Power System Technology
ISSN:1000-3673

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

An Estimate of the Rate of Convergence for the Absolute Summability of Factors of Infinite Series

Suresh Kumar Sahani[1] , Avinash Kumar[2*], [3]Ravi Prakash Dubey
[1]Department of Science and Technology
Rajarshi Janak University, Janakpurdham, Nepal
Email: sureshkumarsahani35@gmail.com[1],
[2]Department of Mathematics
Dr. C.V. Raman University. Bilaspur, India
Email: avinashrathor540@gmail.com[2*]
[3]Department of Mathematics
Dr. C.V. Raman University, Bilaspur, India
Email: drraviprakashdubey@gmail.com[3]

Abstract:

Here in, we provide evidence of three theorems about a special case of the absolute summing-up-factors of infinite series using much less stringent conditions. Some particular results on various absolute summability approaches also been produced. The papers [7], [21], and [22] serve inspirations for our work.

Keywords: absolute matrix summability, summability factors, infinite series.

MSC: 40D15, 40F05, 40G99, 42A24

Introduction:

Let $\{p_\ell\}$ be a sequence of non-zero constants, real or complex with R_ℓ as its non-vanishing ℓ^{th} partial sum much that $R_\ell = p_0 + p_1 + ... + p_\ell \to \infty$ as $\ell \to \infty$ where $R_{-1} = p_{-1} = 0$ (1)

Let V_ℓ denote the ℓ^{th} $(\overline{N}, p_\ell)$ means of the sequence $\{\xi_\ell\}$ of partial sums of a given infinite series $\sum u_\ell$ defined as

$$V_\ell = \frac{1}{R_\ell}\sum_{g=0}^{\ell} p_v \, \xi_\ell \tag{2}$$

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

following author [1], we defined that the series $\sum u_\ell$ is summable $\left|\overline{N}, p_\ell\right|_m$ where $\ell = 1, 2, 3, ...$

if $\sum_{\ell=1}^{\infty}\left(\frac{R_\ell}{p_\ell}\right)^{m-1}\left|V_\ell - V_{\ell-1}\right|^m < \infty$ (3)

and it is summable $\left|\overline{N}, p_\ell; \delta\right|_m$

where δ is a non-negative real number and $k \geq 1$ if (see [2]-[7])

$$\sum_{\ell=1}^{\infty}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1} .\left|V_\ell - V_{\ell-1}\right|^m < \infty \tag{4}$$

(1) If $p_\ell = 1 \forall \ell$ then $\left|\overline{N}, p_\ell\right|_m$ is same as $\left|C, 1\right|_m$.

(2) If $m = 1$ then $\left[\overline{N}, p_\ell\right]_m$ boundedness is same as $\left[\overline{N}, p_\ell\right]$ boundedness

(3) If $\delta = 0$ and $m = 1, \left|\overline{N}, p_\ell; \delta\right|_m$ is same as $\left|\overline{N}, p_\ell\right|_m$ where $m = 1$

(4) If $p_\ell = \frac{1}{\ell + 1}, m = 1$ and $\delta = 0$ then $\left|\overline{N}, p_\ell; \delta\right|_m$ is equivalent to $\left|R, \log \ell, 1\right|$.

Many works dealing with matrix summability of Legendre series, Nȯrlund series, Fourier-Laguerre series,

Borel summability, and ultraspherical series have been done in [24-33].

Known results:

Some works on absolute summability and absolute matrix summability of infinite series and Fourier series have been done [8, 10, 11-20]. In 1972, author [9] proved the following theorem on $\left|C, 1\right|_m$ summability factors of an infinite series.

Theorem A: If $\{\gamma_1, \gamma_2, .., \gamma_\ell, ...\}$ is a convex sequence such that

$\frac{\lambda_1}{1} + \frac{\lambda_2}{2} + ... + \frac{\lambda_\ell}{\ell} + ... < \infty$, and

Power System Technology
ISSN:1000-3673
Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$$\sum_{g=1}^{\ell}\frac{\left|\xi_g\right|^m}{g}=O(\log\ell),(m\geq1) \quad (5)$$

then the infinite series $\sum u_\ell\gamma_\ell$ is summable $|C,1|_m$.

Theorem B: Let $\{\zeta_1,\zeta_2,...,\zeta_\ell,...\}$ be a convex sequence such that $\sum\frac{\zeta_\ell}{R_\ell}$ is bounded.

Let $\{p_\ell,p_2,...,p_\ell,...\}$ be positive, monotonic decreasing sequence of constants real or complex such that $R_\ell=p_1+p_2+...+p_\ell\to\infty$ as $\ell\to\infty$.

Let $\{\chi_1,\chi_2,...,\chi_\ell,...\}$ be positive increasing sequence and $M(z)$ be a positive non-decreasing function of z such that

$$\sum_{g=1}^{\ell}R_g\chi_g\zeta_gM(R_g).\Delta^2\left(\frac{1}{\chi_g}\right)=O(1) \quad (6)$$

and for $\ell\to\infty$,

if $\sum_{g=1}^{\ell}\frac{\left|\xi_g\right|}{R_g}=O(\chi_\ell M(R_\ell))$ (7)

as $\ell\to\infty$, then the infinite series $\sum\frac{\zeta_\ell u_\ell}{\chi_\ell}$ is summable

$\left|\overline{N},p_\ell\right|_m, m\geq1$.

Main theorem:

Our aim is to generalized the above theorem by establishing the following theorem.

Theorem 1: Let $\{\zeta_1,\zeta_2,...,\zeta_\ell,...\}$ be a convex sequence such that $\sum\frac{\zeta_\ell}{R_\ell}$ is bounded.

Let $\{p_1,p_2,...,p_\ell,...\}$ be a positive monotonic decreasing sequence of constants and

$R_\ell=p_1+p_2+...+p_\ell\to\infty$ as $\ell\to\infty$ satisfying the condition

Power System Technology
ISSN:1000-3673

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$$R_\ell = O(\ell\, p_\ell), \text{ as } \ell \to \infty. \tag{8}$$

Let $\{\chi_1, \chi_2, ..., \chi_\ell, ...\}$ be a positive increasing sequence and $M(z)$ be a positive non decreasing function of z such that

$$\zeta_\ell M(R_\ell) = O(1) \tag{9}$$

$$\sum_{g=1}^{\ell} g M(R_g) \left|\Delta^2 \zeta_g\right| = O(1) \tag{10}$$

and $\sum_{g=1}^{\ell} g \left|\zeta_g\right| \left|\chi_g M(R_g)\right| . \left|\Delta^2\left(\frac{1}{\chi_g}\right)\right| = O(1)$, as $\ell \to \infty$ (11)

if $a_\ell = \frac{1}{\ell+1} \sum_{g=1}^{\ell} g\, b_g$ (12)

$$\sum_{g=1}^{\ell} \left(\frac{R_g}{p_\ell}\right)^{\delta m - 1} \left|a_g\right|^m = O\left[\chi_\ell M(R_\ell)\right], \text{ as } \ell \to \infty$$

(13)

and $\sum_{\ell=g>1}^{\infty} \left(\frac{R_g}{p_\ell}\right)^{\delta m-1} . \frac{1}{R_\ell^{-1}} = O\left[\left(\frac{R_g}{p_g}\right)^{\delta m-1} . \frac{1}{p_g}\right]$ then the infinite series $\sum \frac{u_\ell \zeta_\ell}{\chi_\ell}$ is summable $\left|\overline{N}, p_\ell; \delta\right|_m$ for $m \geq 1, \delta \geq 0$.

Lemma: Under the conditions on (8) and (9) of the our main theorem, we have

$$\sum_{\ell=1}^{k} \left|\zeta_\ell\right| M(R_\ell) = O(1), \ell \to \infty \tag{14}$$

$$\sum_{\ell=1}^{\infty} M(R_\ell) \left|\Delta \zeta_\ell\right| < \infty \tag{15}$$

$$\sum_{\ell=1}^{\infty} \ell\, M(R_\ell) \left|\Delta \zeta_\ell\right| = O(1) \text{ as } \ell \to \infty \tag{16}$$

Power System Technology
ISSN:1000-3673

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$KM\left(R_k\right)\zeta_k = O(1)$ as $\ell \to \infty$
(17)

$$\sum_{\ell=1}^{k} \ell\, \zeta_\ell M\left(R_\ell\right) = O(1) \text{ as } \ell \to \infty \tag{18}$$

Proof of the theorem 1:

Let $\{\sigma_\ell\}$ be the sequence of $\left(\overline{N}, p_\ell\right)$ means of the infinite series $\sum \frac{b_\ell \zeta_\ell}{\chi_\ell}$.

Then by definition, we have

$$\sigma_\ell = \frac{1}{R_\ell}\sum_{\theta=0}^{\ell} p_\theta \sum_{\alpha=0}^{\theta} \frac{u_\alpha \zeta_\alpha}{\chi_\alpha}$$

$$= \frac{1}{R_\ell}\sum_{\theta=0}^{\ell}\left(R_\theta - R_{\theta-1}\right)\frac{\zeta_\theta u_\theta}{\chi_\theta} \tag{19}$$

Then for $\ell \geq 1$, we get

$$\sigma_\ell - \sigma_{\ell-1} = \frac{p_\ell}{R_\ell R_{\ell-1}}\sum_{\theta=1}^{\ell} R_{\theta-1}\left(\frac{\zeta_\theta u_\theta}{\chi_\theta}\right),$$

by applying Abel's transformation

$$\sigma_\ell - \sigma_{\ell-1} = \frac{p_\ell}{R_\ell R_{\ell-1}}\sum_{\theta=1}^{\ell-1}\Delta\frac{\left(R_{\theta-1}\zeta_\theta\right)}{\theta.\chi_\theta}\,.\sum_{\alpha=1}^{\theta}\alpha\, u_\alpha + \frac{p_\ell \zeta_\ell}{\ell R_\ell \chi_\ell}.\sum_{\theta=1}^{\ell}\theta\, u_\theta$$

$$= \frac{(\ell+1)\, p_\ell a_\ell \zeta_\ell}{\ell\, R_\ell \chi_\ell} - \frac{p_\ell}{R_\ell R_{\ell-1}}.\left\{\frac{p_\theta a_\theta \zeta_\theta}{\chi_\theta}.\frac{\theta+1}{\theta}\right\} + \frac{p_\ell}{R_\ell R_{\ell-1}}\sum_{\theta=1}^{\ell-1} R_\ell \Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right).\frac{\theta+1}{\theta}.a_\theta + \sum_{\theta=1}^{\ell-1} R_\ell \frac{\zeta_{\theta+1}}{\chi_{\theta+1}} a_\theta \frac{1}{\theta}$$

$$= \sigma_{\ell,1} + \sigma_{\ell,2} + \sigma_{\ell,3} + \sigma_{\ell,4}, \qquad \text{(using (12)).} \tag{20}$$

We have to show that

$\sum_{\ell=1}^{\infty}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1}.\left|\sigma_{\ell,j}\right|^m < \infty$ (Using the help of Murkowski's inequal it's for $j = 1,2,3$ and 4).
(21)

Power System Technology
ISSN:1000-3673

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

Now, we first consider $\sigma_{\ell,1}$, we have

$$\sum_{\ell=1}^{r}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1}.\left|\delta_{\ell,1}\right|^m = O(1)\sum_{\ell=1}^{r}\left|\frac{\zeta_\ell}{\chi_\ell}\right|^{m-1}.\left|\frac{\zeta_\ell}{\chi_\ell}\right|.\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\left|a_\ell\right|^m$$

$$= O(1)\sum_{\ell=1}^{r}\left|\frac{\zeta_\ell}{\chi_\ell}\right|\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\left|a_\ell\right|^m$$

$$= O(1)\sum_{\ell=1}^{r-1}\Delta\left(\left|\frac{\zeta_\ell}{\chi_\ell}\right|\right)\sum_{\theta=1}^{\ell}\left(\frac{R_\theta}{p_\theta}\right)^{\delta m-1}.\left|a_\theta\right|^m + O(1)\left|\frac{\zeta_\ell}{\chi_\ell}\right|.\sum_{\ell=1}^{r}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\left|a_\ell\right|^m$$

$$= O(1)\sum_{\ell=1}^{r-1}\left|\Delta\left(\frac{\zeta_\ell}{\chi_\ell}\right)\right|\chi_\ell M\left(R_\ell\right) + O(1)\left|\frac{\zeta_r}{\chi_r}\right|\chi_r M\left(R_r\right)$$

$$= O(1)\sum_{\ell=1}^{r}\left|\Delta\left(\zeta_\ell\right)\right| M\left(R_\ell\right) + O(1)\sum_{\ell=1}^{r}\left|\zeta_\ell\right| M\left(R_\ell\right) + O(1)\left|\zeta_r\right| M\left(R_r\right)$$

$= O(1)$, as $r \to \infty$ (using the conditions (9), (13), (14), (15) (22)

and Abel's transformation (see [21, 22]).

For $\sigma_{\ell,2}$,

Applying Holder's inequality, including indices m and m^1,

$\frac{1}{m}+\frac{1}{m^1}=1$, we have

$$\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1}.\left|\sigma_{\ell,2}\right|^m = O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}.\left\{\sum_{\theta=1}^{\ell-1}p_\theta\left|a_\theta\right|^m.\left|\frac{\zeta_\theta}{\chi_\theta}\right|^m.\sum_{\theta=1}^{\ell-1}p_v\right\}^{m-1} \text{ (See [21, 22])}$$

$$= O(1)\sum_{\theta=1}^{r}p_\theta\left|a_\theta\right|^m\left|\frac{\zeta_\theta}{\chi_\theta}\right|^{m-1}.\left|\frac{\zeta_\theta}{\chi_\theta}\right|\sum_{\ell=\theta+1}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}$$

$$= O(1)\sum_{\theta=1}^{r}\left|\frac{\zeta_\theta}{\chi_\theta}\right|\left(\frac{R_\theta}{p_\theta}\right)^{\delta m-1}.\left|a_\theta\right|^m$$

Power System Technology
ISSN:1000-3673

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$=O(1)$, as $r\to\infty$ (using the conditions of our theorem and lemma). (23)

Further, we consider $\sigma_{\ell,3}$,

$$\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1}.\left|\sigma_{\ell,3}\right|^m=O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}^m}\left\{\sum_{\theta=1}^{\ell-1}p_\theta\left|\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right)\right|\left|a_\theta\right|\right\}^m$$

$$=O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}^m}.\left\{\sum_{\theta=1}^{\ell-1}\theta\,p_\theta\left|\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right)\right|\left|a_\theta\right|\right\}^m$$

$$=O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}.\left\{\sum_{\theta=1}^{\ell-1}p_\theta\left(\theta\left|\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right)\right|\right)^m.\left|a_\theta\right|^m\right\}.\left\{\frac{1}{R_{\ell-1}}\sum_{\theta=1}^{\ell-1}p_\theta\right\}^{m-1}$$

$$=O(1)\sum_{\theta=1}^{r}\left\{\theta\left|\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right)\right|\right\}^{m-1}.\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right).p_\theta.\left|a_\theta\right|^m\times\sum_{\ell=\theta+1}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}$$

$$=O(1)\sum_{\theta=1}^{r}\theta\left|\Delta\left(\frac{\zeta_\theta}{\chi_\theta}\right)\right|\left(\frac{R_\theta}{p_\theta}\right)^{\delta m-1}.\left|a_\theta\right|^m$$

$$=O(1)\sum_{\theta=1}^{r-1}\left(\theta\left|\Delta\zeta_\theta\right|\left|\frac{1}{\chi_\theta}\right|\right)+\theta\left|\zeta_\theta\right|\left|\Delta\left(\frac{1}{\chi_\theta}\right)\right|\times\sum_{\ell=1}^{\theta}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\left|a_\ell\right|^m+O(1)r\left|\Delta\left(\frac{\zeta_r}{\chi_r}\right)\right|.\sum_{\theta=1}^{r}\left(\frac{R_\theta}{p_\theta}\right)^{\delta m-1}\left|a_\theta\right|^m$$

$$=O(1)\sum_{\theta=1}^{r-1}\left\{\left|\Delta\zeta_{\theta+1}\right|\left|\frac{1}{\chi_\theta}\right|+\theta\left|\Delta^2(\zeta_\theta)\right|\left|\frac{1}{\chi_\theta}\right|+\theta\left|\Delta\zeta_\theta\right|\left|\left(\frac{1}{\chi_\theta}\right)\right|\right\}+$$

$$\left\{\left|\zeta_{\theta+1}\right|\left|\Delta\left(\frac{1}{\chi_\theta}\right)\right|+\theta\left|\Delta\zeta_\theta\right|\left|\Delta\left(\frac{1}{\chi_\theta}\right)\right|+\theta\left|\zeta_\theta\right|\left|\Delta^2\left(\frac{1}{\chi_\theta}\right)\right|\right\}\chi_\theta M(R_\theta)+O(1)$$

$$\left\{r\left|\Delta\zeta_r\right|\left|\frac{1}{\chi_\ell}\right|+r\left|\zeta_r\right|\left|\Delta\left(\frac{1}{\chi_r}\right)\chi_r.M(R_r)\right|\right\}$$

Power System Technology
ISSN:1000-3673
Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$$= O(1)\sum_{\theta=1}^{r-1}|\Delta\zeta_{\theta+1}|M(R_\theta) + O(1)\sum_{\theta=1}^{r-1}\theta|\Delta^2\zeta_\theta|M(R_\theta) + O(1)\sum_{\theta=1}^{r-1}\theta|\Delta\zeta_\theta|M(R_\theta)$$

$$+O(1)\sum_{\theta=1}^{r-1}|\zeta_{\theta+1}|M(R_\theta) + O(1)\sum_{\theta=1}^{r-1}|\zeta_\theta|\chi_\theta M(R_\theta).\left|\Delta^2\left(\frac{1}{\chi_\theta}\right)\right| + O(1)r|\Delta\zeta_r|.M(R_r)$$

$+O(1)r|\zeta_r|.M(R_r)$, (using [21] and [22])

$$= 0(1), \text{ as } r \to \infty. \tag{24}$$

For $\sigma_{\ell,4}$,

$$\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m+m-1}|\sigma_{\ell,4}|^m = O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}^m}.\left\{\sum_{\theta=1}^{\ell-1}\left|\frac{\zeta_{\theta+1}}{\chi_{\theta+1}}\right|p_\theta|a_\theta|\right\}^m$$

$$= O(1)\sum_{\ell=2}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}\left\{\sum_{\theta=1}^{\ell}\left|\frac{\zeta_{\theta+1}}{\chi_{\theta+1}}\right|^m.p_\theta|a_\theta|^m\right\}\left\{\frac{1}{R_{\ell-1}}.\sum_{\theta=1}^{\ell-1}p_\ell\right\}^{m-1}$$

$$= O(1)\sum_{\theta=1}^{\ell-1}\left|\frac{\zeta_{\theta+1}}{\chi_{\theta+1}}\right|^{m-1}.\left|\frac{\zeta_{\theta+1}}{\chi_{\theta+1}}\right|.p_\theta|a_\theta|^m.\sum_{\ell=\theta+1}^{r+1}\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.\frac{1}{R_{\ell-1}}$$

$$= O(1)\sum_{\theta=1}^{r-1}\left|\frac{\zeta_{\theta+1}}{\chi_{\theta+1}}\right|\left(\frac{R_\ell}{p_\ell}\right)^{\delta m-1}.|a_\theta|^m$$

$$= O(1), \text{ as } r \to \infty. \tag{25}$$

Collecting (21), (22), (23) and (24), we get (20).

This completes the proof of the theorem.

An application to trigonometric Fourier series:

Let ψ be a periodic function with the period 2π and Lebesgue integrable over $(-\pi,\pi)$.

Then $\psi(z) \sim \frac{a_0}{2} + \sum_{\ell=1}^{\infty}(a_\ell \cos \ell z + b_\ell \sin \ell z) = \sum_{\ell=0}^{\infty}\phi_\ell(z)$,

Power System Technology
ISSN:1000-3673
Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\psi(z)\,dz,$$

$$a_\ell = \frac{1}{\pi}\int_{-\pi}^{\pi}\psi(z)\cos\ell z\,dz,$$

and $b_\ell = \frac{1}{\pi}\int_{-\pi}^{\pi}\psi(z)\sin\ell z\,dz$.

Let $\xi_\ell = \dfrac{\psi(z+\theta)+\psi(z-\theta)}{2}$

and $\xi_g(\theta) = \dfrac{g}{\theta^g}\int_0^\theta (\theta - v)^{g-1}\xi(v)\,dv$, (where $g > 0$).

If $g = 1$ then

$$\xi_1(\theta) = \frac{1}{\theta}\int_0^\theta \xi(v)\,dv.$$

If $\xi_1(\theta) \in BV(\theta,\pi)$ then

$\theta_\ell(z) = O(1)$ where $\theta_\ell(z)$ is the $(C,1)$ mean of the sequence $\{\ell\,\phi_\ell(z)\}$ (see [23]).

By using the above arguments, the following theorems on absolute matrix summability of the trigonometric Fourier series is obtained.

Theorem 2: If $\xi_1(\theta) \in BV(0,\pi)$ and $\{p_{1,}\, p_2,\ldots,p_\ell,\ldots\}$, $\{\zeta_1,\zeta_2,\ldots,\zeta_\ell,\ldots\}$ and $\{\chi_1,\chi_2,\ldots,\chi_\ell,\ldots\}$ are convex sequences satisfying the conditions (8), (9), (10), (11), (12) and (13) of the given main theorem then the infinite series $\sum \phi_\ell(z)\dfrac{\zeta_\ell(z)}{\chi_\ell(z)}$ is summable $\left|\overline{N}, p_\ell\right|_m, m \geq 1$.

Theorem 3: If $\xi_1(\theta) \in BV(0,\pi)$ and $\{p_1, p_2,\ldots,p_\ell,\ldots\}$, $\{\zeta_1,\zeta_2,\ldots,\zeta_\ell,\ldots\}$ and $\{\chi_1,\chi_2,\ldots,\chi_\ell,\ldots\}$ are convex sequences satisfying all the condition of theorem 2 of author (see [21, page 324]) then the infinite series $\sum p_\ell(z)\zeta_\ell$ is summable $\left|\overline{N}, p_\ell\right|_m, m \geq 1$.

Power System Technology
ISSN:1000-3673
Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

References:

[1] H. Bor, A none on two summability methods, Proc. Amer. Soc., 98 (1986), 81-84.

[2] H. Bor, On local property of $\left|\bar{N}, p_n; \delta\right|_k$, Journal of Mathematical Analysis and Applications, 179 (1993), 646-649.

[3] H. Bor, On the $\left|\bar{N}, p_n\right|_k$ summability factors for infinite series, Proc. Indian Acad. Sci. (Math. Sci) Vol. 101, No. 2, August 1991, 143-146.

[4] H. Bor, On $\left|\bar{N}, p_n\right|$ summability factors of infinite series, Proc. Indian Acad. Scie. (Math. Sci.), Vol. 98, No. 1, June 1988, 53-57.

[5] Y. Okuyama, On the absolute Nörlund summability factors of Fourier series, Bull. Austral. Math. Soc. Vol. 12, 1975, 9-21.

[6] Ș. Yildiz, An absolute matrix summability of infinite series and Fourier series, Bol. Soc., Paran, Vol. 38, No. 7, 2020, 49-58.

[7] L. Leindler, On the Absolute Summability Factors of Fourier Series, Acta Sci.Math.,28, 1967,323-336.

[8] S.M. Mazhar, On the summability factors of infinite series, Publications Math. Debrecen, 13, 1966, 229-236.

[9] S.M. Mazhar, On $|C,1|_k$ summability factors of infinite series, Indian J. Math, 14, 1972, 45-48.

[10] A. Karakas, On absolute matrix summability factors of infinite series, J. Class Annal, 13 (2), 2018, 133-139.

[11] A. Karakas, A new factor theorem for generalized absolute Riesz summability, Carpathian Math. Publ. 11 (2), 2019, 345-349.

[12] H.S. Özarslan, A note on $\left|\bar{N}, p_n; \delta\right|_k$ summability factors, Indian J. Pure Appl. Math., 33 (3), 2002, 361-366.

[13] H.S. Özarslan, On $\left|\bar{N}, p_n; \delta\right|_k$ summability factors, Kyungpook math. J., 43 (1), 2003, 107-112.

[14] H.S. Özarslan, On the local properties of factored Fourier series, Proc. Jangjeon Math. Soc., 9 (2), 2006, 103-108.

[15] H.S. Özarslan, Local properties of factored Fourier series, Int. J. Comp. Appl. Math., 1 (1), 2006, 93-96.

[16] H.S. Özarslan and A. Karakas, A new result on the almost increasing sequences, J. Comput. Anal. Appl., 22 (6), 2017, 989-998.

Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

[17] H.S. Özarslan and B. Kartal, A generalization of a theorem of Bor., J. Inequal. Appl., 179, 2017, 8 pp.

[18] H.S. Özarslan, A new factor theorem for absolute matrix summability, Quest. Math., 42 (6), 2019, 803-809.

[19] H.S. Özarslan, An application of absolute matrix summability using almost increasing and γ-quasi-monotone sequence Kyungpook Math. J., 59 (2), 2019, 233-240.

[20] H.S. Özarslan and A. Karkas, A new study on absolute summability factors of infinite series, Maejo Int. J. Sci., 13, 2019, 257-265.

[21] H. Bor, A new note on the factored infinite series and Trigonometric Fourier series, Comptes Rendus Mathématique (Institute de France, Académie des Sciences). Vol. 359, Issue 3, 2021, 323-328.

[22] H.S. Özarslan, Generalized absolute matrix summability of infinite series and Fourier series, Indian J. Pure Appl. Math., 2022, https://doi.org/10.1007/s 13226-022-00220-1, pp.7.

[23] K.K. Chen, Functions of bounded variation and Cesàro means of Fourier series, Acad. Sinica Record, 1, 1945, 283-289.

[24] S.K Sahani,et al. Some Problems on Approximations of Functions (signals) in Matrix Summability of Legendre series, Nepal Journal of Mathematical Sciences, Vol. 2(1), 2021,43-50.

[25] S.K. Sahani,et al. On the Degree of Approximations of a Function by Nörlund Means of its Fourier Laguerre Series, Nepal Journal of Mathematical Sciences, Vol.1, 2020, 65-70.

[26] S.K. Sahani and L.N. Mishra, Degree of Approximation of Signals by Nörlund Summability of Derived Fourier Series, The Nepali Math.Sc.Report, Vol.38.,No.2,2021,13-19.

[27] S.K. Sahani, et al. On a New Application of Positive and Decreasing Sequences to Double Fourier Series Associated with $(N,p_m^{(1)},p_n^{(2)})$,Journal of Neapl Mathematicial Society,Vol.5,2,2022,58-64.

[28] S.K. Sahani, et al. On Certain Series to Series Transformation and Analytic Continuation by Matrix Method, Nepal Journal of Mathematical Sciences,Vol.3,1,2022, 75-80.

[29] S.K Sahani and V.N. Mishra, Degree of Approximation of Function by Nörlund Summability of Double Fourier Series, Mathematicial Sciences and Applications E-Notes,Vol.11,2, 2023,80-88.

Power System Technology
ISSN:1000-3673
Received: 14-10-2023 Revised: 12-12-2023 Accepted: 20-12-2023

[30] S.K. Sahani, and D. Jha. A Certain Studies on Degree of Approximation of functions by Matrix Transformation, The Mathematics Education, Vol. LV,2, 2021,21-33.

[30] S.K. Sahani, and K.S. Prasad, On a New Application of Almost Non-increasing Sequence to Ultrasphericial Series Associated with $(N,p,q)_k$ Means, Applied Science PeriodicalVol.XXIV,1, 2022,1-11.

[31] S.K. Sahani, et al. On Nörlund Summability of Double Fourier Series, Open Journal of Mathematicial Sciences, Vol.6,1, 2022,99-107.

[32] J.K. Pokharel, N.P. Pahari, and S.K. Sahani, Critical Analyzing on Some Application of Almost Decreasing Sequence to Legendre Series Associated with [B] Sum, Advances in Nonlinear Variational Inequalities, Vol.26, No. 2, 2023, 36-40.

[33] S.K. Sahani, et al. On a New Application of Almost Increasing Sequence to Laguerre Series Associated with Strong Summability of Ultraspherical Series, Nepal Journal of Mathematical Sciences, Vol. 4, No. 2, 2023, 77-82.