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Soft Aωї$_s$–Connected Space in Soft Ideal Topological Spaces

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ABSTRACT

In this paper, we introduce a soft αωЇ$_s$ – Connected Space in soft ideal topological spaces. Furthermore, we introduce to Enlightment and Edification of some properties, theorems and remarks with examples, which is the concept of soft mappings in soft ideal topological spaces.

Keywords: Soft set, soft ideal topological spaces, Soft αωЇ$_s$ -Closed set, Soft connected space, Soft αωЇ$_s$ -connected space.

INTRODUCTION

The concept of soft set theory was introduced by Molodstov [4] and the concept of soft ideal theory, soft local function was introduced by A.Kandil,et.al [1]. The concepts of soft connectedness and compactness was introduced by Zorlutuna et.al [12]. The concepts of soft connectedness in ideal was introduced by A.Kandil [2].

PRELIMINARIES

Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U.

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DEFINITION 2.1 [3] Let D be a non-empty subset of E and a soft set over U is a parameterized family of subsets of an initial universe U. For a particular e ∈ E, F(e) may be considered the set of e-approximate elements of the soft set (F,E) and if e ∉ E, then F(e) = ∅ ,that is, $(F, \mathrm{E}) = \{f(e): e \in Đ \subseteq \mathrm{E}, f: \mathrm{E} \rightarrow P(U)\}$ is called a soft set over U. Then the family of all these soft sets denoted by $SS(U)_E$.

DEFINITION 2.2 [4] Let $Ꞇ_s$ be the collection of soft sets over U. Then $Ꞇ_s$ is said to be a soft topology on U if satisfies the following axioms:

(i). (∅, E), (U, E) belongs to $Ꞇ_s$

(ii). The union of any number of soft sets in $Ꞇ_s$ belongs to $Ꞇ_s$

(iii). The intersection of two number of soft sets in $Ꞇ_s$ belongs to $Ꞇ_s$.

The triplet (U,$Ꞇ_s$,E) is said to be soft topological space and we note that the member of $Ꞇ_s$ are said to be $Ꞇ_s$-soft open sets.

DEFINITION 2.3 [1] Let $Ï_s$ be a non-null collection of soft sets over an initial universe U with the same set of parameter E. Then $Ï_s$ containing $SS(U)_E$ is called as a soft Ideal on U with same set E if,

(i). $(F, E) \in Ī$ and $(G, E) \in Ī$ then $(F, E) \cup (G, E) \in Ī$.

(ii). $(F, E) \in Ī$ and $(G, E) \subseteq (F, E)$ then $(G, E) \in Ī$.

DEFINITION 2.4 [11] A soft topological space (U,$Ꞇ_s$,E) is called Soft Connected Space if U cannot be written as a disjoint soft union of two non-empty soft open sets.

DEFINITION 2.5 [2] A soft ideal topological space (U,Ꞇs,Ïs) is called soft ideal connected if there is no pair of non-empty soft sets (F, E) and (G, E) in Ꞇs, such as $(F, E) \cap (G, E) = \emptyset$ and $(F, E) \cup (G, E) = U$ both (F, E) and (G, E) are soft ideal open.

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SOFT αωЇ$_s$ – SEPARATED SETS IN SOFT IDEAL TOPOLOGICAL SPACES

In this section we introduce and study about soft αωЇ$_s$- separated sets in Soft Ideal topological space and we workout some basic theorems.

DEFINITION 2.1 Two non-empty soft subsets (ƒ,Q) and (Ɠ,Q) of a soft ideal topological space (Ṿ,Ꞇs,Q,Їs) are said to be soft αωЇ$_s$ - separated if (ƒ,Q) ∩ αωЇ$_s$ (Cl(Ɠ,Q)) = αωЇ$_s$ (Cl(ƒ,Q)) ∩ (Ɠ,Q) = { }. If (Ṿ,Q) = (ƒ,Q) ∪ (Ɠ,Q) such that (ƒ,Q) and (Ɠ,Q) are soft αωЇ$_s$ - Separated sets, then (ƒ,Q) and (Ɠ,Q) form a soft αωЇ$_s$ - Separation of (Ṿ,Ꞇs,Q,Їs).

EXAMPLE 2.2 Let Ṿ = {α, β,γ,δ}, Q = {σ_1,σ_2}, Ꞇ$_s$ = {({},Q),(Ṿ,Q),(ƒ$_1$,Q),(ƒ$_2$,Q),(ƒ$_3$,Q) Ї$_s$ = {({},Q),(ƒ$_4$,Q), (ƒ$_5$,Q), (ƒ$_6$,Q)} where (ƒ$_1$,Q) = {{},{α}}, (ƒ$_2$,Q) = {{β},{α,β}}, (ƒ$_3$,Q) = {{Ṿ},{β}}, (ƒ$_4$,Q) = {{},{γ}}, (ƒ$_5$,Q) ={{},{β,δ}}, (ƒ$_6$,Q) ={{β,δ},{β,γ,δ}} are soft sets over Ṿ. Here, {{},{β,δ}} and {{},{γ}} are soft αωЇ$_s$ -Separation and also disjoint of (Ṿ,Ꞇs,Q,Їs).

THEOREM 2.3 For the soft subsets (ƒ,Q) and (Ɠ,Q) be a soft ideal topological space (Ṿ,Ꞇs,Q,Їs) the following statements are equivalent:

(i). (ƒ,Q) and (Ɠ,Q) are soft αωЇ$_s$ - Separated.

(ii). There exist soft αωЇ$_s$ - Closed sets (ƒ$_1$,Q) and (ƒ$_2$,Q) satisfying (ƒ,Q) ⊆ (ƒ$_1$,Q) ⊆ ((Ṿ,Q) − (Ɠ,Q)) and (Ɠ,Q) ⊆ (ƒ$_2$,Q) ⊆ (Ṿ,Q) − (ƒ,Q)).

(iii). There exist αωЇ$_s$ - Open sets (Ɠ$_1$,Q) and (Ɠ$_2$,Q) satisfying (ƒ,Q) ⊆ (Ɠ$_1$,Q) ⊆ (Ṿ,Q) − (Ɠ,Q) and (Ɠ,Q) ⊆ (Ɠ$_2$,Q) ⊆ ((Ṿ,Q) − (ƒ,Q)).

PROOF (i) ⇒ (ii) Assume that (ƒ,Q) and (Ɠ,Q) are soft αωЇ$_s$ - Separated. Then there exist disjoint soft αωЇ$_s$ – Open sets (U,Q) and (V,Q) such that, (ƒ,Q) ⊆ (U,Q), (Ɠ,Q) ⊆ (V,Q) and (U,Q) ∩ (V,Q) = { } then (ƒ$_1$,Q) = (Ṿ,Q) − (V,Q) , (ƒ$_2$,Q) = (Ṿ,Q) − (U,Q) which implies that

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(ƒ$_1$,Q) is the complement of a soft αωÏ$_s$ – Open set, so (ƒ$_1$,Q) is soft αωÏ$_s$ – Closed set. Similarly, (ƒ$_2$,Q) is soft αωÏ$_s$ – Closed. Since (Ɠ,Q) ⊆ (V,Q), we have (Ṿ,Q) − (V,Q) ⊆ (Ṿ,Q) − (Ɠ,Q), so (ƒ$_1$,Q) = (Ṿ,Q) − (Ɠ,Q) and (ƒ,Q) ⊆ (U,Q) = (Ṿ,Q) − (ƒ$_2$,Q), so (ƒ,Q) ⊆ (ƒ$_1$,Q). Similarly, (Ɠ,Q) ⊆ (ƒ$_2$,Q) ⊆ (Ṿ,Q) − (ƒ,Q). Hence, There exist soft αωÏ$_s$ - Closed sets (ƒ$_1$,Q) and (ƒ$_2$,Q) satisfying (ƒ,Q) ⊆ (ƒ$_1$,Q) ⊆ ((Ṿ,Q) − (Ɠ,Q)) and (Ɠ,Q) ⊆ (ƒ$_2$,Q) ⊆ ((Ṿ,Q) − (ƒ,Q)).

(ii) ⇒ (iii) Assume that there exist soft αωÏ$_s$ - Closed sets (ƒ$_1$,Q) and (ƒ$_2$,Q) such that, (ƒ,Q) ⊆ (ƒ$_1$,Q) ⊆ (Ṿ,Q) − (Ɠ,Q) and (Ɠ,Q) ⊆ (ƒ$_2$,Q) ⊆ (Ṿ,Q) − (ƒ,Q) then (Ɠ$_1$,Q) = (Ṿ,Q) − (ƒ$_2$,Q) and (Ɠ$_2$,Q) = (Ṿ,Q)− (ƒ$_1$,Q), (ƒ$_2$,Q) is soft αωÏ$_s$ – Closed which implies that (Ɠ$_1$,Q) is soft αωÏ$_s$ – Open and (ƒ$_1$,Q) is soft αωÏ$_s$ – Closed which implies that (Ɠ$_2$,Q) is soft αωÏ$_s$ – Open. Since (Ɠ,Q) ⊆ (ƒ$_2$,Q) then (Ṿ,Q) − (ƒ$_2$,Q) ⊆ (Ṿ,Q) − (Ɠ,Q), so (Ɠ$_1$,Q) ⊆ (Ṿ,Q) − (Ɠ,Q) similarly, (ƒ,Q) ⊆ (ƒ$_1$,Q) which implies that (Ɠ$_2$,Q) ⊆ (Ṿ,Q) − (ƒ,Q). Also (ƒ,Q) ⊆ (Ṿ,Q) − (ƒ$_2$,Q) = (Ɠ$_1$,Q), (Ɠ,Q) ⊆ (Ɠ$_2$,Q). Hence, There exist αωÏ$_s$ -Open sets (Ɠ$_1$,Q) and (Ɠ$_2$,Q) satisfying (ƒ,Q) ⊆ (Ɠ$_1$,Q) ⊆ ((Ṿ,Q) − (Ɠ,Q)) and (Ɠ,Q) ⊆ (Ɠ$_2$,Q) ⊆ ((Ṿ,Q) − (ƒ,Q)).

(iii) ⇒ (i) Assume that there exist soft αωÏ$_s$ - Closed sets (ƒ$_1$,Q) and (ƒ$_2$,Q) such that, (ƒ,Q) ⊆ (Ɠ$_1$,Q) ⊆ (Ṿ,Q) − (Ɠ,Q) and (Ɠ,Q) ⊆ (Ɠ$_2$,Q) ⊆ ((Ṿ,Q) − (ƒ,Q)), (Ɠ$_1$,Q) ∩ (Ɠ$_2$,Q) ⊆ ((Ṿ,Q) − (Ɠ,Q)) ∩ ((Ṿ,Q) − (ƒ,Q)) = (Ṿ,Q) − ((ƒ,Q) ∪ (Ɠ,Q)) so ((Ɠ$_1$,Q) ∩ (Ɠ$_2$,Q)) ∩ ((ƒ,Q) ∪ (Ɠ,Q)) = Φ. Then (ƒ,Q) ⊆ (Ɠ$_1$, Q), (Ɠ,Q) ⊆ (Ɠ$_2$,Q), ((Ɠ$_1$,Q) ∩ (Ɠ$_2$,Q)) = Φ. Thus (ƒ,Q) and (Ɠ,Q) are contained in disjoint soft αωÏ$_s$ – Open sets which implies that (ƒ,Q) and (Ɠ,Q) are soft αωÏ$_s$ - Separated.

PROPOSITION 2.4 Let (ƒ,Q) and (Ɠ,Q) be a soft subsets of a soft ideal topological space (Ṿ,Ꞇs,Q,Ïs). If (ƒ,Q) and (Ɠ,Q) are soft αωÏ$_s$ -Separated, { } ≠ (C,Q) ⊆ (ƒ,Q) and { } ≠ (D,Q) ⊆ (Ɠ,Q), then (C,Q) and (D,Q) soft are αωÏ$_s$ – Separated.

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PROOF Since (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Separated sets, (Ƒ,Q) ∩ αωÏ$_s$Cl(Ǥ,Q) = { } and αωÏ$_s$Cl(Ƒ,Q) ∩ (Ǥ,Q) = { }. By hypothesis (C,Q) ⊆ (Ƒ,Q), we αωÏ$_s$Cl(C, Q) ∩ (D,Q)) = { }. Similarly, we have (C,Q) ∩ αωÏs(Cl(D,Q))= { }. Therefore, (C,Q) and (D,Q) are soft αωÏ$_s$-Separated sets.

THEOREM 2.5 For a soft αωÏ$_s$ - Closed soft subset (S,Q) of a soft ideal topological space (Ṿ,Ꞇs,Q,Ïs), then (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Separated sets, such that (S,Q) = (Ƒ,Q) ∪ (Ǥ,Q), then (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Closed sets.

PROOF Let (S,Q) = (Ƒ,Q) ∪ (Ǥ,Q), where soft αωÏs(Cl(Ƒ,Q)) ∩ (Ǥ,Q))={ } = (Ƒ,Q) ∩ αωÏ$_s$ (Cl(Ǥ,Q)). Now, (S,Q) ∩ αωÏs(Cl(Ƒ,Q)) = ((Ƒ,Q) ∪ (Ǥ,Q)) ∩ αωÏs(Cl(Ƒ,Q)) = (Ƒ,Q). As the intersection of soft αωÏ$_s$ - Closed sets is soft αωÏ$_s$ - Closed, (Ƒ,Q) is soft αωÏ$_s$ - Closed. Similarly (Ǥ,Q) is soft αωÏ$_s$ – Closed.

THEOREM 2.6 Let (Ƒ,Q) and (Ǥ,Q) be a nonempty soft subsets of a soft ideal topological space (Ṿ,Ꞇs,Q,Ïs)

The following statements hold:

(i). If (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Separated and (Ƒ$_1$,Q) ⊆ (Ƒ,Q), (Ǥ$_1$,Q) ⊆ (Ǥ,Q), then (Ƒ$_1$,Q) and (Ǥ$_1$,Q) are so.

(ii). If (Ƒ,Q) ∩ (Ǥ,Q) = { } such that (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Closed (soft αωÏ$_s$ - Open), then (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Separated.

(iii). If (Ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Closed (soft αωÏ$_s$ - Open) and if (H,Q) = (Ƒ,Q) ∩ (Ṿ,Q) − (Ǥ,Q) and (G,Q)= (Ǥ,Q) ∩ ((Ṿ,Q) − (Ƒ,Q)), then (H,Q) and (G,Q) are soft αωÏ$_s$ - Separated.

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PROOF. (i). Since (ƒ$_1$,Q) ⊆ (ƒ,Q), αωÏs(Cl(ƒ,Q)) ⊆ αωÏs(Cl(ƒ,Q)). Then (Ǥ,Q) ∩ αωÏs(Cl(ƒ,Q)) = { } implies (Ǥ$_1$,Q) ∩ αωÏs(Cl(ƒ,Q)) = { } and (Ǥ$_1$,Q) ∩ αωÏs(Cl(ƒ$_1$,Q)) = { }. Similarly (ƒ$_1$,Q) ∩ αωÏs(Cl(Ǥ$_1$,Q)) = { }. Hence (ƒ$_1$,Q) and (Ǥ$_1$,Q) are soft αωÏ$_s$ - Separated.

(ii). Since (ƒ, Q)=αωÏs(Cl(ƒ,Q)), (Ǥ,Q)=αωÏs(Cl(Ǥ,Q)) and (ƒ,Q) ∩ (Ǥ,Q) = { }, αωÏs(Cl(ƒ,Q)) ∩ (Ǥ,Q) = { } and αωÏs(Cl(Ǥ,Q)) ∩ (ƒ,Q) = { }. Hence (ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Separated sets. If (ƒ,Q) and (Ǥ,Q) are soft αωÏ$_s$ - Open, then their complements are soft αωÏ$_s$ - Closed.

(iii). If (ƒ,Q) and (Ǥ,Q) are soft αωÏs - Open, then (Ṿ,Q) − (ƒ,Q) and (Ṿ,Q)− (Ǥ,Q) are soft αωÏ$_s$ - Closed. Since (H,Q) ⊆ (Ṿ,Q) − (Ǥ,Q), αωÏs(Cl(H,Q)) ⊆ αωÏs(Cl((Ṿ,Q) − (Ǥ,Q)) = (Ṿ,Q) − (Ǥ,Q) and so αωÏs(Cl(H,Q)) ∩ (Ǥ,Q) = { }. Thus (G,Q) ∩ αωÏs(Cl(H,Q)) = { }. Similarly, (H,Q) ∩ αωÏs(Cl(G,Q)) = { }. Hence, (H,Q) and (G,Q)are soft αωÏ$_s$ - Separated sets.

THEOREM 2.7 The soft subsets (ƒ,Q) and (Ǥ,Q) be a soft ideal topological space (Ṿ,Ꞇs,Q,Ïs) are soft αωÏs - Separated if and only if there exist (U,Q), (V,Q) ∈ αωÏsO(Ṿ,Q) such that (ƒ,Q) ⊆ (U,Q), (Ǥ, Q) ⊆ (V,Q) , (ƒ,Q) ∩ (V,Q) = { } and (Ǥ,Q) ∩ (U,Q)= { }.

PROOF. Let (ƒ,Q) and (Ǥ,Q) be soft αωÏs - Separated sets. Then (V,Q) = (Ṿ,Q) − αωÏs(Cl(ƒ,Q)) and (U,Q) = (Ṿ,Q) − αωÏs(Cl(Ǥ,Q)). Then (U,Q), (V,Q) ⊆ αωÏsO(Ṿ,Q) such that (ƒ,Q) ⊆ (U,Q), (Ǥ,Q) ⊆ (V,Q) , (ƒ,Q) ∩ (V,Q) = { } and (Ǥ,Q) ∩ (V,Q) = { }. On the other hand, let (U,Q), (V,Q) ∈ αωÏsO(Ṿ,Q) such that (ƒ,Q) ⊆ (U,Q), (Ǥ,Q) ⊆ (V,Q), (ƒ,Q) ∩ (V,Q) = { } and (Ǥ,Q) ∩ (U,Q) = { }. Since (Ṿ,Q) − (V,Q) and (Ṿ,Q) − (U,Q) are soft αωÏs - Closed sets, αωÏs(Cl(ƒ,Q)) ⊆ (Ṿ,Q) − (V,Q) ⊆ (Ṿ,Q) − (Ǥ,Q) and αωÏs(Cl(Ǥ,Q)) ⊆ (Ṿ,Q) − (U,Q) ⊆ (Ṿ,Q) − (ƒ,Q). Thus αωÏs (Cl(ƒ,Q)) ∩ (Ǥ,Q)= { } and αωÏs(Cl(Ǥ,Q)) ∩ (ƒ,Q)) = { }.

THEOREM 2.8 Let (Ṿ,Ꞇs,Q,Ïs) be a soft ideal topological space. If (ƒ,Q) and (Ǥ,Q) are soft αωÏs - Separation of (Ṿ,Q) itself, then (ƒ,Q) and (Ǥ,Q) are soft αωÏs - Closed sets of (Ṿ,Ꞇs,Q,Ïs).

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PROOF Since (Ƒ,𝑄) and (Ɠ,𝑄) are soft αωЇs - Separated, (Ƒ,𝑄) ∩ αωЇs(Cl(Ɠ,𝑄)) = αωЇs (Cl(Ƒ,𝑄)) ∩ (Ɠ,𝑄) = { }. Then (Ƒ,𝑄) ∩ αωЇs(Cl(Ɠ,𝑄)) = { } if and only if (Ɠ,𝑄) is soft αωЇs - Closed in (Ƒ,𝑄) ∪ (Ɠ,𝑄) = (Ṿ,𝑄).

SOFT $\alpha\omega Ї_s$ – CONNECTED SPACE IN SOFT IDEAL TOPOLOGICAL SPACES

In this section, we introduce and study about soft αωЇs-connected spaces and also investigate some of their basic properties.

DEFINITION 3.1 A subset (Ƒ,𝑄) of a soft ideal topological space (Ṿ,Ꞇs,𝑄,Їs) is said to be soft αωЇs - Connected if it cannot be expressed as the union of two soft αωЇs - Separated sets. Otherwise, the set (Ƒ,𝑄) is called soft αωЇs - disconnected.

EXAMPLE 3.2 Let Ṿ = {α, β,γ,δ}, Q = {σ_1,σ_2}, $Ꞇ_s$ = {({},𝑄),(Ṿ,𝑄),($Ƒ_1$,𝑄),($Ƒ_2$,𝑄),($Ƒ_3$,𝑄) $Ї_s$ = {({},𝑄),($Ƒ_4$,𝑄), ($Ƒ_5$,𝑄), ($Ƒ_6$,𝑄)} where ($Ƒ_1$,𝑄) = {{},{α}}, ($Ƒ_2$,𝑄) = {{β},{α,β}}, ($Ƒ_3$,𝑄) = {{Ṿ}, {β}}, ($Ƒ_4$,𝑄) = {{},{γ}}, ($Ƒ_5$,𝑄) ={{},{β,δ}}, ($Ƒ_6$,𝑄) ={{β,δ},{β,γ,δ}} are soft sets over Ṿ. Here, {{},{α,δ}} and {{γ},{β,γ}} are soft αωЇs-connected because it cannot be expressed as the union of two soft αωЇs-separated sets.

LEMMA 3.3 Let (Ƒ,𝑄) ⊆ (Ɠ,𝑄) ∪ (C,𝑄) such that (Ƒ,𝑄) be a nonempty αωЇs - Connected set in a soft ideal topological space (Ṿ,Ꞇs,𝑄,Їs) and (Ɠ,𝑄), (C,𝑄) be a soft αωЇs - Separated sets. Then only one of the following conditions hold:

1. (Ƒ,𝑄) ⊆ (Ɠ,𝑄) and (Ƒ,𝑄) ∩(C,𝑄)= { }.

2. (Ƒ,𝑄) ⊆ (C,𝑄) and (Ƒ,𝑄) ∩ (Ɠ,𝑄) = { }.

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PROOF Since (Ƒ,Q) ∩ (C,Q)= { }, (Ƒ,Q) ⊆ (Ǥ,Q) . Also, if (Ƒ,Q) ∩ (Ǥ,Q) = { }, then (Ƒ,Q) ⊆ (C,Q). Since (Ƒ,Q) ⊆ (Ǥ,Q) ∩ (C,Q), then both (Ƒ,Q) ∩ (Ǥ,Q) = { } and (Ƒ,Q) ∩ (C,Q)= { } cannot hold simultaneously. Similarly, suppose that (Ƒ,Q) ∩ (Ǥ,Q) ≠ { } and (Ƒ,Q) ∩ (C,Q) ≠ { }, then by Theorem 2.6 (1), (Ƒ,Q) ∩ (Ǥ,Q) and (Ƒ,Q) ∩ (C,Q) are soft αωЇs - Separated sets, such that (Ƒ,Q) = ((Ƒ,Q) ∩ (Ǥ,Q)) ∪ ((Ƒ,Q) ∩ (C,Q)) which contradicts with the soft αωЇs - Connectedness of (Ƒ,Q). Hence, the conditions (1) and (2) must hold.

THEOREM 3.4 If a soft αωЇs - Connected set (S,Q) of a soft ideal topological space (Ṿ,Ƭs,Q,Їs) is contained in (Ƒ,Q) ∪ (Ǥ,Q), where (Ƒ,Q) and (Ǥ,Q) are soft αωЇs - Separated sets, then either (S,Q) ⊆ (Ƒ,Q) or (S,Q) ⊆ (Ǥ,Q) .

PROOF We have (S,Q) = ((S,Q) ∩ (Ƒ,Q)) ∪ ((S,Q) ∩ (Ǥ,Q)) where (S,Q) ∩ (Ƒ,Q) and (S,Q) ∩ (Ǥ,Q) are soft αωЇs - Separated sets. So either (S,Q) ∩ (Ƒ,Q) = { } or (S,Q) ∩ (Ǥ,Q) = { } and hence either (S,Q) ⊆ (Ǥ,Q) or (S,Q) ⊆ (Ƒ,Q).

THEOREM 3.5 Every soft αωЇs – Connected space is soft connected.

PROOF Let (Ṿ,Ƭs, Q,Їs) be a soft αωЇs – Connected space. Suppose that (Ṿ,Ƭs,Q,Їs) is not soft connected. Then (Ṿ,Q) = (Ƒ,Q) ∪ (Ǥ,Q) where (Ƒ,Q) and (Ǥ,Q) are disjoint non-empty open soft subsets of (Ṿ,Ƭs,Q,Їs). Then (Ƒ,Q) and (Ǥ,Q) are soft αωЇs - Open and (Ṿ,Q) = (Ƒ,Q) ∪ (Ǥ,Q) where (Ƒ,Q) and (Ǥ,Q) are disjoint non-empty and soft αωЇs - Open sets in (Ṿ,Ƭs,Q,Їs). This contradicts the fact that (Ṿ,Ƭs,Q,Їs) is soft αωЇs – Connected and so (Ṿ,Ƭs,Q,Їs) be a soft connected.

The converse of the above Theorem need not be true as seen from the following Example.

EXAMPLE 3.6 Let Ṿ = {α, β,γ,δ}, Q = {σ_1,σ_2}, $Ƭ_s$ = {({},Q),(Ṿ,Q),($Ƒ_1$,Q),($Ƒ_2$,Q),($Ƒ_3$,Q) $Ї_s$ = {({},Q),($Ƒ_4$,Q), ($Ƒ_5$,Q), ($Ƒ_6$,Q)} where ($Ƒ_1$,Q) = {{},{α}}, ($Ƒ_2$,Q) = {{β},{α,β}}, ($Ƒ_3$,Q) = {{Ṿ},{

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β }}, (Ƒ4,Q) = {{},{ γ }}, (Ƒ5,Q) ={{},{ β,δ }}, (Ƒ6,Q) ={{ β,δ },{ β,γ,δ }} are soft sets over Ṿ. Here, {{},{ α,γ }}and {{},{ β,δ }} are not soft αωΪs – connected.

THEOREM 3.7 A subset (M,Q) be a soft ideal topological space (Ṿ,Ƭs,Q,Ϊs) is soft αωΪs - Connected if there exists a soft αωΪs - Connected set (C,Q) satisfying (C,Q) ⊆ (M,Q) ⊆ αωΪs (Cl(C,Q)).

PROOF Let (M,Q) = (Ƒ,Q) ∪ (Ǥ,Q), where (Ƒ,Q) and (Ǥ,Q) are soft αωΪs - Separated sets. Then either (C,Q) ⊆ (Ƒ,Q) and (C,Q) ⊆ (Ǥ,Q) and hence either (M,Q) ⊆ αωΪs(Cl(C,Q)) ⊆ αωΪs(Cl(Ƒ,Q)) ⊆ ((Ṿ-(Ǥ,Q)) or (M,Q) ⊆ (Ṿ, Q) $-$ (Ƒ,Q)). Therefore, either (Ǥ,Q) = { } or (Ƒ,Q) = {}.

THEOREM 3.8 If {(M,Q)$\alpha : \alpha \in \Delta$ } is a family of soft αωΪs - Connected subsets of a soft ideal topological space (Ṿ,Ƭs,Q,Ϊs) satisfying the property that any two of which are not soft αωΪs - Separated, then (M,Q) = ({(∪,Q)$\alpha : \alpha \in \Delta$}(M,$Q$)$\alpha$) is soft αωΪs - Connected.

PROOF Let (M,Q) = (Ƒ,Q) ∪ (Ǥ,Q) , where (Ƒ,Q) and (Ǥ,Q) are soft αωΪs - Separated sets. Then for each $\alpha \in \Delta$ either (M,Q)α ⊆ (Ƒ,Q) or (M,Q)α ⊆ (Ǥ,Q) . Since any two members of the family {(M,Q)$\alpha \in \Delta$} are not soft αωΪs - Separated, either (M,Q)α ⊆ (Ƒ,Q) for each $\alpha \in \Delta$ or (M,Q)α ⊆ (Ǥ,Q) for each $\alpha \in \Delta$. So either (Ǥ,Q) = { } or (Ƒ,Q) = { }.

COROLLARY 3.9 If ({(∪,Q)$\alpha \in \Delta$}(M,Q)α),where each Mα is soft αωΪs - Connected in a soft ideal topological space (Ṿ,Ƭs,Q,Ϊs) and ∩$\alpha \in \Delta$(M,Q)α ≠ { } for each $\alpha \in \Delta$, then (M,Q) is soft αωΪs - Connected.

PROOF Suppose {(∪,Q)$\alpha \in \Delta$}(M,Q)α is not soft αωΪs - Connected. Then we have {(∪,Q)$\alpha \in \Delta$}(M,Q)α = (H,Q) ∪ (G,Q) where (H,Q) and (G,Q) are soft αωΪs - Separated sets in (Ṿ,Q).

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Since $\cap_{\alpha\in\Delta}(M,Q)_\alpha \neq$ { }, we have a point Ṿ in $\cap_{\alpha\in\Delta}(M,Q)_\alpha$. Since (Ṿ,$Q$) ∈ $\cap_{\alpha\in\Delta}$ $(M,Q)_\alpha$, either (Ṿ,Q) ∈ (G,Q) or (Ṿ,Q) ∈ (H,Q). Suppose that (Ṿ,Q) ∈ (H,Q). Since (Ṿ,Q) ∈ $(M,Q)_\alpha$ for each $\alpha\in\Delta$, then $(M,Q)_\alpha$ and (H,Q) intersect for each $\alpha\in\Delta$. Since (H,Q) and (G,Q) are disjoint $(M,Q)_\alpha$ ∈ (H,Q) for all $\alpha\in\Delta$ and hence $(\cup,Q)_{\alpha\in\Delta}$ ⊆ (H,Q). This implies that (G,Q) is empty. This is a contradiction. Suppose that (Ṿ,Q) ∈ (G,Q). By similar way, we have that (H,Q) is empty. This is a contradiction. Thus ({$(\cup,Q)_{\alpha\in\Delta}$}$(M,Q)_\alpha$) is soft αωÏs - Connected.

THEOREM 3.10 For a soft ideal topological space (Ṿ,Ꞇs,Q,Ïs), then the following statements are equivalent:

1. (Ṿ,Q) is soft αωÏs - Connected.

2. (Ṿ,Q) cannot be expressed as the union of two non-empty disjoint soft αωÏs - Open sets.

3. (Ṿ,Q) contains no nonempty proper soft subset which is both soft αωÏs - Open and soft αωÏs - Closed.

PROOF (i) ⇒ (ii) Suppose that (Ṿ,Q) is soft αωÏs - Connected and if (Ṿ,Q) can be expressed as the union of two nonempty disjoint soft sets (ƒ,Q) and (Ǥ,Q), such that (ƒ,Q) and (Ǥ,Q) are soft αωÏs - Open sets. Consequently (ƒ,Q) ⊆ (Ṿ,Q) − (Ǥ,Q). Then soft αωÏs(Cl(ƒ,Q)) ⊆ αωÏs (Cl((Ṿ,Q) − (Ǥ,Q)) = (Ṿ,Q) − (Ǥ,Q) . Therefore, αωÏs(Cl(ƒ,Q)) ∩ (Ǥ,Q) = { }. Similarly, we can prove (ƒ,Q) ∩ αωÏs(Cl(Ǥ,Q)) = { }. This is a contradiction to the fact that (Ṿ,Q) is soft αωÏs - Connected. Therefore, (Ṿ,Q) cannot be expressed as the union of two nonempty disjoint soft αωÏs - Open sets.

(ii)⇒(iii) Suppose that (Ṿ,Q) cannot be expressed as the union of two nonempty disjoint soft sets (ƒ,Q) and (Ǥ,Q), such that (ƒ,Q) and (Ǥ,Q) are soft αωÏs - Open sets. If (Ṿ,Q) contains a nonempty proper soft subset (ƒ,Q) which is both soft αωÏs - Open and soft αωÏs

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- Closed. Then (V̤,Q) = (Ƒ,Q) ∪ ((V̤,Q) − (Ƒ,Q)). Hence (Ƒ,Q) and (V̤,Q) − (Ƒ,Q) are disjoint soft αωЇs - Open sets whose union is (V̤,Q). This is the contradiction to our assumption. Hence, (V̤,Q) contains no nonempty proper soft subset which is both soft αωЇs - Open and soft αωЇs - Closed.

(iii) ⇒ (i) Suppose that (V̤,Q) contains no nonempty proper soft subset which is both soft αωЇs - Open and soft αωЇs - Closed and (V̤,Q) is not soft αωЇs - Connected. Then (V̤,Q) can be expressed as the union of two nonempty disjoint soft sets (Ƒ,Q) and (Ɡ,Q) such that ((Ƒ,Q) ∩ (αωЇs(Cl(Ɡ,Q))) ∪ (αωЇs(Cl(Ƒ,Q))) ∩ (Ɡ,Q)) = { }. Since (Ƒ,Q) ∩ (Ɡ,Q) = { }, (Ƒ,Q) = (V̤,Q) − (Ɡ,Q) and (Ɡ,Q) = (V̤,Q) − (Ƒ,Q). Since soft αωЇs(Cl(Ƒ,Q)) ∩ (Ɡ,Q) = { }, αωЇs(Cl(Ƒ,Q)) ⊆ (V̤,Q) − (Ɡ,Q). Hence, αωЇs(Cl(Ƒ,Q)) ⊆ (Ƒ,Q). Therefore, (Ƒ,Q) is soft αωЇs - Closed. Similarly, (Ɡ,Q) is soft αωЇs - Closed. Since (Ƒ,Q) = (V̤,Q) − (Ɡ,Q), (Ƒ,Q) is soft αωЇs - Open. Therefore, there exists a nonempty proper soft set (Ƒ,Q) which is both soft αωЇs - Open and soft αωЇs - Closed. This is a contradiction to our assumption. Therefore, (V̤,Q) is soft αωЇs - Connected.

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