Power System Technology
ISSN:1000-3673

Received: 16-08-2025 Revised: 05-09-2025 Accepted: 05-10-2025

A Comparative Study between Various Elevation Systems, Accuracy Estimation, Determination of Plumb Line Deflection in Northeastern Syria

Sheikh Ahmad Ali *, Ramadan A. Al-Razak, Labbabidi Abdul Rahman

Department of Surveying Engineering, Faculty of Civil Engineering, Aleppo University, Syria.ali.sheikh.ahmad@alepuniv.edu.sy

Abstract

The Syrian leveling network was established in the first half of the nineteenth century by three different parties (French – Russian – Syrian) and it covers approximately 70% of the area of Syria.

The lack of development and updating of this network since its establishment, its inconsistency in some areas, the damage it has suffered during the years of war, and the loss of its points in many locations constitute a major challenge that hinders the densification of this network, especially at reconstruction project sites in Syria or in utilizing from modern surveying technologies such as the Global Positioning System (GPS).

In this research, the basic heights used in surveying were reviewed, and the normal heights (H^N) calculated from the Earth's gravity field were proposed as an alternative to orthometric heights computed by spirit leveling and referenced to mean sea level. A digital elevation model was constructed from the available points in the study area to derive the normal height at any point, and then this study was generalized to a large part of Syria where gravity measurements (g) or gravity anomaly maps are available. Finally, the deflection of the vertical was determined in the study area and its effect on the horizontal positioning of points was assessed.

Keywords: gravity, normal height, dynamic height, undulation, vertical deflection

1. Introduction

The height system is a one-dimensional coordinate system used to express the metric distance (height) of a specific point from the Earth's surface up to its projection on a reference surface. The definition and quantity of these heights vary according to the chosen reference surface and the path along which the height is measured.

In Syria, the orthometric height system referenced to the mean sea level has been used to construct the Syrian height network, which covers approximately 70% of Syria's area. Three different parties (French, Russian, and Syrian) worked on building this network during the first half of the nineteenth century. The accuracy of the network is given by: $2.5\sqrt{D}$ cm, where D is the distance between two points in kilometers.

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ISSN:1000-3673
Received: 16-08-2025 Revised: 05-09-2025 Accepted: 05-10-2025

In this research, a statistical study was conducted on the differences between heights calculated from terrestrial gravity measurements (dynamic and normal heights) and orthometric heights referenced to mean sea level from available data in the study area, along with accuracy estimation. Additionally, a digital elevation model was constructed for the most accurate height to derive the elevation of any point and to generalize this approach in regions of Syria where terrestrial gravity measurements and gravity anomaly maps are available. Finally, the deflection of the vertical in the study area was determined using terrestrial gravity data, along with an accuracy assessment.

2. Research Objective

The research aims to review the (various) fundamental heights used in surveying, particularly those heights calculated using the Earth's gravity potential, and to select the best among them in terms of accuracy to serve as an alternative to the heights of the Syrian height network points calculated by geometric leveling (spirit leveling) and referenced to the mean sea level (MSL). This is intended for areas that are unoccupied or where the height network points have been lost or damaged, as well as at reconstruction project sites in Syria after the war. Subsequently, a digital elevation model will be constructed to derive the height at any point within the study area, and then the study methodology (approach) will be generalized to all regions in Syria where terrestrial gravity measurements and gravity anomaly maps are available. Additionally, these heights will be utilized when measuring with the Global Positioning System (GPS) and for calculating the geoid undulation (N) when needed.

3. Research Methodology

3.1 Study Area

The study area is located in the northeastern part of Syria, between latitudes 36.5° and 37.5°, and longitudes 40° and 42.5°, covering an approximate area of 12,000 km² (see Figure 1). This area was selected due to its importance and the abundance of gravity measurements available, aimed at exploring the oil and gas fields concentrated in this region.

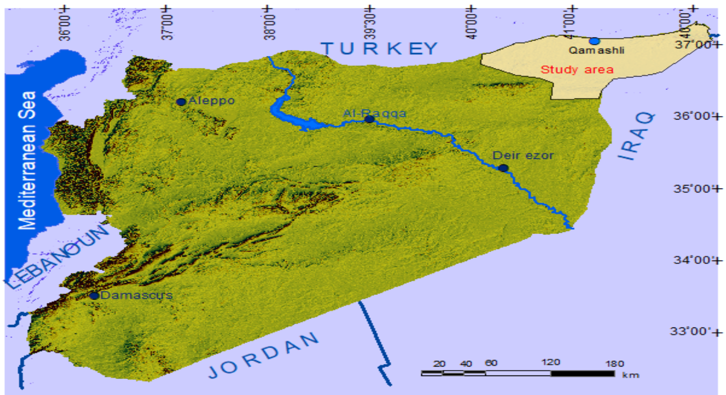

Figure 1, show the location of study area from Syria

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3.2 Available Data

The data were obtained from the Syrian Petroleum Company, which began oil exploration activities in northeastern Syria starting in 1960. The dataset includes 1,000 field measurement points distributed almost uniformly across the study area. Figure 2 illustrates the distribution of some gravity measurement points.

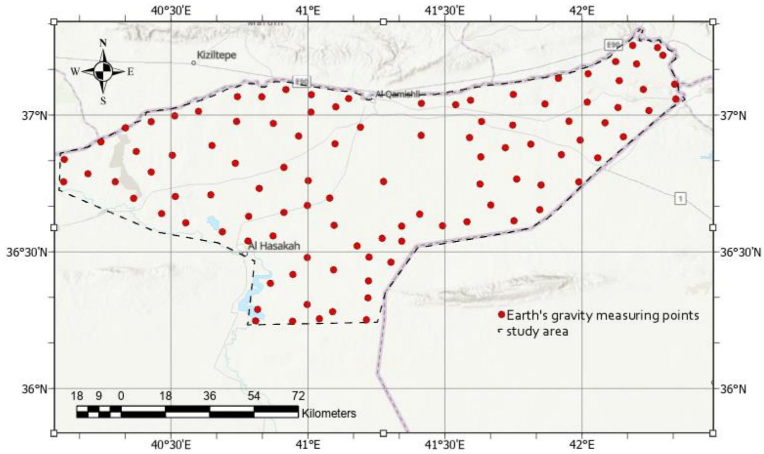

Figure 2, shows the distribution of gravity measurement points across study area.

3.3 Height Systems

The choice of the reference surface and the path along which distance (height) is measured are two important factors in determining the type of height (as mentioned in the introduction). There are two main types of height systems height systems that ignore the Earth's gravity field and therefore use straight paths, such as (Geometric Heights) obtained via Global Positioning System (GPS) measurements. The reference surface for this type of height is the (reference ellipsoid). The other type is height systems whose definitions are related to equipotential surfaces and Earth's gravity field lines, thus following curved paths, known as physical heights. Among the most important heights used in surveying (W. E. Featherstone, M. Kuhn,1987) are:

3.4 Dynamic Heights

Dynamic heights are easy to calculate (if the geopotential number is known) using the following formula (Hirt C, Featherstone WE, Claessens SJ 2011):

$$H^{dyn} = \frac{C}{\gamma_{45}}$$

Where $\gamma_{45} = 980.61977\ Gal$

Since they retain the same properties as the geopotential number, they correctly predict fluid flow and provide closure for the zero- leveling loop. It is worth mentioning that the practical use of dynamic heights is rare. (KARACA, Onur ,2016)

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3.5 Normal Heights

The normal height is defined as the distance along the vertical line from point Q, the projection of point P on the telluroid surface, to point Q' on the reference ellipsoid surface. It is worth mentioning that the telluroid surface is an auxiliary surface with unequal potential, and the distance ξ between point P (which lies on the Earth's surface) and its projection on the telluroid surface Q is called the height anomaly. This distance corresponds to the separation between the quasi-geoid and the reference ellipsoid. Normal heights can be calculated because they do not require knowledge of the Earth's internal structure, which is an advantage of Molodensky's theory. He proposed calculating normal heights H* in 1954 using the following formula (Wen Bin Shen, Jin Li, Bin Tian, and Jiancheng Han,2010):

$$H = \frac{C}{\gamma.a}.(1 + f + m - 2f.Sin(\varphi)^2.\frac{C}{\gamma.a} + \frac{C}{\gamma.a^2}$$

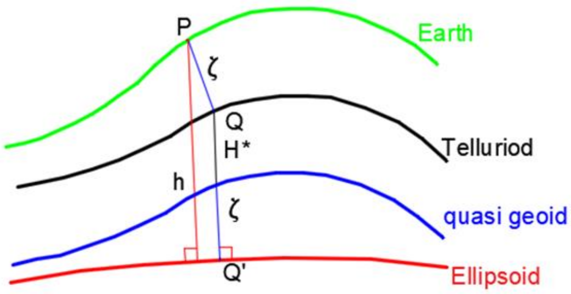

Figure 3, shows the normal heigh

3.6 Geophysical Potential Number

The geophysical potential numbers (C) cannot be observed directly because there is no instrument capable of actually measuring gravitational potential. Therefore, they are practically calculated using geophysical potential differences, which is the difference between the potential on the geoid surface W_0 and the potential Wp at the point of interest P, assuming that $w_0 = u_0 = 62636851.7146\ m^2/s^2$ on the EGM-2008 surface, as shown by the equation (Nima TR8350.2,Third Education Amidment1 .2000):

$C=W_0-W_P$

Geophysical potential numbers are measured in units of geophysical potential and do not contain units of length.

Table (1) shows the dynamic and normal heights for a sample of points in the study area and their differences from the orthometric heights referenced to mean sea level. (Heiskanen, W. A. & Moritz, H, 1967)

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$$W_P = V_P + Q = V_P + \frac{1}{2}.\omega^2.r^2.COS\varphi^2$$

Where:

φ: latitude angle, ω: The Earth's rotational speed, r: The radius of the circle passing through point P.

$$V = \frac{GM}{r}\left[1 + \sum_{n=2}^{n_{max}}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n \bar{P}_{nm}(\sin(\varphi).(\bar{C}_{nm}.\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right]$$

Where: V the Gravitational potential, GM: Earth's gravitational constant, r: Distance to the center of the Earth, a: Major axis radius of the WGS 84 ellipsoid, n, m: Degree and order of spherical harmonic, λ: longitude, P (n, m) fully Legendre Polynomials C¯. nm, S¯. nm: fully Normalized coefficients (W. E. FEATHERSTONE1 and M. C. DENTITH2, (2007))

P#	**φ** (deg)	**λ** (deg)	H (ortho)m	g (Gal)	Wp	C
1	36.78717	40.19689	360.60	979.77713	62633328.76	3533.88
2	36.99907	40.5138	451.27	979.76865	62632435.48	4422.446
3	36.70415	40.51589	435.24	979.75515	62632588.71	4265.352
4	37.06228	41.14863	462.26	979.76617	62632310.85	4530.148
5	36.92511	40.96618	391.31	979.7725	62633014.37	3834.838
6	36.64552	40.91242	336.96	979.78325	62633563.25	3302.208
7	36.25681	41.04186	358.70	979.75079	62633351.07	3515.26
8	36.5969	41.49131	357.08	979.76725	62633381.84	3499.384
9	36.84878	41.63095	375.00	979.77072	62633213.21	3675.00
10	37.11471	42.3383	423.25	979.75917	62632717.61	4147.85

Table No (1) shows the geophysical effort numbers.

Table No (2) shows the dynamic and orthometric elevations in meter of a sample of points in the study area and the differences between them and the orthometric elevations relative to sea level.

P#	H (ortho)	H (normal)	H (dynamic)	H (ortho) - H (normal)	H (ortho) – H (dynamic)
1	360.6	360.6621	360.3721	-0.06	0.23
2	451.27	451.3456	450.9848	-0.08	0.29

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3	435.24	435.3232	434.9649	-0.08	0.28
4	462.26	462.3357	461.9678	-0.08	0.29
5	391.31	391.3745	391.0627	-0.06	0.25
6	336.96	337.021	336.747	-0.06	0.21
7	358.7	358.7784	358.4733	-0.08	0.23
8	357.08	357.1473	356.8543	-0.07	0.23
9	375	375.0633	374.763	-0.06	0.24
10	423.25	423.3147	422.9825	-0.06	0.27

Table No. (2) shows the differences between the normal and dynamic elevation and the orthometric elevation for points sample from the study area.

3.7 Geometric Heights

The geometric height (h) is the distance measured along the vertical line from a point P on the Earth's surface to its projection P_0 on the reference surface, as illustrated in Figure (4). Unlike the heights mentioned in previous sections, this height is defined independently of the Earth's gravity field; that is, it is a purely geometric quantity. Geometric heights are obtained by measurements from Global Navigation Satellite Systems (GNSS)

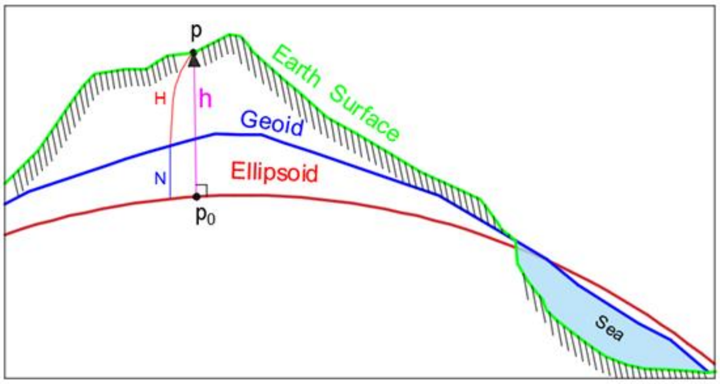

Figure 4, shows the geometric height and orthometric height

(Eteje, S. O.*, Oduyebo, O. F., Oluyori, 2019).

3.8 Statistical Analysis of Errors

After calculating the normal and dynamic heights and determining the differences between them and the actual orthometric heights, these differences were studied, analyzed, and their accuracy was assessed by determining a number of statistical indicators. The following is a summary of the statistical study (Bolshakov O.D, Markozee U.E., Golbev V.V, 1989):

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Source	sum $(\overline{H_i})$	MIN (H)	MAX (H)	RMSE
H-normal	-6.63	-0.098	-0.051	±0.06
H-dynamic	23.88	0.17	0.33	±0.24

Table No. (3) shows the results of the statistical study

Geometric heights relate to orthometric heights by the equation h - H = N, where N is the geoid undulation.

A digital elevation model (DEM) for heights has been created, as shown in Figure (5), and tested by interpolating the heights of several points in the study area.

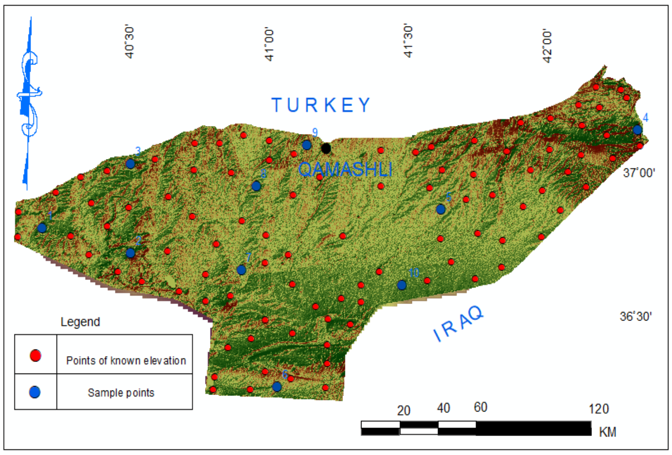

Figure 5, shows the sample of points at which the third-degree polynomial was tested

Normal heights have been improved through a third-degree polynomial used to convert normal heights to orthometric heights with accuracy 1cm, expressed by the equation:

$$-2.09648869102979e-09\,\boldsymbol{X}^3 + 2.15921952102134e-06\,\boldsymbol{X}^2 + 0.999162137596216\mathbf{X} + 0.0545122574947451$$

4. Calculation of Plumb Line Deflection in the Study Area

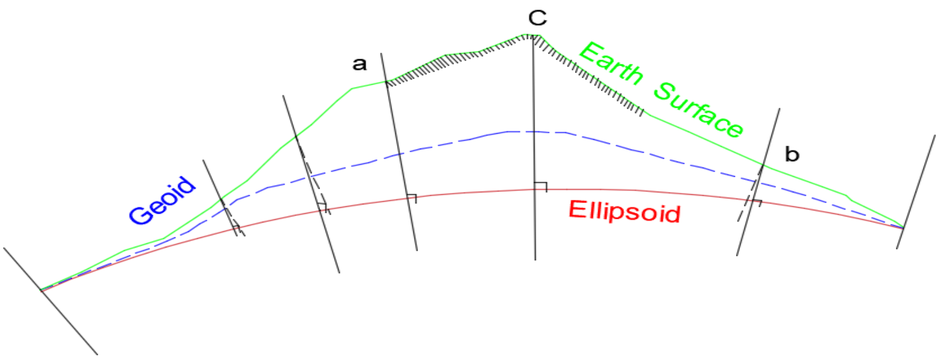

Figure 6, shows the angle of deflection of the plumb line at different locations

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The plumb line deflection is defined as the angle between the direction of the Earth's gravity (plumb line) and the normal to the (reference ellipsoid). This deflection occurs due to the inequality of gravitational forces and their directions at all points on the Earth's surface, caused by variations in the density of masses within the Earth and the imperfect nature of the reference ellipsoid in terms of dimensions, shape, orientation, and compression at the poles.

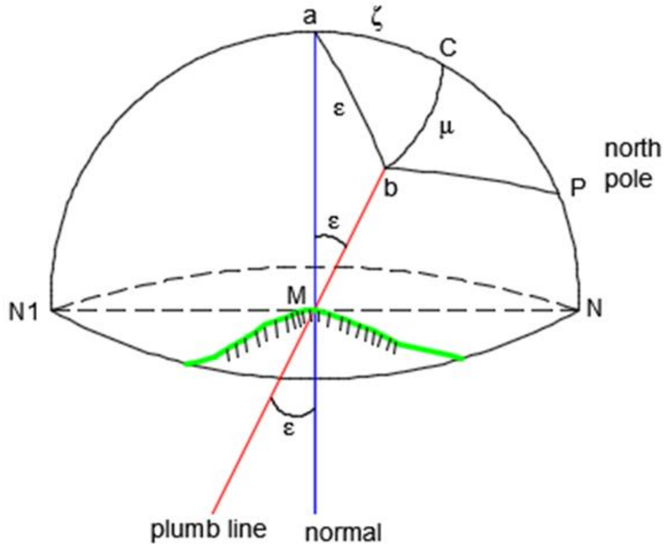

Figure7, shows the spherical dome and the angle of deflection of the plumb line

The plumb line deflection increases at the boundaries between seas, oceans, and land, as well as in high mountainous areas at mountain edges, as illustrated in Figure (6). It gradually decreases until it almost disappears in flat regions.

The plumb line deflection consists of two components: the first is north-south in the meridian plane, and the second is east-west in the prime vertical plane (prime circle). Figure (7) shows the total deflection and its two components, where a spherical cap with a radius of one length unit was created at a point M on the Earth's surface (Adam Łyszkowicz,Adam,2010).

Transitioning from astronomical coordinates (B, L, A) to geodetic coordinates (φ, λ, α) requires knowledge or calculation of the plumb line deflection according to the following formulas (Defu Che, Hang Li, Shengjun Zhang and Baodong Ma,2021)

$$\xi = B - \varphi$$

$$\mu = (L - \lambda).COSB$$

$$A - \alpha = (L - \lambda).SINB$$

$$\varepsilon = \sqrt{\xi^2 + \mu^2}$$

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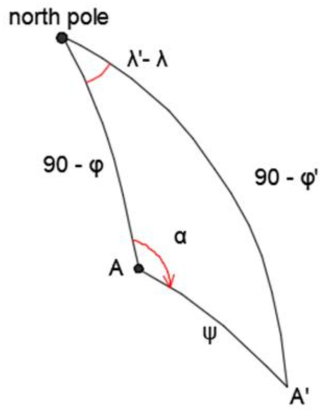

Figure 8, relation between geographical and polar coordinates on sphere

Since there are no astronomical measurements in the study area, the deflection of the plumb line can be calculated using gravity measurements.

The Stokes formula allows the calculation of the deflection of the plumb from gravity anomalies according to the following formulas (Hofmann-Wellenhof, B. & Moritz, H., 2005):

$$\xi = \frac{1}{4\pi\gamma_0}\int_{\alpha=0}^{\alpha=2\pi}\int_{\psi=0}^{\psi=\pi}\Delta g(\psi,\alpha).cos\alpha.\frac{dS(\psi)}{d\psi}.sin\psi\, d\psi\, d\alpha$$

$$\eta = \frac{1}{4\pi\gamma_0}\int_{\alpha=0}^{\alpha=2\pi}\int_{\psi=0}^{\psi=\pi}\Delta g(\psi,\alpha).sin\,\alpha.\frac{dS(\psi)}{d\psi}.sin\psi\, d\psi\, d\alpha$$

$$\frac{dS(\psi)}{d\psi} = -\frac{\cos\left(\frac{\psi}{2}\right)}{2\sin\left(\frac{\psi}{2}\right)^2} + 8sin\psi - 6\cos\left(\frac{\psi}{2}\right) - 3\frac{1-\sin\left(\frac{\psi}{2}\right)}{sin\psi} + 3sin\psi\ln[\sin\left(\frac{\psi}{2}\right) + \sin\left(\frac{\psi}{2}\right)^2]$$

$$tan\alpha = \frac{cos\varphi'\sin(\lambda'-\lambda)}{cos\varphi sin\varphi' - sin\varphi\, cos\varphi'\cos(\lambda'-\lambda)}$$

$$\gamma_0 = \frac{a.\gamma_e * \cos\varphi^2 + b.\gamma_p * sin\varphi^2}{\sqrt{a^2\, cos\varphi^2 + b^2\, sin\varphi^2}}$$

where a,b are the radii of the WGS-84 ellipsoid, γ_e, γ_p are the theoretical gravity at the equator and the theoretical gravity at the pole, and φ is the geodetic latitude angle of the measurement point. Table (4) shows the values of the plumb line deviation for a number of points in the study area. (L M Sabri, Bambang Sudarsono, and Rina Dwi Indiana, 2019)

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P#	**φ** (deg)	**λ** (deg)	*ξ* (sec)	***η*** (sec)	***θ*** (sec)
1	36.78717	40.19689	-0.01	-0.34	0.34
2	36.99907	40.5138	0.24	-0.75	0.79
3	36.70415	40.51589	-0.12	-0.66	0.67
4	37.06228	41.14863	4.72	-1.62	4.99
5	36.92511	40.96618	2.06	-4.33	4.79
6	36.64552	40.91242	-0.76	-1.24	1.45
7	36.25681	41.04186	-0.32	-0.10	0.34
8	36.5969	41.49131	-1.83	1.59	2.43
9	36.84878	41.63095	0.37	3.28	3.30
10	37.11471	42.3383	0.19	0.54	0.57

Table No. (4) shows the values of the deflection sample of points

The total deviation values were represented by contour lines as shown in Figure No (9).

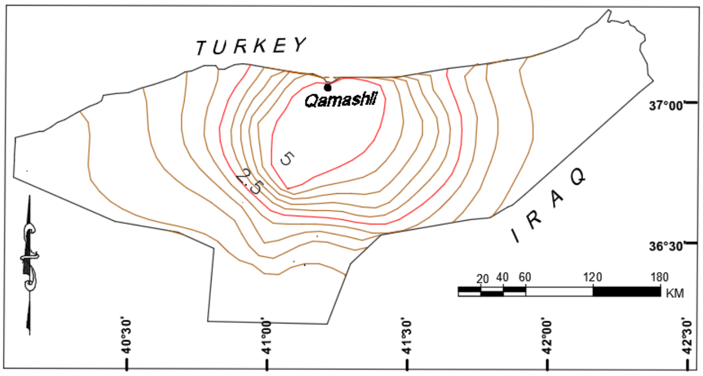

Figure 9, show contours shows the contour lines of the total deflection

5. Results and Discussion

In this research, dynamic heights and normal heights were calculated using physical potential numbers. According to the statistical study results shown in Table (3), it was found that the accuracy of dynamic heights is ± 24 cm and that of normal heights is ± 06 cm Therefore, normal heights can be used in engineering projects that do not require high accuracy when true orthometric heights are not available.

A third-degree polynomial was used to improve the normal heights from 6cm to 1cm because the differences are small.

The average elevation of the study area is approximately H = 450 m. Consequently, the effect of the maximum plumb line deflection in the study area (2.5 seconds) on horizontal point positions equals: 0.000011 × 450 = 0.005 m.

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Received: 16-08-2025 Revised: 05-09-2025 Accepted: 05-10-2025

which is a negligible value. Therefore, GPS positioning measurements can be used in engineering projects without applying plumb line deflection corrections.

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