable resources such as water (Garcia et al., 2022).

Despite the multitude of strategies proposed so far for managing water resources in cities, a generally accepted classification system to manage urban water resources including the management of urban water distribution and supply and demand is not yet to be developed (Guthrie et al., 2017) since solutions that work in a particular city to manage water resources may be specific to the city and fail in other cities (Liu et al., 2011). Therefore, taking advantage of others' experiences can prove itself quite effective in urban water resource distribution management. For instance, while some small cities have access to abundant high-quality water, other cities such as Dubai and Abu Dhabi have made large investments to turn salt water into potable water (Noiva., 2016).

The present study investigates two optimization mathematical models to solve urban water resource distribution management problems. Optimal distribution of resources among different urban users is one of the most important priorities in managing urban water resources. water distribution management among urban districts is the main concerns of water and sewage company managers in Iran. Given their predominant fabric, urban districts typically demand the highest-quality water. The scarcity of high-quality water in Qom to cover all the needs of consumption districts has led urban water management planners to take measures for optimizing the distribution of available water resources among consumption districts.

Management of Urban water resource distribution among urban consumption districts is a process aimed at the allocation of finite water resources among districts that compete to acquire maximum advantage of said resources. This process is prominent, particularly in cases where the spatial, temporal, and quality distribution of the available water resources are disproportionate to the needs of the stakeholders. Optimized distribution of resources among various urban users is one of the most important priorities in urban water resource management.

Linear optimization mathematical models can be used to ensure the optimized distribution of water resources among the stakeholders (Roozbahani., 2015). The use of mathematical models has become widely popular in planning water resource distribution, reducing water resource supply costs for urban stakeholders, and minimizing environmental risks over recent years (Zarghami and Hajykazemian., 2013).

Researchers have long used mathematical models in important managerial decisions. For instance, (Swierez, 1995) modeled urban water distribution in mathematical terms. The



present study investigates the current state of water distribution among consumption districts using the first model and seeking to minimize the costs of the water and sewage company that is in charge of supplying and distributing water among consumption districts. The second model is proposed to minimize water and sewage company costs while observing consumer rights. Given that various resources provide different qualities of water and all consumer districts demand the highest quality. Therefore, one of the conditions of the second model is the supply of high-quality water based on the demand of consumption areas.

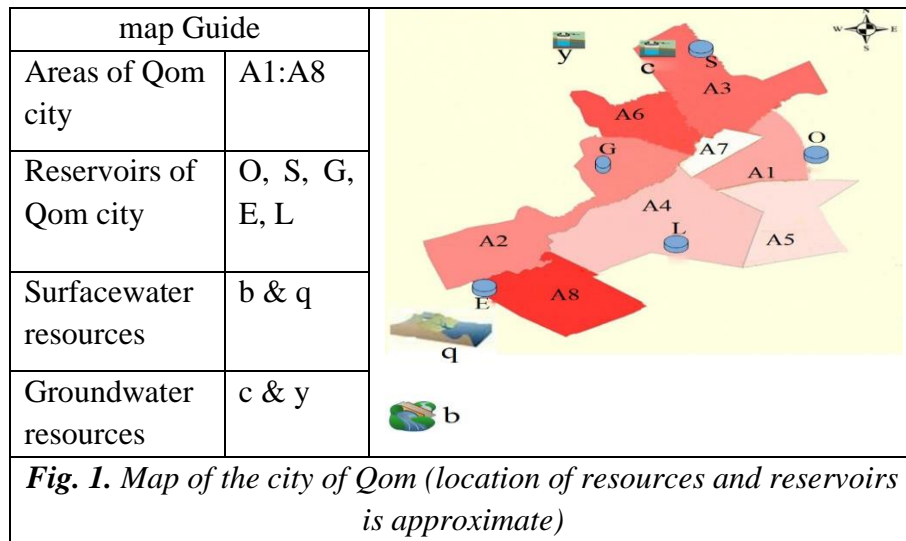
2. Methodology

2.1. Case study

This research was conducted in the city of Qom. The city of Qom is located 120 km south of the capital of Iran and in a low water area. This city is one of the most important cities in Iran and the Islamic world, so it has been called the religious capital of Iran.

The reasons for choosing the city of Qom as the city under study is that Qom is one of the important cities of Iran that is developing, since this city is located in a low water area, for this reason, the managers have made a decision in the direction of the sustainable development of the city. has imported high-quality surface water sources from other provinces. In the city of Qom, as in all other cities of Iran, water is not provided to the consumer at the full price, and the government charges the minimum price per cubic meter of consumption from the citizens by subsidizing it. Ab Shahr wants to cover all the needs of consumers at the lowest cost. Another reason for choosing the city of Qom as the city under study is that many cities in Iran have conditions similar to the city of Qom, so conducting this research will be useful for such cities as well.

The city of Qom includes eight urban districts and possesses four water resources to respond to the needs of the districts, including two surface (b and q) and two groundwater (c and y) resources. The city also possesses five reservoirs of O, S, G, E, and L. Table 1 demonstrates the maximum water supplied by each reservoir to be allocated to urban needs, the cost of purchasing water from each resource per cubic meter, and the capacity of each reservoir (given that Qom is situated in a desert area and lacks sufficient high-quality water resources to fully satisfy the needs of the stakeholders, the water and sewage company of the city has been forced to take initiative importing high-quality water from other provinces. Given the sensitives in Iran regarding the transfer of water resources from one city or province to another and the security concerns pointed out by the respective experts, the exact location and names of the resources are not disclosed and the resources are alternatively referred to with abbreviations. Figure. 1). Tables 2 and 3 demonstrate the costs of water transfer (from surface and groundwater resources) to reservoirs. Table 4 indicates the costs of transferring water from reservoirs to consumption districts. Finally, Table 5 shows the water requirements of the districts over the days of the week.



All the figures and the tables about the case study area were obtained from Qom water and sewage company and expert opinions in this company.

2.2. Research Methodology

Water and sewage companies purchase the water required to be distributed among the consumption areas from the regional water organization which provides them with a specific amount of water from each water resource. After treatment, water from ground and surface resources are transferred to reservoirs (the combination of water from ground and surface resources is called composite water). The water company then pumps the water from reservoirs to consumption areas to cover their needs. Figure 2 demonstrates the water distribution chain from the resource to the consumption areas.

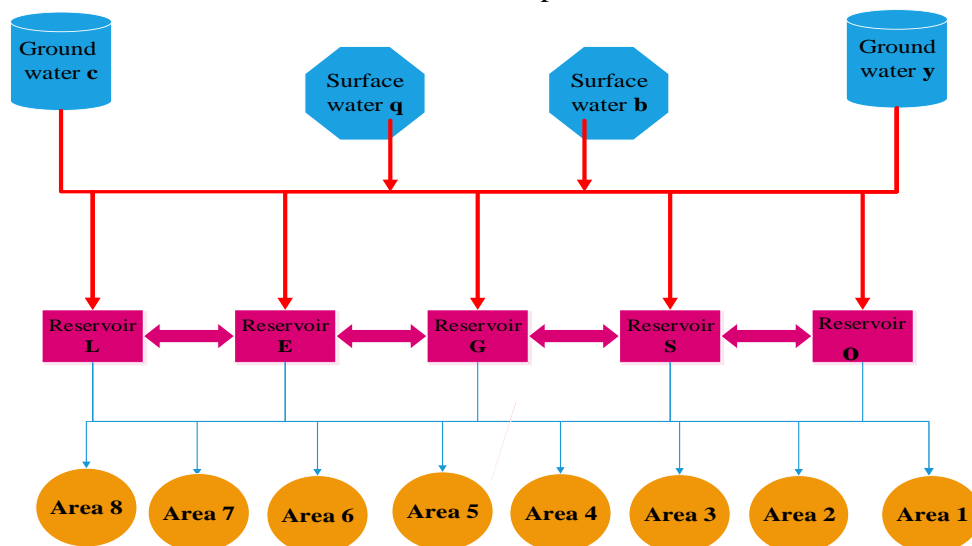


Figure 2: a schematic demonstration of the Qom water distribution chain







Figure 2 guide	
Symbol of ground water resources There are two resources of ground water in the city of Qom.	
Symbol of surface water resources There are two resources of surface water in the city of Qom.	
Symbol of water reservoirs There are 5 water reservoirs in Qom city.	
Consumption areas The city of Qom consists of 5 consumption areas.	

Figure 2 shows the water distribution chain (from water supply resources to consumption areas of Qom city). As it is known, water is transferred from surface and ground resources to reservoirs. Surface and ground water are combined in the reservoirs and then the water and sewage company transports the water from the reservoirs for distribution to the consumption areas.

The present study seeks to solve the two scheduling models designed for optimal water allocation to Qom consumption areas to answer the following question through the first model:

How much water is needed to be transferred from each surface and ground resources that, while considering resources' limitation of allocation to consumption districts, the needs of the districts are satisfied and the costs of the water company kept at the minimum?

Besides the question above, the second model answers the following question:

Given that urban consumers (consumption districts) demand the highest-quality water, would it be possible to use the mathematical model for water distribution from various resources with various water qualities to both satisfy the requirements of the consumers in terms of water quality and ensure that optimal resource allocation to consumption districts is carried out at the lowest cost for the company?

By presenting mathematical optimization models, the present study aims for optimal urban water distribution management through optimal water allocation to urban users. The economic approach of urban water resource management seeks to minimize the costs of water allocation to various forms of consumption. In addition to minimizing the allocation costs and considering the requirement of satisfying the quantitative needs of all consumption districts. The supply of high-quality water required by the consumption district will be considered in the second model.

The first and second mathematical models used in the present study are general models, which indicates that they can be implemented in any city with diverse water resources where



the available resources are inadequate to respond to urban needs two models explained in the following were implemented in Qom city (of Iran). The indices used in models, decision-making variables, target functions, and limitations will be elaborately discussed as follows.

2.3. Model assumptions (the following are the assumptions of both models)

- ✓ The capacity of the treatment plant and other facilities used to transfer water from the resources to consumption districts in Qom exceeds the needs of the city since the population increase of the city over the years to come is considered in the construction of such facilities.
- ✓ The needs of the consumption districts are specified.
- ✓ Planning for water purchases takes place daily. Seven days of the week have been modeled in both the first and the second models.
- ✓
- ✓ if the water consumption of the eight urban districts of Qom exceeds the supply capacity of ground and surface water resources or Unexpected failure will occur We will not be able to fully meet the needs of consumption districts (which is why Qom's water and sewage company has considered the aforementioned and set the water requirement volume at a level that would respond to the needs of the city even in case of high consumption).
- ✓ Combining the water from ground and surface resources would cause no problem in Qom since the water from all respective resources is potable.
- ✓ The stakeholders in the present study include the urban water users of the eight consumption districts in Qom and each district has been considered a single stakeholder.
- ✓ Other considerations such as water evaporation from the resource, water network failure, and other issues have been assumed ineffective over the weekly schedule proposed by the present study.
- ✓ The unit of cost variables and parameters used in the present study is IRR while the unit of volume variables and parameters is cubic meters. It must be noted that all data on the parameters used in the present study have been collected from the experts at Qom Water and Sewage Company.
- ✓ It was assumed that water resources, including surface and ground water resources, do not transfer water to the reservoirs at the same time, so that water from only one resource is transferred to the reservoir at any given time. The same principle rules water transfer from the resource to consumption districts.
- ✓ The present study measures water quality with an index called EC, which is a general qualitative index of water quality encompassing all quality measures such as electric conductivity investigated by the experts and is expressed on a scale of zero to 100. For instance, a resource with an EC of 100 has a higher quality than one with an EC of 70.
- ✓ The EC of the water resulting from the mixture of ground and surface water resources in the reservoir is assumed to be the mean EC of its constituents. For instance, the water in a



reservoir containing 1m^3 of water with an EC of 90 and 1m^3 of water with an EC of 50 would have an EC of 70.

✓ It has been assumed that surface and ground waters are transferred from the resources to the reservoirs early in the day and from the reservoirs to consumption districts until the end of the day so that no residual water remains in the reservoir for the following day.

2.4. Description of the model 1

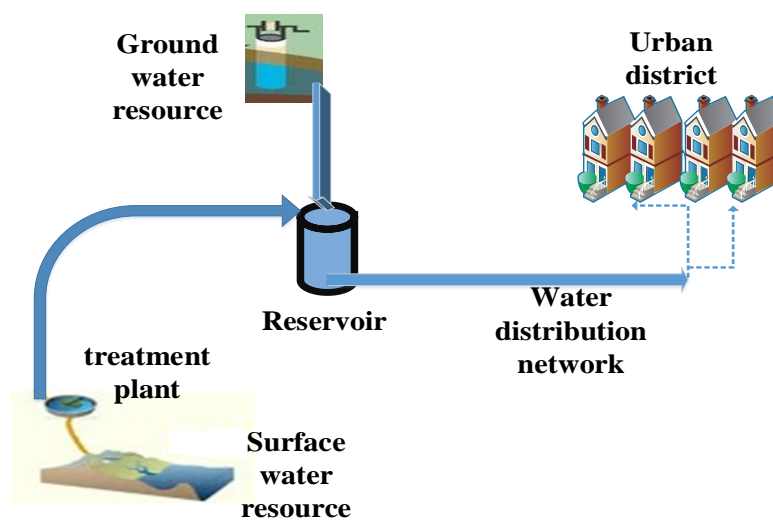


Figure 3: graphic demonstration of the first model

Figure 3 explicitly demonstrates the Qom water supply and distribution chain. As Figure 3 illustrates, water is transferred from the surface water resource to the treatment plant and then reservoirs. In the case of groundwater resources, the water is transferred from resources to reservoirs. In reservoirs, surface and ground water are mixed and composite water is formed. And finally, as much as the needs of the consumption areas, the composite water is transferred to the consumption areas.

It should be noted that in both models of this research, only the costs of water transfer from surface sources to reservoirs are needed, therefore, the water purification stage is not included in the mathematical models. The following equation expresses this matter.

The cost of transferring one cubic meter of water from the surface resource to the reservoir= the cost of transferring one cubic meter of water from the surface resource to the treatment plant + the cost of treating one cubic meter of surface water + the cost of transferring one cubic meter of water from the treatment plant to the reservoir.

The first model seeks to reduce the costs of the water and sewage company. this company is in charge of purchasing water from the ground and surface water resources. The waters from these resources come at variable costs due to their quality, ease of access, distance from the city, and security issues for urban consumption. The water and sewage company purchases



this water from Qom's regional water organization and pays a specific fee for every cubic meter of water purchased from each of the ground and surface resources, and proceeds to transfer the purchased water from the resource to the reservoirs, at which stage it shoulders the costs of transferring the water from resource to the reservoirs. Given that each resource and reservoir is situated in a specific location, transferring water from each resource to each reservoir costs differently. In other words, items such as pumping, facility repair and maintenance, human forces, and water treatment costs are factored into the fees of transferring every cubic meter of water from the desired resource to the desired reservoir, which is paid by the water and sewage company. Thus, all the aforementioned costs sum up to a specific amount expressed as the costs of transferring one cubic meter of water from the desired resource to the desired reservoir, which is referred to as the water transfer cost from ground or surface resource to reservoir per cubic meter. This is why the treatment stage has not been considered separately in the water supply and distribution chain. The following step of the water supply and distribution chain is transferring the composite water from the reservoir to consumption districts. The present study has assumed the needs of the Qom consumption district to be predetermined. Thus, after water is transferred from ground and surface resources and mixed in the reservoirs, the composite water must be pumped to the consumption district to fully satisfy their needs. The cost to carry each cubic meter of water from each reservoir to each consumption district is also specified for the company. The company should eventually decide from which resources water is to be transferred to which reservoirs, and from which reservoirs each district should get its water.

In the first model, the water and sewage company seeks to reduce the costs of the water supply chain. In the first model, this objective, expressed in mathematical terms in the way that surface and groundwater are transferred to the closest reservoirs in order to minimize the costs. The composite water would then be transferred from these reservoirs to the city to cover all water consumption needs. The aforementioned has been expressed in mathematical terms as follows.

2.4.1. Mathematical model 1 (The model formulation)

2.4.1.1. Indices

i: Indices of surface water resources (there are two surface water resources named *q* and *b* in Qom city).

ip: Indices of ground water resources (there are two ground water resources named *c* and *y* in Qom city).

j: Indices of reservoirs (there are 5 reservoirs named O, S, G, E and L in Qom city).

a: Indices of consumption areas (Qom city has 8 areas named a_8, \dots, a_2, a_1

r: scheduling days: (days of the week).



2.4.1.2 Decision variables

- M_{irj} The amount of water transferred from resource i on day r to reservoir j
- MU_{iprj} The amount of water transferred from resource ip on day r to reservoir j
- ME_{jra} The amount of water transferred from reservoir j on day r to consumption district a
- MB_{jr} The amount of composited water in the reservoir j on day r

2.4.1.3 Modeling parameters

- P_i The cost of buying one cubic meter of water from the surface resource i
- U_{ip} The cost of buying one cubic meter of water from the ground resource ip
- C_{irj} The cost of transferring one cubic meter of water from the surface resource i on day r to the reservoir j
- CU_{iprj} The cost of transferring one cubic meter of water from the ground resource ip on day r to the reservoir j
- CB_{jra} The cost of transferring one cubic meter of water from the reservoir j on day r to the consumption area a
- VB_{ar} The amount of water needed by area a on day r
- S_{ir} Sales capacity of surface resource i on day r
- SU_{ipr} Sales capacity of ground resources ip on day r
- F_j The reservoir capacity j

2.4.1.4 Objective function

$$\begin{aligned} Min(TC1) = & \sum_i P_i \sum_i \sum_r \sum_j M_{irj} + \sum_i \sum_r \sum_j C_{irj} \sum_i \sum_r \sum_j M_{irj} \\ & + \sum_{ip} U_{ip} \sum_{ip} \sum_r \sum_j MU_{iprj} + \sum_{ip} \sum_r \sum_j MU_{iprj} \sum_{ip} \sum_r \sum_j CU_{iprj} \\ & + \sum_j \sum_r \sum_a CB_{jra} \sum_j \sum_r \sum_a ME_{jra} \end{aligned}$$

2.4.1.5 Model constraints

$$\sum_j M_{irj} \leq S_{ir} \quad \forall i, \forall r \quad (1)$$

$$\sum_j MU_{iprj} \leq SU_{ipr} \quad \forall ip, \forall r \quad (2)$$

$$\sum_i M_{irj} + \sum_{ip} MU_{iprj} = MB_{jr} \quad \forall j, \forall r \quad (3)$$

$$MB_{jr} \leq F_j \quad \forall j, \forall r \quad (4)$$



$$\sum_a ME_{jra} \leq MB_{jr} \quad \forall j, \forall r \quad (5)$$

$$\sum_j ME_{jra} \geq VB_{ar} \quad \forall a, \forall r \quad (6)$$

The value of the variables must be greater than zero, or in other words:

$$\begin{aligned} (TC1) &\geq 0 & MB_{jr} &\geq 0 & M_{irj} &\geq 0 \\ ME_{jra} &\geq 0 & MU_{iprj} &\geq 0 \end{aligned}$$

2.4.1.6 Explaining Objective function

Regarding the model's target function, surface and groundwater resources are not situated in a specific point, and high-quality resources in adjacent cities or provinces are sometimes used to supply the city water. Therefore, the distance of the resource from the city impacts the final costs paid by the water and sewage company (due to pumping, maintenance, human force, etc. costs). Hence, the first model seeks to minimize costs for the water and Sewage Company by transferring water from the closest resource to the closest reservoirs and supplying the needs of the districts with water from the reservoirs closest to them.

2.4.1.7 Explaining problem constraints

The resource constraint mentioned earlier indicates that the water and sewage company can transfer a specific amount of water from each surface water resource to the reservoirs within each day, which must not pass the capacity of said resource.

The second constraint indicates the same in the case of groundwater resources. In other words, environmental considerations insist that each groundwater resource can only provide the authorized amount of water within a day, and the water and sewage company would thus not be allowed to transfer more water from any resource to the reservoirs than its capacity.

The third constraint indicates that the amount of surface water transferred from the surface resource to each reservoir plus the amount of water from groundwater resources transferred to the same reservoir indicates the amount of composite water in the reservoir.

The fourth constraint is concerned with the capacity of the reservoirs. Given that each reservoir has a fixed capacity, the amount of composite water in a reservoir must not exceed its capacity.

The fifth constraint indicates that the amount of water transferred from the reservoir to Qom consumption districts over a day must not exceed the amount of composite water in a reservoir on that day.

According to the managers at the water and sewage company, complete satisfaction of urban water consumer needs is among the goals of water and sewage companies in Iran. This concern is more prominent in Qom given its particular religious position in the country and world and its situation in a relatively deprived region in terms of water resources. The



seventh constraint is concerned with the aforementioned. In other words, the seventh constraint indicates that the amount of composite water allocated from the reservoirs to each district over a day should not be less than the districts' daily needs, which suggests that all the needs of the stakeholders must be met.

2.5. Description of the model 2

Water supply resources are among the most central components of the water supply chain in cities. Thus, the water consumed in urban districts must be supplied from reliable resources with high-quality water. However, not all cities possess sufficient high-quality resources to address all the needs of urban districts. Separating potable and non-potable water in urban regions is among the strategies adopted in this regard in many cities of the world given that only a small portion of urban stakeholders' water needs is potable water and their other needs such as washing, bathing, and hygiene can be satisfied with non-potable water. There are even some service and industrial centers in some cities and urban outskirts where allocating potable water to non-potable uses imposes significant costs on the Water and Sewage Company and wastes high-quality freshwater resources. Still, the Iranian government currently cannot burden the costs of allocating two separate meters to drinking and non-drinking purposes in many cities.

However, experts argue that potable and non-potable water currently cannot be separated for urban water users. Still, citizens expect the water they receive through the urban water network to be of the highest quality (the present study measures water quality through the EC index).

It would not be possible for the water and sewage company given the limited volume of high-quality water resources to meet all needs of consumption districts with high-quality water. Thus, we recommend the allocation of an EC figure to each reservoir based on the Qom water supply and distribution chain (EC_j represents the EC of an empty reservoir). Thus, the EC of the composite water stored in a reservoir should not be less than EC_j (in other words, each reservoir will have a fixed EC written on a sign installed on site, which should constantly be equal to or less than the EC of the composite water stored in the reservoir). Moreover, citizens demand water from reservoirs with the highest EC to satisfy their needs. Hence, every urban district will demand the highest possible water quality according to its predominant fabric including residential or industrial-service. As a result, the present study seeks to allocate the minimum required EC to each district based on expert opinions. It must be noted that the EC of the urban district might be variable since districts containing service and industrial centers would require a different minimum quality of water than those mainly made up of residential areas.

In this study, we have stipulated the condition of providing citizens' water needs that provides the quality required for citizens to drink.



The first model stipulated the water and sewage company to fully cover the water needs of consumption districts. In the first optimization model we modeled the current state of water resource distribution among consumption districts. The second model, i.e., the proposed model, seeks to help planning managers optimize water resource distribution among stakeholders in terms of qualitative and quantitative water indices. In this model, we will rank water resources based on their quality (using the EC index), and assign a score on a scale of zero to 100 to each resource based on expert opinions. Figure 2 demonstrates the water supply and distribution chain in this model. The present model is a proposed model adopting the constraints of the first while investigating the quality of water from ground and surface resources and requiring the company to deliver the water with minimum acceptable standards to consumption districts. The second model has been expressed in mathematical terms as follows:

2.5.1 Mathematical model 2 (The model formulation)

2.5.1.1 Indices

i: Indices of surface water resources (there are two surface water resources named *q* and *b* in Qom city).

ip: Indices of ground water resources (there are two ground water resources named *c* and *y* in Qom city).

j: Indices of reservoirs (there are 5 reservoirs named *O*, *S*, *G*, *E* and *L* in Qom city).

a: Indices of consumption areas (Qom city has 8 areas named *a8*, ..., *a2*, *a1*).

r: scheduling days: (days of the week).

2.5.1.2 Decision variables

- M_{irj} The amount of water transferred from resource *i* on day *r* to reservoir *j*
- MU_{iprj} The amount of water transferred from resource *ip* on day *r* to reservoir *j*
- ME_{jra} The amount of water transferred from reservoir *j* on day *r* to consumption district *a*
- MB_{jr} The amount of composited water in the reservoir *j* on day *r*

2.5.1.3 Modeling parameters

- P_i The cost of buying one cubic meter of water from the surface resource *i*
- U_{ip} The cost of buying one cubic meter of water from the ground resource *ip*
- C_{irj} The cost of transferring one cubic meter of water from the surface resource *i* on day *r* to the reservoir *j*
- CU_{iprj} The cost of transferring one cubic meter of water from the ground resource *ip* on day *r* to the reservoir *j*
- CB_{jra} The cost of transferring one cubic meter of water from the reservoir *j* on day *r* to



	the consumption area a
VB_{ar}	The amount of water needed by area a on day r
S_{ir}	Sales capacity of surface resource i on day r
SU_{ipr}	Sales capacity of ground resources ip on day r
F_j	The reservoir capacity j
ECA_a	Expected EC of consumption area " a "
ECJ_j	the EC of empty reservoir j
ECL_i	The EC of surface water resource i
$ECIP_{ip}$	The EC of ground water resource ip

2.5.1.4 Objective function

$$\begin{aligned} Min(TC2) = & \sum_i P_i \sum_i \sum_r \sum_j M_{irj} + \sum_i \sum_r \sum_j C_{irj} \sum_i \sum_r \sum_j M_{irj} \\ & + \sum_{ip} U_{ip} \sum_{ip} \sum_r \sum_j MU_{iprj} + \sum_{ip} \sum_r \sum_j MU_{iprj} \sum_{ip} \sum_r \sum_j CU_{iprj} \\ & + \sum_j \sum_r \sum_a CB_{jra} \sum_j \sum_r \sum_a ME_{jra} \end{aligned}$$

2.5.1.5 Model constraints

$$\sum_j M_{irj} \leq S_{ir} \quad \forall i, \forall r \quad (8)$$

$$\sum_j MU_{iprj} \leq SU_{ipr} \quad \forall ip, \forall r \quad (9)$$

$$\sum_i M_{irj} + \sum_{ip} MU_{iprj} = MB_{jr} \quad \forall j, \forall r \quad (10)$$

$$MB_{jr} \leq F_j \quad \forall j, \forall r \quad (11)$$

$$\sum_i (M_{irj} ECL_i) + \sum_{ip} (MU_{iprj} ECIP_{ip}) \geq \sum_i (M_{irj} ECJ_j) + \sum_{ip} (MU_{iprj} ECJ_j) \quad \forall j, \forall r \quad (12)$$

$$\sum_a ME_{jra} \leq MB_{jr} \quad \forall j, \forall r \quad (13)$$

$$\sum_j (ME_{jra} ECJ_j) \geq \sum_j (ME_{jra} ECA_a) \quad \forall a, \forall r \quad (14)$$

$$\sum_j ME_{jra} \geq VB_{ar} \quad \forall a, \forall r \quad (15)$$



The value of the variables must be greater than zero, or in other words:

$$\begin{aligned}(TC2) &\geq 0 & MB_{jr} &\geq 0 & M_{irj} &\geq 0 \\ ME_{jra} &\geq 0 & MU_{iprj} &\geq 0\end{aligned}$$

2.5.1.6 Explaining Objective function

In the objective function of the second model, we are looking for the fact that surface water is transferred to the nearest reservoir and also ground water is also transferred to the nearest reservoir. The water consumption of each area should be supplied from the nearest reservoir. In other words, each of the regions should receive the composite water they need from the nearest reservoir. In fact, in the second model, the water and sewage company seeks to minimize the water supply costs of urban districts.

2.5.1.7 Explaining problem constraints

The Eighth constraint indicates that the water and sewage company can transfer a specific amount of water from each surface water resource to the reservoirs within each day, which must not pass the capacity of said resource.

The ninth constraint indicates the same in the case of groundwater resources. In other words, environmental considerations insist that each groundwater resource can only provide the authorized amount of water within a day, and the water and sewage company would thus not be allowed to transfer more water from any resource to the reservoirs than its capacity.

The tenth constraint indicates that the amount of surface water transferred from the surface resource to each reservoir plus the amount of water from groundwater resources transferred to the same reservoir indicates the amount of composite water in the reservoir.

The Eleventh constraint is concerned with the capacity of the reservoirs. Given that each reservoir has a fixed capacity, the volume of water transferred to it each day must not exceed its capacity.

The twelfth constraint is concerned with water transfer from the resource to the reservoir while taking quality into account. Each reservoir demands composite water with specified minimum quality. In other words, the combined quality of water transferred to each reservoir should not be lower than the quality figure assigned to the empty reservoir.

The thirteenth constraint indicates that the amount of water transferred from the reservoir to Qom consumption districts over a day must not exceed the amount of composite water in a reservoir on that day.

The fourteenth constraint is concerned with water transfer from reservoirs to consumers while taking quality into account. Each district requires a minimum water quality, which is expressed in this constraint. The twelfth constraint indicates that the quality of the water transferred from the resources to reservoirs over each day should be equal to or more than the quality set for the desired reservoir, while the fourteenth constraint indicates that the quality



of the water transferred from reservoirs to each consumption district over each day should be equal to or more than the quality set for the desired district.

The fifteenth constraint indicates that the amount of composite water allocated from the reservoirs to each district over a day should not be less than the districts' daily needs, In other words, that all the needs of the stakeholders must be met.

2.6 Studied districts and model parameters

The city of Qom includes eight urban districts and possesses four water resources to respond to the needs of the districts, including two surface (b and q) and two groundwater (c and y) resources. The city also possesses five reservoirs of O, S, G, E, and L. Table 1 demonstrates the maximum water supplied by each reservoir to be allocated to urban needs, the cost of purchasing water from each resource per cubic meter, and the capacity of each reservoir (given that Qom is situated in a desert area and lacks sufficient high-quality water resources to fully satisfy the needs of the stakeholders, the water and sewage company of the city has been forced to take initiative importing high-quality water from other provinces. Given the sensitivities in Iran regarding the transfer of water resources from one city or province to another and the security concerns pointed out by the respective experts, the exact location and names of the resources are not disclosed and the resources are alternatively referred to with abbreviations). Tables 2 and 3 demonstrate the costs of water transfer (from surface and groundwater resources) to reservoirs. Table 4 indicates the costs of transferring water from reservoirs to consumption districts. Finally, Table 5 shows the water requirements of the districts over the days of the week.

Table 1: cost of purchasing water from each resource, the sales capacity of each resource, and the capacity of the reservoirs

cost of buying one cubic meter of water from a surface resource q	1000	Daily sales capacity of surface water resource y	51840
cost of buying one cubic meter of water from a surface resource b	1500	capacity of reservoir O	90000
cost of buying one cubic meter of water from a ground resource c	700	capacity of reservoir S	70000
cost of buying one cubic meter of water from a ground resource c	900	capacity of reservoir G	30000
Daily sales capacity of surface water resource q	190080	capacity of reservoir E	100000
Daily sales capacity of surface water resource b	561600	capacity of reservoir L	120000
Daily sales capacity of ground water resource c	159840		



Table 2: cost of transferring one cubic meter of water from surface resource i on day r to reservoir j

surface water resource.day	reservoir				
	O	S	G	E	L
q.1, q.2, q.3, q.4, q.5, q.6, q.7	648	720	624	420	564
b.1, b.2, b.3, b.4, b.5, b.6, b.7	1268	1340	1244	1040	1184

Table 3: cost of transferring one cubic meter of water from ground resource ip on day r to reservoir j

ground water resource.day	Reservoir				
	O	S	G	E	L
q.1, q.2, q.3, q.4, q.5, q.6, q.7	168	70	322	294	126
b.1, b.2, b.3, b.4, b.5, b.6, b.7	306	14	432	670	404

Table 4: cost of transferring one cubic meter of water from reservoir j on day r to consumption district a

day. reservoir	Consumption district							
	a1	a2	a3	a4	a5	a6	a7	a8
O.1, O.2, O3, O4, O5, O6, O.7	70	218	146	98	70	218	70	146
S.1, S.2, S.3, S.4, S.5, S.6, S.7	146	142	70	174	146	142	146	222
G.1, G.2, G.3, G.4, G.5, G.6, G.7	218	70	142	186	217	70	218	138
E.1, E.2, E.3, E.4, E.5, E.6, E.7	146	138	210	118	146	138	146	70
L.1, L.2, , L.3, L.4, L.5, L.6, L.7	98	186	184	70	98	186	98	98

Table 5: The amount of water needed by each district in every day

Consumption district	Days of the Week						
	1	2	3	4	5	6	7
A1	38412	31689.9	31689.9	40332.6	42253.2	42253.2	34570.8
A2	37941.6	31301.82	31301.82	39838.8	41735.76	41735.76	34147.44
A3	34272.6	28274.89	28274.89	35986.23	37699.86	37699.86	30845.34
A4	38551	31804.57	31804.57	40478.55	42406.1	42406.1	34695.9
A5	15680.2	12936.16	12936.16	16464.21	17248.22	17248.22	14112.18
A6	42671.2	35203.74	35203.74	44804.76	46938.32	46938.32	38404.08
A7	8325	6868.123	6868.123	8741.25	9157.5	9157.5	7492.5
A8	24378	20111.85	20111.85	25596.9	26815.8	26815.8	21940.2

3. Results



This section initially discusses the results of the first model and then proceeds to explain the results obtained from the second model. To facilitate comparison, Tables 8 and 9 contain the general results and the details of both models' solutions in GAMS were not included. For instance, the total amount of water transferred from surface resources to reservoirs over a week has been reported instead of reporting the amount of water transferred from surface resource i to reservoir j from the first to the seventh day of the week separately.

3.1 Results of the first model

The following table indicates the results of implementing the first model in Qom.

Table 6: results of the first model

The amount of water transferred from surface resources i to reservoirs j over a week	178792.86
The amount of water transferred from ground resources ip to reservoirs j over a week	1454782.106
Total water transferred from surface resources i and ground resources ip to reservoirs j over a week	1633574.966
The amount of composite water generated in reservoir j over a week	1633574.966
The amount of water transferred from reservoir j to consumption districts over a week	1633574.966
Total weekly water demands of the consumption districts	1633574.966
The percentage of water supply of consumption districts from surface water resources over a week	10.95
The percentage of water supply of consumption districts from ground water resources over a week	89.05
Total costs for the water and sewage company over a week (IRR billion)	106.863

As Table 6 suggests, all the needs of Qom consumption districts have been covered over the week. Results of implementing the first model in Qom indicated that the city supplied approximately 89% of the water consumed in its eight districts from groundwater resources and around 11% from the surface resource for obvious reasons. Qom is situated in a rather deprived area in terms of water resources and the small portion of water supplied through surface resources is due to the high costs of water transfer, which is itself due to the large distance between the city and surface water resources. Besides, in some cases, water and sewage company directors prefer to exploit groundwater resources because of their higher security since water from surface resources has to be imported from other regions.

3.2 Results of the second model



The following table indicates the results of implementing the second model in Qom.

Table 7: results of the second model

The amount of water transferred from surface resources i to reservoirs j over a week	731789.9792
The amount of water transferred from ground resources ip to reservoirs j over a week	901784.9868
Total water transferred from surface resources i and ground resources ip to reservoirs j over a week	1633574.966
The amount of composite water generated in reservoir j over a week	1633574.966
The amount of water transferred from reservoir j to consumption districts over a week	1633574.966
Total weekly water demands of the consumption districts	1633574.966
The percentage of water supply of consumption districts from surface water resources over a week	44.80
The percentage of water supply of consumption districts from ground water resources over a week	55.20
Total costs for the water and sewage company over a week (IRR billion)	131.539

Table 7 demonstrates a general overview of the results of the second model. As Table 7 indicates, the amount of water transferred from surface and groundwater resources to reservoirs is equal to the consumption of the districts. Results of the second model suggest that the water and sewage company takes around 45% of the water delivered to consumption districts from surface resources and approximately 55% from groundwater resources. The share of the surface resources has grown from 11% in the first model to 45% in the second model due to the higher quality of water obtained from these resources since the second model requires the company to ensure that the quality of the water delivered to consumption districts meets their quality demands. Providing the districts with higher-quality water was revealed to increase the costs from 106.863 to 131.539 billion IRR. It must be mentioned that the total water demand across the consumption districts of Qom was 1633574.966 m³ which was obtained from surface and underground water sources.

4. Discussion

Hydro-economic models can serve as valuable tools for the better understanding of water allocation systems as well as the improvement of decision-making and water resources management (Alamanos et al., 2019).



In this research, the aim is to provide management solutions to the planners and decision makers of the water and sewage company in line with the sustainability of the city. Since all cities have their own conditions, many of the solutions offered for the sustainability of cities in terms of water resources may not be suitable for the city of Qom (as many cities in Iran have conditions similar to the city of Qom).

For example, using rain water or allocating separate meters for drinking and non-drinking purposes in Qom city is not possible in terms of financial resources allocation.

The city of Qom is developing due to its special location. Since the water resources of the city of Qom do not meet the needs of urban areas, therefore, in the direction of sustainable development, imported water resources should be used. The water and sewage company must respond to all the needs of the citizens by managing the optimal distribution of water resources between consumption areas. The following situations can be imagined for the supply of drinking water in urban areas:

4.1 The first situation: supplying all the needs of consumption areas by relying on domestic sources.

The benefits of this plan in Iran are very high because these resources are specific to the same region, so this plan has high operational security. With the implementation of this plan, any conflicts and possible wars in the future that will be over water will be avoided.

It is not possible to implement this project in Qom city because the water resources of Qom city do not meet all the needs of consumption areas. In case of implementation of this project, the hours of access of citizens to water must be determined by scheduling. However, due to the location of Qom city, managers do not have any decision on water cutoff, rationing, and timing of access to water at certain times, because the goal of the managers of the water and sewage company is to cover all the needs of the consumption areas.

Due to the fact that the water resources of the city of Qom are not sufficient for all the needs of the consumption areas, also since the city of Qom is a developing city, therefore, in the direction of sustainable development, it should use imported water resources. In this regard, the managers of Qom Water and Wastewater Company have access to two sources of imported water for allocation to the areas of Qom city. One of these sources alone is capable of covering all the needs of the consuming areas. It is available in Qom. Therefore, the second situation can be planned in this way.

4.2 The second situation: providing all the needs of the consumption areas from the best water source that the water and sewage company has access to.

In the city of Qom, source B has a very high quality, this source is imported and has the ability to meet all the needs of the consumption areas. If we want to provide all the needs of consumption areas from this source, this plan will have advantages and disadvantages. If this plan is implemented, the city areas will meet all their needs from the best water source, which will satisfy the citizens. Disadvantages of this plan will be the very high cost of its



implementation. If this plan is implemented, the security of water supply to the citizens will be in danger because it may be difficult to access this source in the future. Access to high-quality water will cause dissatisfaction among citizens. Therefore, the water and sewage company must supply water from sources that are highly safe. Since the implementation of this project has a very high cost, the water and sewage company seeks to cover all the needs of the consumption areas with the lowest cost. Therefore, the third situation can be planned as follows.

4.3 The third situation: providing all the needs of consumption areas with the lowest cost.

All sources of water supply in Qom are suitable for drinking and other uses in the consumption areas, but these sources are different in terms of quality. Some of these sources are of higher quality than others. Consumers pay different prices for each of these resources. Also, the cost of transporting water from sources inside the city is lower than imported sources, in other words, if the water and sewage company wants to supply water to consumption areas from sources close to it, it will pay less than if it wants to supply water to consumption areas from sources. and supply tanks further away. The water and sewage company in Qom wants to provide all the needs of the consumption areas with the lowest cost. This situation has been investigated in detail in the mathematical model of the first optimization of this research. The benefits of implementing this plan for the company is the lowest cost. And one of its disadvantages is not meeting the needs of citizens from the best water source. For this reason, we have examined situation 4.

4.4 The fourth situation: providing all the needs of the consumption areas at the lowest cost, provided that the minimum quality required by the consumption areas is provided.

Urban areas in Iran and the city of Qom are not necessarily residential areas, some urban areas are commercial and industrial. The city of Qom is the cultural capital of Iran, and the pilgrimage court is located in an area of this city. This area is full of Iranian and foreign travelers in most seasons, so the quality of water provided in this area should be higher than other areas. For this reason, not all regions need the same water quality. Therefore, we have assigned their desired quality number to each region so that the water supplied for the use of that region is less than the desired quality

Not that area. This situation should be implemented with the lowest cost, that's why we have discussed the fourth situation completely in the second mathematical optimization model.

In the following chart, in terms of implementation cost, plans that can be implemented within a week are compared with each other.

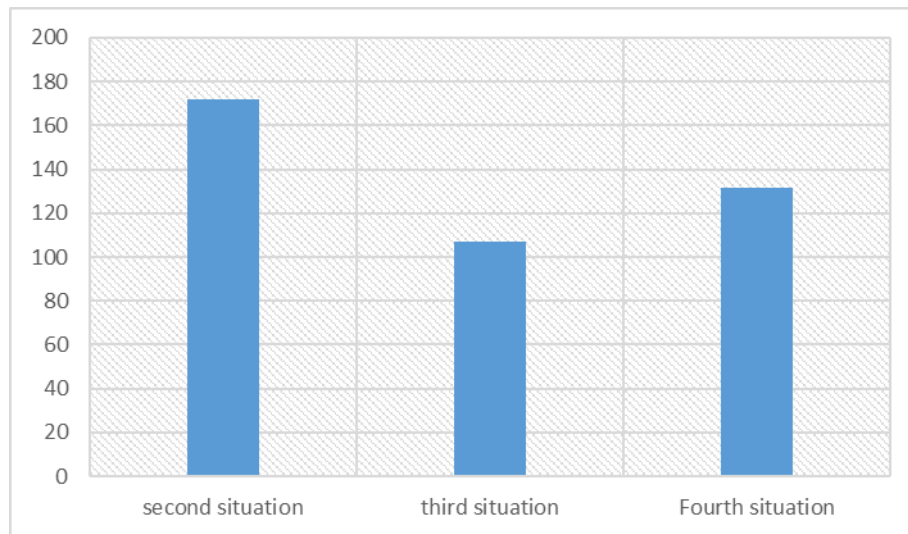


Figure 5. Comparison of the cost of implementing each situation in one week (figures in billions of Rials)

5. Conclusion

Using two linear mathematical models solved by GAMS software, The present study addressed the problem of urban water distribution management in consumption districts which is among the most prominent problems urban water planning managers face. The first model modeled the current state of water resource distribution using a mathematical optimization model and sought to reduce the costs burdened by the water and sewage company. The constraint of meeting all the needs of consumption districts were also included in the first model. Finally, the total cost of the water and sewage company was estimated at IRR 106.863 billion for one week of supplying water to the consumption areas.

The second model is a linear mathematical optimization model solved using GAMS software. This model proposes the current state to be optimized. For this purpose, a quality index was assigned to each reservoir indicating the minimum quality of the composite water stored in it. The composite water transferred from the reservoirs to each consumption district was also required to meet the minimum quality threshold set for the desired district. The final costs for the Water and Sewage Company were eventually estimated at IRR 131.539 billion using this model, which indicated a steep increase in the costs of the company as expected. Thus, the water and sewage company can use the results of the present study to decide whether it is willing to pay this extra cost under the current economic situation.

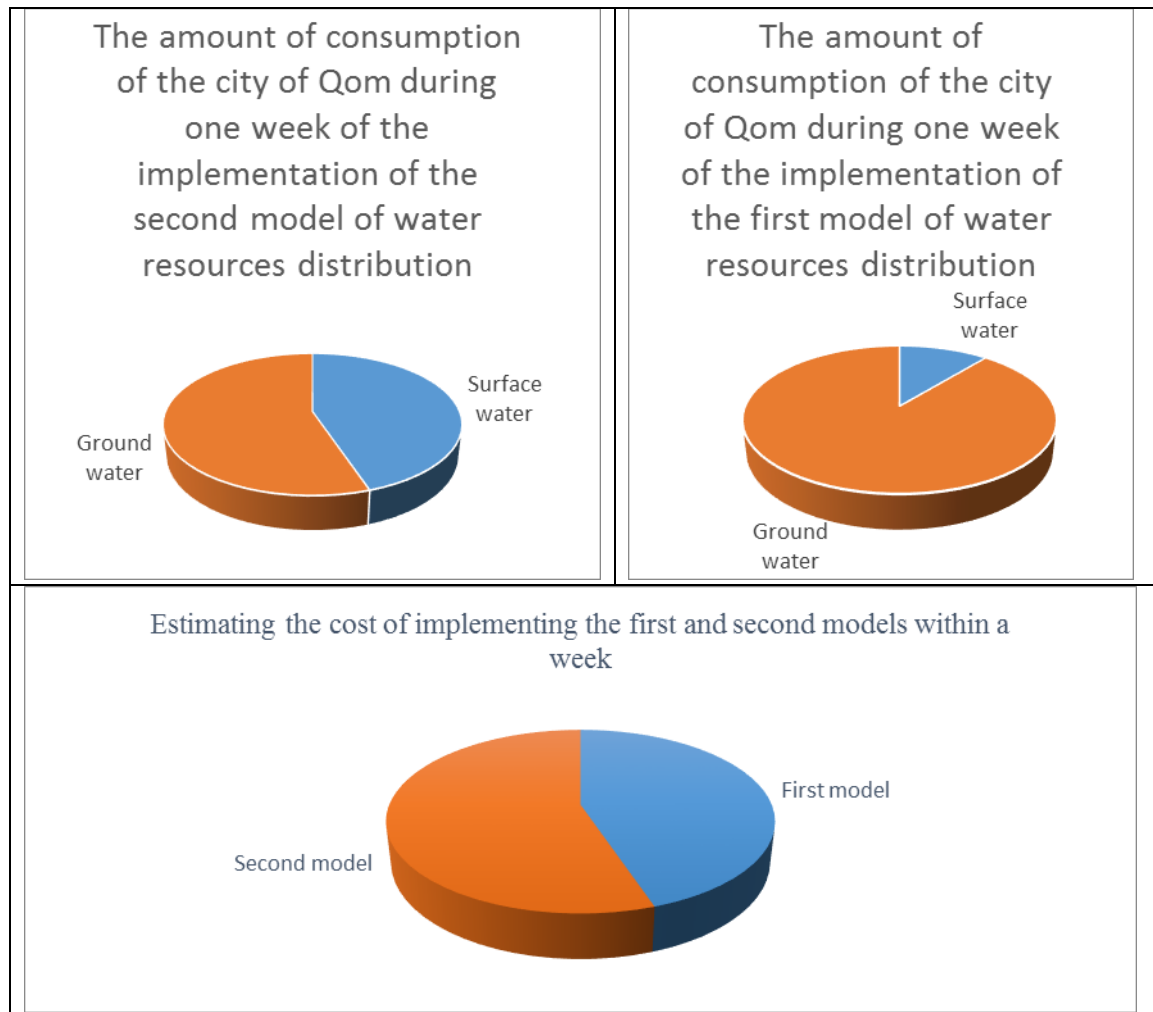


Figure 1: Comparison of the performance of the first and second models

As mentioned earlier, researchers have proposed various strategies such as using recycled water and rainwater and using separate meters for potable and non-potable water to manage urban water resources and distribute them among urban users and stakeholders optimally. Given that implementing each of these recommendations would require a thorough knowledge of the economic conditions governing the studied region, the use of modern portfolio theory concepts is recommended in a city such as Qom to provide managers with useful results in this regard.

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