



## Intelligent Control Design for Standard Search Pattern of a Quadcopter Under Prevailing Wind Gust

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**Abstract:** - The majority of applications require Quadcopter to fly in the outdoor environment where the velocity and direction of the wind can influence its movement from one point to another point. This situation will pose a challenge of designing an automatic flight control system that controls the Quadcopter to overcome the effects of wind, this paper mainly concentrates on mathematical modeling of Quadcopter dynamics and the design of an Adaptive control strategy that will generate corresponding control commands to maintain the Quadcopter in its designated path/trajectory even in the presence of wind. The idea behind designing Adaptive control is to make the Quadcopter autonomous and to minimize the deviations from the designated path in comparison to conventional controllers like PID. In this paper, the path/trajectory which is to be fed to the Quadcopter is generated using two different approaches one is with the help of trigonometric functions and the other based on position profile. Two types of wind disturbances are mathematically modeled using standard wind gust function in the MATLAB environment. Comparative analysis of the performance of both the controllers is performed by inducing wind disturbance in the geometric axis as X-axis, Y-axis as well as XY-axis. A detailed description of outputs obtained while the Quadcopter following different trajectories by employing both the control strategies is provided.

**Keywords:** Unmanned Aerial Vehicle (UAV), Trajectory Following, Wind Gust, Adaptive Back Stepping, Automatic Flight Control System.

### 1. Introduction (Quadcopter)

In recent days autonomous vehicles are playing a major role in many fields. Autonomous vehicles can sensor systems can persuade the environment and navigate without human interaction. a UAV has the caliber of operating the variable degrees of autonomy which is power boosted aerial vehicle that does not contain the onboard crew. A remote pilot or onboard autonomous control system can be operating the control of the UAV. Therefore, despite conventional human interacted vehicles, UAVs spare the risk of human lives particularly in special operations like search, rescue operations, firefighting, and dangerous military operations.

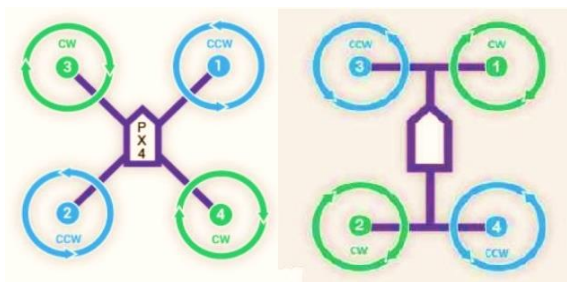


Generally, controlling of UAV is done by the pilot from the GCS (Ground Control Station). But there are some missions in which the UAV has to fly autonomously. In such situations, the path in which the aerial vehicle has to travel is fed to it either in the form of waypoints or geometric paths based on the reference frame it is using. Trajectory means the path that a moving object follows through space as a function of time. For this reliability should be placed on the control mechanism by which the UAV movement is being controlled. The controlling methodology requires the exact data of position and attitude which are measured by the gyroscope, an accelerometer, sonar sensors, laser sensors, and global positioning system (GPS)

### ***Quadcopter configurations and working:***

The two-broad configuration of Quadcopter is “X” and “H” which are specified below in fig (1). In both the configurations the pair of rotors diagonal to each other are made to rotate in the opposite direction for the other pair. This is to avoid the aerodynamic torque which will be generated when all the rotors rotate in the same directions, making the Quadcopter unstable.

The Quadcopter possesses 6 DoFs (Degree of Freedom): the 6 degrees of freedom are Three translational motions (along X, Y, and Z-axis) and three rotational (Roll, yaw, and Pitch) motions. which are achieved by using four control inputs. In real-time, the requirements are obtained through the controllers, the inputs are Applied voltages for operated rotor rotations. these voltages can result in estimating the rotor RPM (Revolutions per Minute).



a) 'X' configuration b) 'H' configuration.

**fig (1): Configurations of Quadcopter**

Four basic forces are acting on Quadcopter provided in fig (2),

1. **Gravity ( $mg$ )** – The force that pulls the quadcopter downwards due to its mass.
2. **Lift ( $F_t \cos\theta$ )** – This is the upward reaction force generated by the rotation of propellers.
3. **Thrust ( $F_t \sin\theta$ )** – component of overall force which drives the Quadcopter to move from one point to other.
4. **Drag ( $F_D$ )** – The force acting in the opposite direction of motion.



Rotors generate a tremendous amount of thrust which is always perpendicular to the motion of the quadcopter body and torque is added to the quadcopter which makes the Quadcopter rotate based on its spin direction, RPM, and distance from the center of the body. The specifically directed motion or spin can be acquired by variation in the relative speeds and spin of individual rotors, making them move from one point to another. From the force diagram specified in fig (2) it is evident that  $F_t \sin\theta$  drives Quadcopter, against drag force ( $F_D$ ).

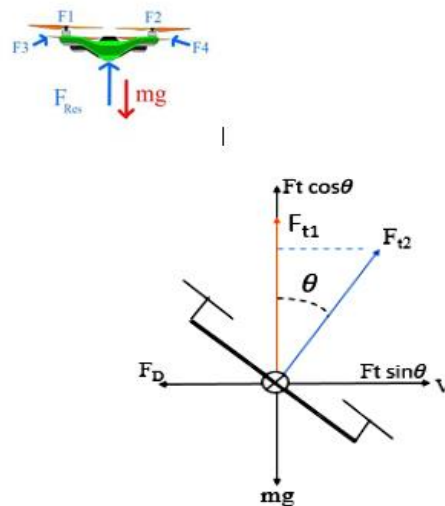


fig (2): Force diagram

$$F = ma = F_t \sin\theta - F_D \quad (1)$$

And  $F_t \cos\theta$  holds Quadcopter to be in the air acting against weight ( $mg$ ). For hovering the criteria is  $\theta = 0$  making  $F_t = mg$ .

$$F_t \cos\theta = L \text{ (Lift)} \quad (2)$$

$$\theta = \cos^{-1} \left( \frac{mg}{F_t} \right) \quad (3)$$

$$F_{t2} \sin\theta = ma = T \text{ (Thrust)} \quad (4)$$

The major movements of the quadcopter body (altitude, pitch, roll, and yaw) can be effectively controlled with the help of possible controlling of individual rotor speed and spin. Displacement of the copter body is generated by the total effective thrust generated by the four rotors whose directions are changing based on its attitude of the quad-rotor: Forward/Backward motion, Sideways Left/Right motion, Left/Right rotation motion, and upward/Downward motion as shown in fig (3).

**Forward & backward motion:** These motions are controlled by **Pitch**, generated with increasing of the relative speed of motor 3 for motor 1 while keeping the other at a constant speed and vice versa respectively.

**Sideways Left/Right motion:** These motions are controlled by **Roll**, generated by increasing the relative speed of motor 2 for motor 4 while keeping the other at a constant speed and vice versa respectively.



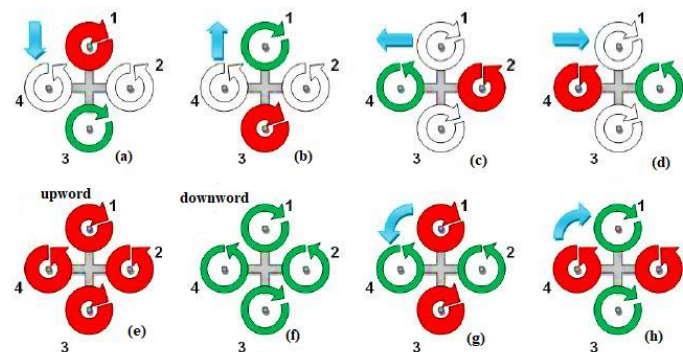
**Upward/Downward motion:** These are generally called **climb** and **Descent** respectively, obtained by increasing and decreasing the speed of all the rotors equally. Maintaining flight at a fixed point is called **hovering**, which occurs when all the four motors are operated at a constant speed to generate lift force equal to the weight.

**Left/Right rotation motion:**

These motions are controlled by **Yaw**, obtained by relatively increasing the speed of the motors 1 and 3 combined to 2 and 4 and vice versa.

**Trajectory following:**

The Trajectory following concerns movement along a time-parameterized reference line that defines the time evolution of a position. The degree of difficulty involved in achieving this goal includes designing control laws that allow the Quadcopter to follow a certain track which can be generated using two different approaches. One is based on trigonometric functions and the other is based on positional composition.



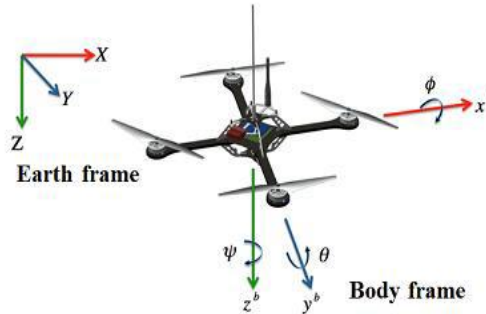
**fig (3): configurations of quadcopter dynamics**

**Effect of wind:** Wind will have considerable effects on the attitude, position, and velocity of unmanned aerial vehicles when deployed in the outdoor environment as the wind is very dynamic and random. To study the influence of wind we have modeled disturbances using the standard wind gust model present in the MATLAB environment. A nonlinear adaptive controller is designed such that is robust to constant force disturbances and model imperfection. To mitigate the deviation from the specified trajectory an adaptive controller is designed that can reduce them to a greater extent when compared to the conventional control techniques like PID which operates based on static gains.

## 2. DYNAMICS

### a. Dynamic of Quadcopter

The dynamic model of the quad-copter body, the control inputs  $U_1$ ,  $U_2$ ,  $U_3$  and  $U_4$  and their relation with the motor dynamics can be established by using 6-DOF equations.



**fig (4): Quadcopter body axis schematic**

After this, a fully functional dynamic model of the quad-copter is generated in the MATLAB environment. The thrust and angular velocity are related as follows:

$$F_i = k_i w_i^2 [N] \quad (5)$$

here, the relation between angular velocity and thrust generated by the propeller is defined by  $k_n (5.7 \times 10^{-8} \frac{N}{rpm})$ . The torque  $T_i$  generated by propellers can be related to thrust and expressed as

$$T_i = k_m F_i [N, m] \quad (6)$$

In Equation 6,  $k_m$  is relates to the optimal compromise between the thrust and torque generated by the propellers of the quad-copter. Then the Value of  $k_m = 0.0618 m$ . Since  $k_m$  and  $k_n$  are constants, it gives the relationship between thrust and torque generated by propellers and angular velocity of propellers. Quadcopter having four control inputs which are expressed symbolically as  $U_1, U_2, U_3$  and  $U_4$ . The mathematical definitions of  $U_1, U_2, U_3$  and  $U_4$  are as follows:

$$U_1 = f_1 + f_2 + f_3 + f_4 \quad (7)$$

$$U_2 = f_4 - f_2 \quad (8)$$

$$U_3 = f_3 - f_1 \quad (9)$$

$$U_4 = T_2 + T_4 - T_1 - T_3 \quad (10)$$

### ***Translational velocities-mathematical transformation***

The translational velocity of the quad-copter which yield from the components of the body of fixed frame and earth fixed frame can be related with the help of three sequential rotations of the body. the usual order of yaw, pitch, roll ( $\phi, \theta, \varphi$ ) is to be used for making three successive transformations, we can write the transformation matrix  $L_{BI}$  that transforms  $F_I$  to  $F_B$  as follows:

$$L_{BI} = L(\phi)L(\theta)L(\varphi) \quad (11)$$



$$L_{BI} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & s(\phi) \\ 0 & -s(\phi) & c(\phi) \end{bmatrix} \begin{bmatrix} c(\theta) & 0 & -s(\theta) \\ 0 & 1 & 0 \\ s(\theta) & 0 & c(\theta) \end{bmatrix} \begin{bmatrix} c(\varphi) & s(\varphi) & 0 \\ -s(\varphi) & c(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

For feasibility, sine and cosine trigonometric functions can be denoted with  $s$  and  $c$  respectively then the Equation (12), yields to the final form of  $L_{BI}$  as follows:

$$L_{BI} = \begin{bmatrix} c(\theta) c(\varphi) & c(\theta) s(\varphi) & -s(\theta) \\ s(\phi) s(\theta) c(\varphi) - c(\phi) s(\varphi) & s(\phi) s(\theta) s(\varphi) - c(\phi) c(\varphi) & s(\phi) c(\theta) \\ c(\phi) s(\theta) c(\varphi) + s(\phi) s(\varphi) & c(\phi) s(\theta) s(\varphi) - c(\phi) c(\varphi) & c(\phi) c(\theta) \end{bmatrix} \quad (13)$$

Let  $V_I$  is the component of the translational velocity of Quadcopter in frames of  $F_B$  and  $F_I$  respectively using the transformation matrix. then the Equation (13), yields  $V_B$  as follows:

$$V_I = \xi [\dot{x}, \dot{y}, \dot{z}] \quad (14)$$

Therefore

$$\xi = [L_{BI} V_B] \quad (15)$$

$$V_B = L_{BI} V_I \quad (16)$$

### Angular velocities- mathematical transformation

the angles of roll, pitch, and yaw  $[\phi, \theta, \varphi]$  are called "Euler angles." respectively. The derivatives of Euler angles which are defined in positions and orientation vectors of the Quadcopter for the center of mass, presenting interns of Launch Point Inertial Frame ( $F_I$ ). For a change in time, the rate of change of Euler angles is as follows:

$$\dot{\eta} = [\dot{\phi}, \dot{\theta}, \dot{\varphi}]^T \quad (17)$$

transformation matrix is  $L_{BI}$  non-singular, therefore inverse of  $L_{BI}$  exists. On multiplying both sides of Equation (16) with  $L_{IB}^{-1}$  we can obtain  $V_I$  follows:

$$V_I = L_{IB}^{-1} V_B \quad (18)$$

From the same terminology used in Equation (13), we can obtain  $L_{EB}$  (transformation Matrix) that transforms ( $F_B$  to  $F_I$ ) as following:

$$V_E = L_{EB} V_B \quad (19)$$

Then, from Equations (17) and (18), one can find that:

$$L_{IB} = L_{BI}^{-1} \quad (20)$$

Instead of finding the complex inverse of  $L_{BI}^{-1}$  one can easily use the orthogonal transformation property of the matrix to find the matrix  $L_{IB}$ . In Equation (12), every matrix in



the equation is orthogonal and also the product orthogonal. So, it is way easy to find the inverse of  $L_{BI}$  as follows:

$$L_{BI}^{-1} = L_{IB}^T \quad (21)$$

Then, by using Equations (11), (12), and (13),  $L_{EB}$  which transforms  $F_B$  to  $F_I$  is obtained as following:

$$L_{IB} = \begin{bmatrix} c(\theta) c(\varphi) & s(\phi) s(\theta) c(\varphi) - c(\phi) s(\varphi) & c(\phi) s(\theta) c(\varphi) + s(\phi) s(\varphi) \\ c(\theta) s(\varphi) & s(\phi) s(\theta) s(\varphi) - c(\phi) c(\varphi) & c(\phi) s(\theta) s(\varphi) - c(\phi) c(\varphi) \\ -s(\theta) & s(\phi) c(\theta) & c(\phi) c(\theta) \end{bmatrix} \quad (22)$$

### b. Non-Linear Mathematical Model

Every quad-copter can be described with the aerodynamic equations of non-linear type along with the dynamics of the actuator. The model is represented as follows:

$$\ddot{x} = (\cos\varphi \sin\theta \cos\phi + \sin\varphi \sin\phi) U_1/m \quad (23)$$

$$\ddot{y} = (\sin\varphi \sin\theta \cos\phi + \cos\varphi \sin\phi) U_1/m \quad (24)$$

$$\ddot{z} = (-g + \cos\varphi \cos\theta) U_1/m \quad (25)$$

$$\ddot{\phi} = (I_z - I_y / I_x) \theta\varphi - (U_2/I_x) \quad (26)$$

$$\ddot{\theta} = (I_z - I_x / I_y) \phi\varphi - (U_3/I_y) \quad (27)$$

$$\ddot{\varphi} = (I_x - I_y / I_z) \theta\phi - (U_4/I_z) \quad (28)$$

Where X, Y, and Z are coordinates of the center of mass w.r.t Inertial frame  $\phi$ ,  $\theta$  and  $\varphi$  are the Euler angles p, q and r are the body rates, m in the Quadcopter mass,  $I_x$ ,  $I_y$  and  $I_z$  are the moments of inertia, and  $U_1$ ,  $U_2$ ,  $U_3$  and  $U_4$  are the forces respectively the actuator dynamic equations are be defined as follows for the copter:

$$U_1 = b_1(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (29)$$

$$U_2 = b_1(-\Omega_2^2 + \Omega_4^2) \quad (30)$$

$$U_3 = b_1(-\Omega_1^2 + \Omega_3^2) \quad (31)$$

$$U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \quad (32)$$

Where b is the thrust co-efficient, d is the drag coefficient,  $b_1$  is the radius of the Quadcopter and  $\Omega_i$  is the speed of propeller of motor  $i$ .

### 3. PID AND ADAPTIVE CONTROLLER



### a. PID Controller.

PID controllers are being used in a broad range of applications. The necessary control signals to obtain the desired output are generated based on the error signal which is the difference between the desired value and the obtained value. The general structure of the PID controller is shown below in fig (5).

As the Quadcopter is a MIMO system the employed design strategy includes designing 4 individual PID controllers which are tuned to different gain values to control the attitude (one controller for each angle) and altitude independently as specified below in fig (6). Where the inner loop deals with controlling the required attitude angles and the outer loop make the quadcopter reach the designated altitude.

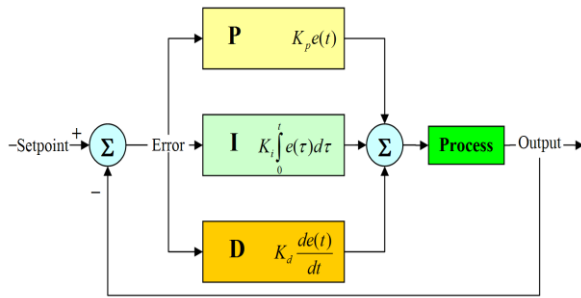


fig (5): General Structure of PID Controller.

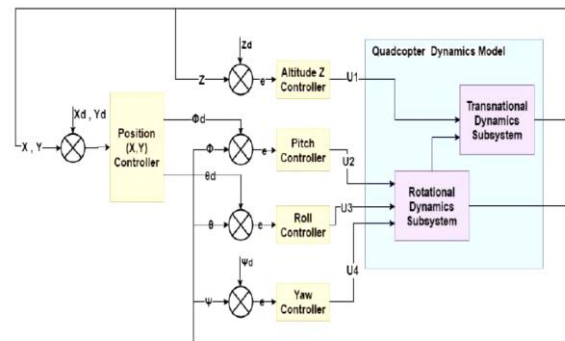


fig (6): Block diagram of FCS of a Quadcopter based on PID.

**Altitude control:**  $U_1$  is the thrust force control variable defined as:

$$U_1 = \left( K_{Pz} e_z + K_{Iz} \int e_z - K_{Dz} \left( \frac{d}{dt} \right) e_z \right) \quad (33)$$

Where  $K_{Pz}$ ,  $K_{Iz}$  and  $K_{Dz}$  are three altitude parameters of the PID controller and,  $e_z = z_{des} - z_{mes}$  is the difference between the measured altitude and the desired altitude called the altitude error.

**Roll control:**  $U_2$  is the roll moment control variable defined as follows:

$$U_2 = \left( K_{P\phi} e_\phi + K_{I\phi} \int e_\phi - K_{D\phi} \left( \frac{d}{dt} \right) e_\phi \right) \quad (34)$$

Where  $K_{P\phi}$ ,  $K_{I\phi}$  and  $K_{D\phi}$  are three roll parameters of the PID controller.  $e_\phi$  is the altitude error defined as  $e_\phi = \phi_{des} - \phi_{mes}$ , which is the difference between desired roll angle and the measured roll angle.

**Pitch control:**  $U_3$  is the pitch moment control variable defined as:



$$U_3 = \left( K_{P\theta} e_\theta + K_{I\theta} \int e_\theta - K_{D\theta} \left( \frac{d}{dt} \right) e_\theta \right) \quad (35)$$

Where  $K_{P\theta}$ ,  $K_{I\theta}$  and  $K_{D\theta}$  are three pitch attitude parameters of the PID controller. And  $e_\theta$  is the attitude error defined as  $e_\theta = \theta_{des} - \theta_{mes}$  is the difference between desired pitch angle and the measured pitch angle.

**Yaw control:**  $U_4$  is the yaw moment control variable defined as:

$$U_4 = \left( K_{P\varphi} e_\varphi + K_{I\varphi} \int e_\varphi - K_{D\varphi} \left( \frac{d}{dt} \right) e_\varphi \right) \quad (36)$$

Where  $K_{P\varphi}$ ,  $K_{I\varphi}$  and  $K_{D\varphi}$  are three attitude parameters of the PID controller.  $e_\varphi$  is the attitude error, defined as  $e_\varphi = \varphi_{des} - \varphi_{mes}$  which is the difference between the desired yaw and the measured yaw. Eq. (23) and (24) are the linear accelerations of the quad-copter in x and y directions respectively:

$$\ddot{x} = (\theta \cos\varphi + \phi \sin\varphi) U_1 / m \quad (37)$$

$$\ddot{y} = (\theta \sin\varphi + \phi \cos\varphi) U_1 / m \quad (38)$$

Now the Equation (37) and (38) are put in matrix notation:

$$[\ddot{x} \ \ddot{y}] = \begin{bmatrix} \sin\varphi & \cos\varphi \\ -\cos\varphi & \sin\varphi \end{bmatrix} \begin{bmatrix} \varphi \\ \theta \end{bmatrix} \quad (39)$$

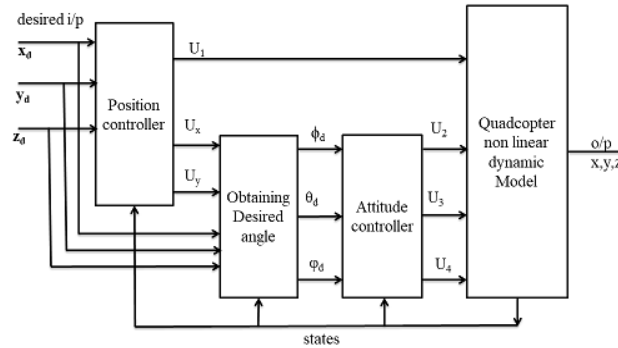
The desired pitch and roll:

$$[\phi_d \ \theta_d] = \frac{U_1}{m} \begin{bmatrix} -\sin\varphi & -\cos\varphi \\ \cos\varphi & -\sin\varphi \end{bmatrix} \quad (40)$$

### ***b. Adaptive Controller using Backstepping method***

Adaptive Backstepping is an effective method based on the Lyapunov approach comprising a recursive and nonlinear design method that is capable of updating the parameters corresponding to quad-copter which is effective to deal with the uncertainties in the system. The main motto of the adaptive backstepping method is to create a recursive law to formulate the stabilizing functions taking some of the state variables as virtual control inputs.

The situations under which the system operates may sometimes make the dynamics and kinematics of the system vary unpredictably over time. In such scenarios, unlike traditional controllers, there is every need to design a control strategy that adapts to the prevailing changes and makes the system behave in a desired manner. Thus, the design of an adaptive controller can be realized by parameter estimation over time.



**fig (7): Adaptive back-stepping controller used for non-linear Quadcopter dynamic model**

### ***Back-stepping controller***

The design of the non-linear backstepping controller specified in this section based on the simplified nonlinear dynamic model of the Quadcopter described in the above section aims to track the desired trajectory that is fed to the Quadcopter. The formulation of the back-stepping controller is attained piecewise by developing attitude and position control laws individually.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{z} \\ \ddot{z} \\ \dot{y} \\ \ddot{y} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_4 x_6 a_1 + b_1 U_2 \\ x_4 \\ x_2 x_6 a_3 + b_2 U_3 \\ x_6 \\ x_4 x_2 a_5 + b_3 U_4 \\ x_8 \\ -g + \frac{\cos(x_1) \cos(x_3) U_1}{m} - K_x \frac{x_{12}}{m} \\ x_{10} \\ U_x \frac{U_1}{m} - K_x \frac{x_8}{m} \\ x_{12} \\ U_y \frac{U_1}{m} - K_y \frac{x_{10}}{m} \end{bmatrix} \quad (41)$$

Where,

$$a_1 = \frac{I_x - I_z}{I_x}; \quad a_3 = \frac{I_z - I_x}{I_y}; \quad a_5 = \frac{I_x - I_y}{I_z}$$



$$b_1 = \frac{d}{I_x}; \quad b_2 = \frac{d}{I_y}; \quad b_3 = \frac{d}{I_z}$$

in addition,  $U_1, U_2, U_3$  and  $U_4$  and  $K_x, K_y, K_z$  represents the control inputs and aerodynamic drag coefficients for translation motion respectively.

### Step 1: Computing desired angles

This section deals with the estimation of the desired Euler angles, inputs of the attitude controller  $x_{1d}$ ,  $x_{3d}$  and  $x_{5d}$  which specifies the attitude of the quadcopter which is to maintain to drive towards and reach the destination are obtained. By observing the orientation specified in fig 8 and applying the basic trigonometric formulae the desired yaw angle ( $x_{5d}$ ) is obtained as:

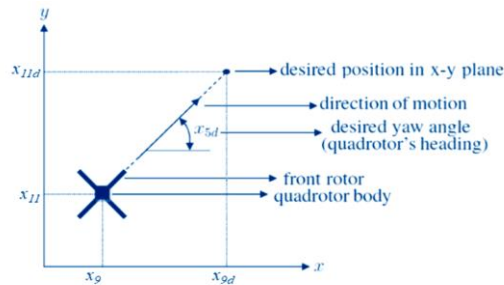


fig (8): visualization in XY plane

$$x_{5d} = \arctan \left[ \frac{x_{11d} - x_{11}}{x_{9d} - x_9} \right] \quad (42)$$

Here, the actual position in the  $XY$  plane is represented as  $x_9$  and  $x_{11}$ . Whereas, the desired position in the  $x$ - $y$  plane which is nothing but the next position in the given trajectory is represented as  $x_{9d}$  and  $x_{11d}$ . After this, the desired roll angle  $x_{3d}$  and desired pitch angle  $x_{1d}$  angles are obtained as shown below:

$$U_x = (\cos(x_1) \sin(x_3) \cos(x_5) + \sin(x_1) \cos(x_5)) \quad (43)$$

$$U_y = (\cos(x_1) \sin(x_3) \sin(x_5) + \sin(x_1) \cos(x_5)) \quad (44)$$

By performing necessary trigonometric and algebraic operations the desired roll angle  $x_{1d}$  is defined as:

$$x_{1d} = a \tan 2(\sin(\phi_c) \cos(\phi_c)) \quad (45)$$

The desired pitch angle  $x_{3d}$  is also achieved by following a similar strategy:

$$x_{3d} = a \tan 2(\sin(\theta_c) \cos(\theta_c)) \quad (46)$$



## Step2: Attitude control

The control inputs  $U_2 U_3 U_4$  used to control the attitude of the quadcopter are formulated in this section by choosing appropriate Lyapunov function which is itself a positive definite and consisting of first-order derivatives of time and the error terms are made to converge to zero based on the Lyapunov theory.

The first error term for “Euler angle”  $\phi(x_1)$  is written as:

$$z_1 = x_{1d} - x_1 \quad (47)$$

The corresponding Lyapunov function can be written as:

$$V(z_1) = \left(\frac{1}{2}\right)z_1^2 \quad (48)$$

The first-order time derivative of  $V(z_1)$  resulted in:

$$\dot{V} = z_1 \dot{z}_1 = z_1(x_{1d} - \dot{x}_1) \quad (49)$$

By substituting equation (41),

$$V(\dot{z}_1) = z_1(x_{1d} - \dot{x}_1) \quad (50)$$

Here, if  $x_2$  is taken as  $x_2 = x_{1d} + c_1 z_1$  then for  $c_1 > 0$ ,  $\dot{V}(z_1)$  is a negative semi-definite. From this,  $x_{2d}$  is chosen as follows:

$$x_{2d} = \dot{x}_{1d} + c_1 z_1 \quad \text{for } c_1 > 0 \quad (51)$$

It can be understood that, desired  $\phi(x_{2d})$  can be obtained by controlling  $x_2$  ( $\phi$ ). Therefore, to obtain desired  $x_2(x_{2d})$  another error term is needed that converges to zero.

$$z_2 = x_{2d} - x_2 \quad (52)$$

To obtain  $x_{1d}$  and  $x_{2d}$ , both the error terms  $z_1$  and  $z_2$  should converge to zero. To achieve this, another Lyapunov function is considered,

$$\dot{V}(z_1, z_2) = \left(\frac{1}{2}\right)(z_1^2 + z_2^2) \quad (53)$$

Again, the first-time derivative of equation (48) is obtained as:

$$V(z_1, z_2) = z_1 \dot{z}_1 + z_2 \dot{z}_2 \quad (54)$$

By utilizing equations (46) and (47),  $z_2$  is rewritten as:

$$z_2 = \dot{x}_{1d} + c_1 z_1 - x_2 \quad (55)$$



from equation (45),  $z_2$  becomes:

$$z_2 = \dot{z}_1 + c_1 z_1 \quad (56)$$

Applying time derivative to equation (50),  $\dot{z}_2$  is obtained as:

$$\dot{z}_2 = \ddot{x}_{1d} + \dot{c}_1 - \dot{x}_2 \quad (57)$$

By replacing equations (51) and (52) into equation (49),  $\dot{V}(z_1, z_2)$  is obtained as:

$$\dot{V}(z_1, z_2) = z_1(z_2 - c_1 z_1) + z_2(\ddot{x}_{1d} + c_1 z_1 - \dot{x}_2) \quad (58)$$

Substituting the value of  $\dot{x}_2$  specified in equation (41), the final form of  $\dot{V}(z_1, z_2)$  is:

$$\dot{V}(z_1, z_2) = z_1 z_2 - c_1 z_1 + z_2 \ddot{x}_{1d} + z_2 c_1 - c_1 z_1 z_2 - z_2 x_4 x_6 a_1 - z_2 b_1 U_2 \quad (59)$$

$U_2$  in the above equation is one of the required control inputs. By employing Lyapunov theory and neglecting the least significant terms like  $\ddot{x}_{1d}$  control input  $U_2$  is written as:

$$U_2 = [(b_1)(\ddot{x}_{1d} + c_1 z_1 d - c_2 z_2 + z_1 - a_1 x_4 x_6)] \quad (60)$$

In a similar approach control inputs  $U_3$  and  $U_4$  can be also defined like  $U_2$  as below:

$$U_3 = \left[ \left( \frac{1}{b_2} \right) (\ddot{x}_{3d} + c_3 z_3 d - c_4 z_4 + z_3 - a_2 x_2 x_6) \right] \quad (61)$$

$$U_4 = \left[ \left( \frac{1}{b_1} \right) (\ddot{x}_{5d} + c_5 z_5 d - c_6 z_6 + z_5 - a_3 x_2 x_4) \right] \quad (62)$$

Again, to obtain  $U_3$  and  $U_4$ ,  $\ddot{x}_{3d}$  and  $\ddot{x}_{5d}$  are also considered zero and the involved error terms are specified below:

$$z_3 = x_{3d} - x_3 \quad (63)$$

$$z_4 = x_{4d} - x_4 \quad (64)$$

$$z_5 = x_{5d} - x_5 \quad (65)$$

$$z_6 = x_{6d} - x_6 \quad (66)$$

Where,  $x_{4d}$  and  $x_{6d}$  represents  $\theta$  and  $\varphi$  respectively and considered as negative semi-definite.

$$x_{4d} = \dot{x}_{3d} + c_3 z_3 \quad (67)$$

$$x_{6d} = \dot{x}_{5d} + c_5 z_5 \quad (68)$$



### Step 3: Position Control

Here, the control  $U_1$  which is responsible for the translation motion of the quadcopter in the  $x$ ,  $y$ , and  $z$  directions is obtained by following the same methodology. It is to be noted that motion in  $x$  and  $y$  directions are also affected by  $U_x$  and  $U_y$ , respectively.

$U_1$  is obtained following the Lyapunov method by making the error between desired and actual altitude values converge to zero. The error in altitude is defined as:

$$z_7 = x_{7d} - x_7 \quad (69)$$

The appropriate Lyapunov function  $V(z_7) = \left(\frac{1}{2}\right)z_7^2$  is chosen. The first-order time derivatives, substitutions, and choice of error terms are performed in the same way as defined in step 2 i.e. attitude control. After performing all the necessary operations  $U_1$  is obtained as,

$$U_1 = \left[ \left( \frac{1}{\left(\frac{1}{m}\right) \cos(x_1) \cos(x_3)} \right) (-c_8 z_8 + g + x_{7d}'' + c_7 \dot{z}_7 + z_7) \right] \text{ for } c_7 > 0 ; c_7 > 0 . \quad (70)$$

To obtain better performance of the controlled, the second-order derivative of the desired altitude ( $x_{7d}''$ ) is made to influence the control command.

In a similar approach control inputs  $U_x$  and  $U_y$  can be also derived from  $U_2$  as shown below:

$$U_x = \left[ \left( \frac{m}{U_1} \right) \left( z_9 + x_{9d}'' + c_9 z_{10} - c_9^2 z_9 + \frac{K_x x_{10}}{m} + c_{10} z_{10} \right) \right] \text{ for } c_9 > 0 ; c_{10} > 0 . \quad (71)$$

$$U_y = \left[ \left( \frac{m}{U_1} \right) \left( z_{11} + x_{11d}'' + c_{11} z_{12} - c_{11}^2 z_{11} + \frac{K_y x_{12}}{m} + c_{12} z_{12} \right) \right] \text{ for } c_{11} > 0 ; c_{12} > 0 \quad (72)$$

The error terms in the above equations are defined as:

$$z_9 = x_{9d} - x_9 \quad (73)$$

$$z_{10} = x_{10d} - x_{10} \quad (74)$$

$$z_{11} = x_{11d} - x_{11} \quad (75)$$

$$z_{12} = x_{12d} - x_{12} \quad (76)$$

The terms  $x_{7d}$  (desired  $z$ ),  $x_{9d}$  (desired  $x$ ) and  $x_{11d}$  (desired  $y$ ) are nothing but the inputs (desired or next location of the quadcopter) given to define the trajectory.

$$x_{10d} = x_{9d} + c_9 \dot{z}_9 \quad \text{for } c_9 > 0 \quad (77)$$

$$x_{12d} = x_{11d} + c_{11} z_{11} \quad \text{for } c_{11} > 0 \quad (78)$$



the functions  $U_x$  and  $U_y$  are used to find desired Euler angles  $x_{1d}$  and  $x_{3d}$  which are fed as inputs to the attitude controller.

**Table 1: Optimized control parameters of Adaptive backstepping**

<i>Control constant</i>	<i>Value</i>
$c_1$	100
$c_2$	100
$c_3$	100
$c_4$	100
$c_5$	100
$c_6$	100
$c_7$	3
$c_8$	3
$c_9$	10
$c_{10}$	10
$c_{11}$	10
$c_{12}$	10

**Table 2: Optimized control gains of Adaptive backstepping**

Control gain parameter	$K_{dx}$	$K_{px}$	$K_{dy}$	$K_{py}$
Values	8	10	8	6

## TRAJECTORY & WIND DISTURBANCE GENERATION

### A. Trajectory generation

The trajectory is the locus of the points, which represents the position of an object traced over time. Trajectory provides the information of starting, ending points, and also the position of the object at a specific instant of time. To make the quadcopter move autonomously from one location to other the information of the path to be followed is fed in the form of waypoints which are defined using any of the reference frames.

### Different ways to define trajectories

In this project, the path in which the Quadcopter has to travel is expressed using two different ways as specified below.

#### 1) Function-based

In this method, the trajectory is developed by writing a MATLAB code using trigonometric functions sine and cosine. Using this approach two trajectories are generated to mimic the patterns employed for surveillance and construction monitoring i.e. circular and spiral.

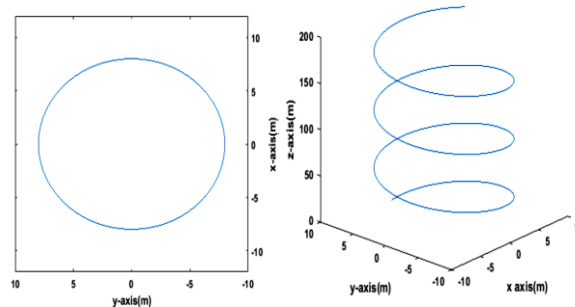


fig (9): Circle & Spiral trajectory.

## 2) Position profile based on time

In real-time Quadcopter has to follow the trajectories that are specified in terms of waypoints (position concerning time) which are generally not smooth. The below shown complex trajectories are used in this project to analyze the performance of the controller while taking turns. To generate these trajectories path planning algorithms like A\* have been used which can generate an optimum path that is free from an obstacle in the given environment. The working of the path planning algorithm is beyond the scope of this project work as we have used only the obtained paths. Out of the wide range of available paths, the below shown in fig (10) was chosen to know the behavior of the controller while taking sharp turns [14]

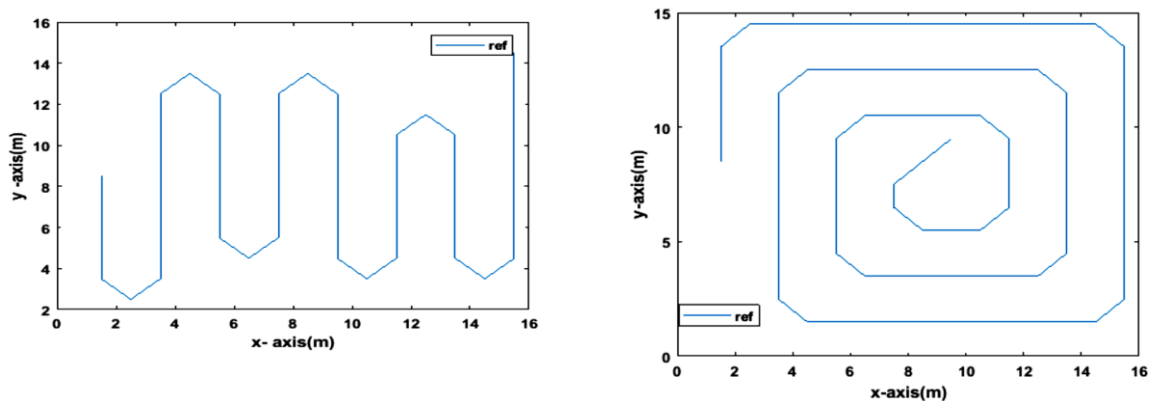
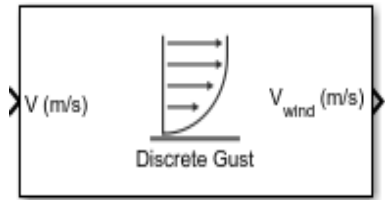


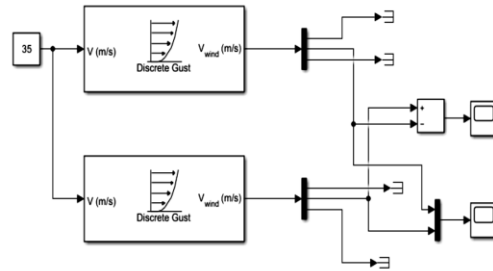
Fig (10): Zig-Zag & Converging Position profile based on the standard search patterns.

## B. Wind disturbance

The Discrete Wind Gust Model block implements the mathematical representation of gust as specified in the Military Specification MIL-F-8785C. This block generates wind gusts of constant velocity continuously, using this as it is will not fulfill our project requirement. To generate wind disturbance for a specific duration with the desired velocity a Simulink model is designed as shown in fig (12) producing 2 types of wind disturbances.

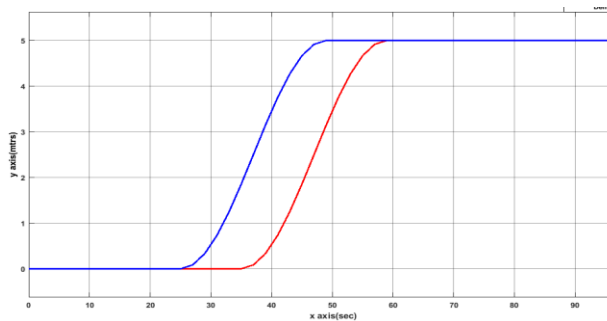


**fig (11):** wind gust block in Simulink.

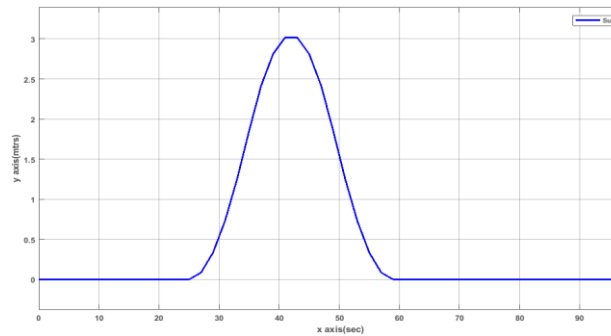


**fig (12):** Simulink diagram for wind disturbance generation.

The obtained wind disturbances with different slopes and different gust intervals specified in fig(13-b) & (14-b) are termed type-1 and type-2 wind disturbances. Different gust intervals for the two disturbances allow them to influence the quadcopter position at different instants. This provides flexibility for the designer to make them influence at his point of interest and thereby evaluating the controller performance accordingly.

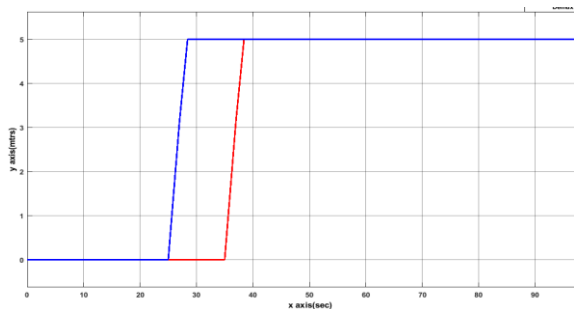


**(a) i/p of wind gusts initial at different time**

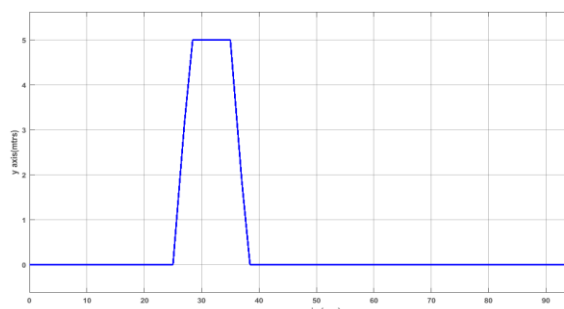


**(b) o/p of wind disturbance**

**fig (13):** type 1 wind gust



**a) i/p of wind gusts initiated at different time instant time.**



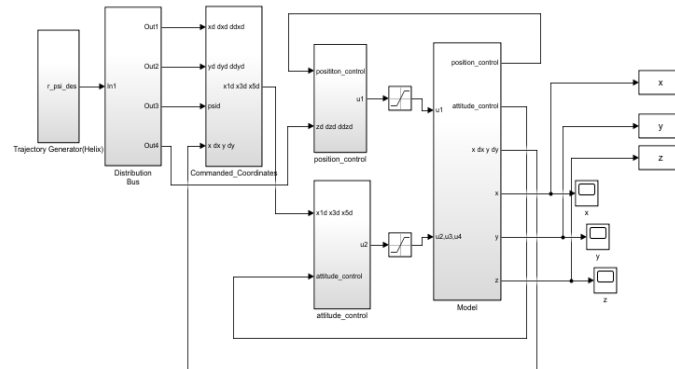
**b) o/p of wind disturbance**

**Fig (14):** type 2 wind gust



## 4. RESULTS AND DISCUSSIONS

The entire design of the Flight control system and the trajectories of the quadcopter along with wind disturbances is carried out in a MATLAB Simulink environment.

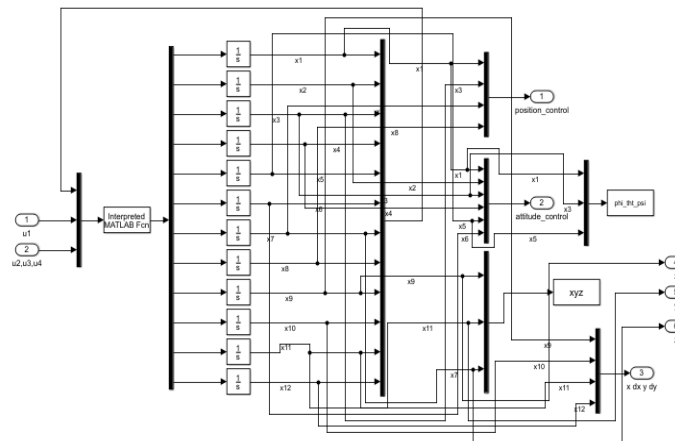


**Fig (15): Simulink model of Adaptive Backstepping Controller for Quadcopter.**

The abovefig provides a top-levelview of blocks corresponding to the model, controller trajectory input, and their interconnections. Here trajectory generator block acquires input trajectory from the workspace. In the next stage, desired attitudes are calculated based on the desired positions. Later these are fed to the controller section where the control inputs  $U_1, U_2, U_3$  and  $U_4$  are generated and given as input to the model. Finally, the position of the quadcopter is carried into the workspace by using “to workspace” blocks named  $x, y,$  and  $z$ .

### A. Model-Based Design

Model-based design enables quick and cost-effective development of dynamic systems like control systems, signal processing, and communications systems, here a development life cycle is considered for system design.



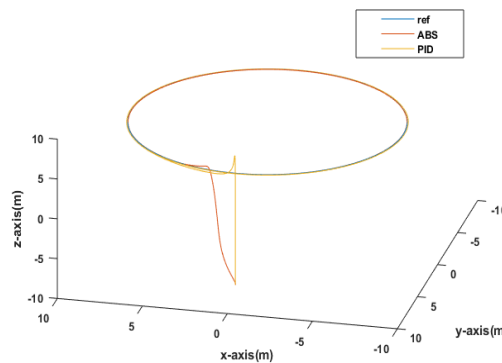
**Fig (16): Simulink model for Quadcopter.**



The control inputs are  $U_1, U_2, U_3$  and  $U_4$  assigned in a script then applied to Simulink using functional blocks the obtained outputs are the states of the Quadcopter along with these derivatives of first & second order corresponding to position  $[x, y, z]$ , velocities  $[d_x, d_y, d_z]$ , angles  $[\phi, \theta, \varphi]$  and angular velocities  $[\dot{\phi}, \dot{\theta}, \dot{\varphi}]$ . Some of the outputs are taken as feedback to provide as an input to the controller.

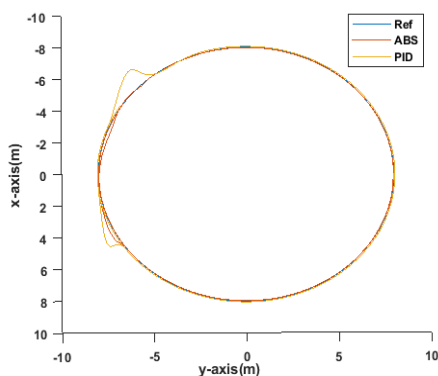
### B. Comparison between Adaptive backstepping and PID controller

This section mainly concentrates on providing the comparison between the PID and Adaptive backstepping controller while tracking the given trajectory. The reference and the followed trajectories are generated using the variables exported to the workspace from Simulink.

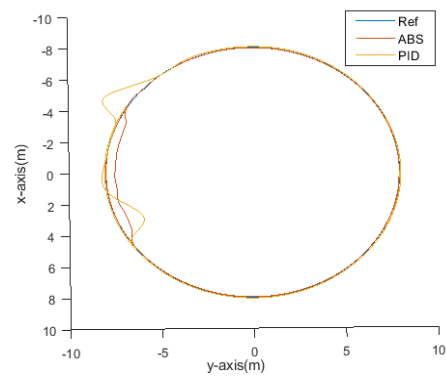


**Fig (17): Circular trajectory followed using PID and back-stepping without wind disturbance**

The above fig shows the reference trajectory and trajectory followed by the quadcopter by using PID and backstepping controller without the influence of wind gust. Here the reference trajectory is similar to the circular trajectory it is evident that using PID, the time required for the quadcopter to reach the given trajectory is less compared to backstepping but the initial deviations from the trajectory are less in case of backstepping.



**Fig (18) Circular trajectory followed for scenario 1 wind disturbance**

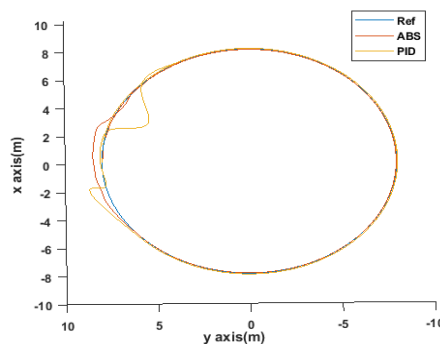


**Fig (19): Circular trajectory followed for scenario 2 wind disturbance**



The above fig shows that obtained trajectories followed by Quadcopter in the presence of wind disturbance specified in scenario1. The wind disturbance is made to effect during the simulation time of 25 to 35 sec. It is observed that the deviation from the reference trajectory is more in the case of the PID Controller.

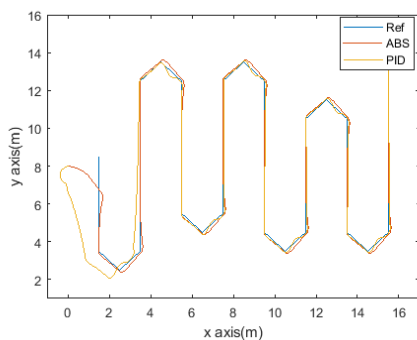
The above fig(19) shows that the wind is made to affect the velocity of the quad in the y-axis; the deviations when compared to fig (17) are notably high in fig (19).



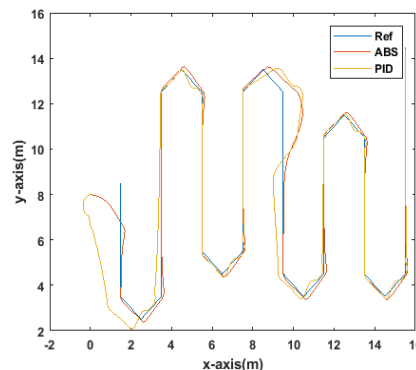
**Fig (20): Circular trajectory followed for Scenario 3 wind disturbance**

For scenario 2 wind disturbances the effects are shown in Fig (20). The wind disturbance is made to effect in both X & Y axis but the obtained the result is shown that the PID deviation is more and steeling time is less than the adaptive controller. This provides sufficient evidence to comment that the Quadcopter is less prone to wind disturbance than those effects in the X-direction.

### Position profile1 based trajectory:



**Fig (21): Comparison of position profile1 based trajectory followed without wind disturbance**



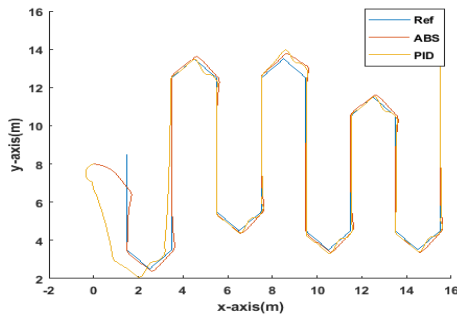
**Fig (22): Comparison of position Profile1 based trajectory followed for Scenario 1 wind disturbance**

The initial disturbance observed in fig (21) is due to the mismatch of the initial positions of the Quadcopter and the starting point of the trajectory. Despite the gap, the Adaptive backstepping (ABS) controller is making Quadcopter get into the reference trajectory earlier

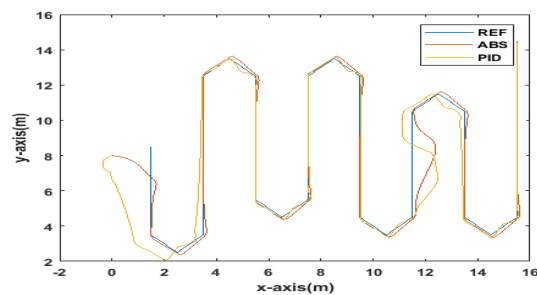


than compared to PID. Also, where ever sharp turning is encountered in the path the Quadcopter is expressing a drift from the reference trajectory.

Taking the shift encountered in fig (21) into consideration, further analysis is carried out by making the wind disturbance effect while the Quadcopter is taking a turn. The obtained result is shown in fig (22) demonstrates the similar entities of earlier.



**Fig (23): Comparison of position profile 1 based trajectory followed for Scenario 2 wind disturbance**

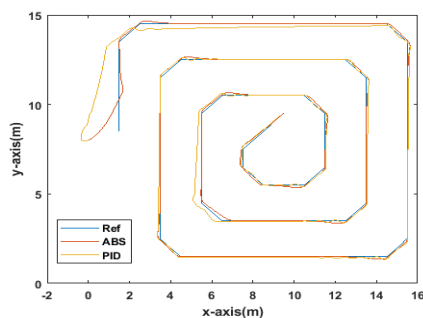


**Fig (24): Comparison of position profile 1 based trajectory followed for scenario 3 wind disturbance**

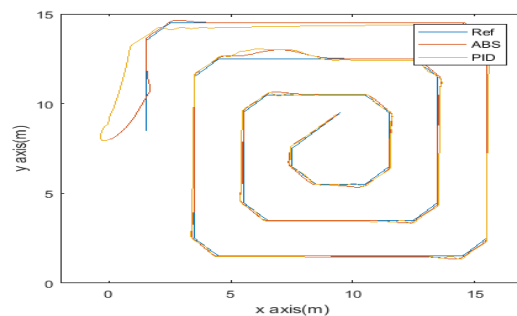
The same type of wind is applied but the disturbance of scenario 2 fig (23) shows that the influence of wind in y-direction on the quadcopter is less when compared to the x-direction. The shows that the controller is compensating for the effect of wind in a better fashion utilizing position control.

Fig (24) shows the obtained results when scenario 3 wind disturbance is applied during the simulation time from 71 to 74 sec. Keeping the result shows in fig (22) and (23) in mind. The wind disturbance applied at time instant where the motion of quadcopter is in a straight line here also deviations in PID is more than backstepping.

### **Position profile 2 based trajectory:**



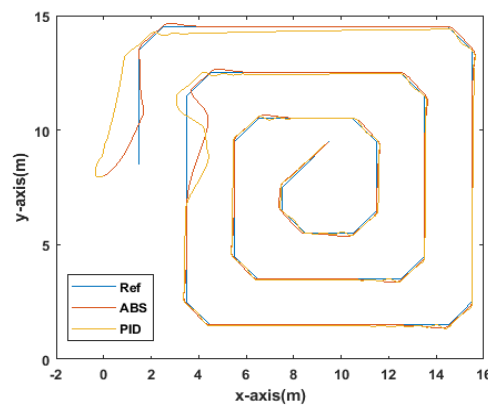
**Fig (25): Comparison of position profile 2 based trajectory followed for scenario 1 wind disturbance**



**Fig (26): Comparison of position profile 2 based trajectory followed for scenario 2 wind disturbance**



The choice of this type of trajectories is made due to the reason that these types of trajectories are specified as the standard search patterns. From Fig (25), it is evident that the PID controller is taking a huge time to be in line with the given trajectory. The situation is even the same with the wind disturbance also. Here the wind disturbance is made to effect during 51 to 54 sec. It is to be noted that a small grid size is chosen to general sharp turning to analyze the controller performance.



**Fig (27): Comparison of position profile2 based trajectory followed for scenario3 wind disturbance**

Consolidating all the results obtained after applying different wind disturbances specified in scenario1, 2 & 3 to different trajectories at different instants of time. The influence of wind disturbance is more on Quadcopter during maneuvering. And the Adaptive backstepping controller is making Quadcopter to be in the reference trajectory as clearly as possible.

## 5. Conclusion

In this paper, the working of quadcopter and mathematical equations related to the dynamics of the quadcopter are presented. Simulink model depicting the dynamics of the quadcopter is designed based on the obtained mathematical relations. An automatic flight control system designed using PID and Adaptive Backstepping techniques. Performance evaluation of these controllers is carried out feeding different trajectories obtained using two different approaches (using trigonometric functions and position profile) in the presence of wind. The wind disturbance is designed using the standard discrete wind gust model provided in the Simulink library, disturbance is made to affect the velocity of the quadcopter in both X, Y-axis, and XY axis at different instants of time. Reference trajectory and the deviations that occurred while following it are presented. Based on the simulation results it is observed that the deviations from the reference trajectory are less using Adaptive Backstepping control than PID. Hence it is concluded that Adaptive backstepping control is performing better in case of wind disturbances.



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