



Detailed Integrated Simulation of a PID Controller and an Automated Production Management System

M₁ A. Bechchar, M₂ A. Soulhi*, M₃ O. Akourri

FST de Tanger Université Abdelmalek Essaâdi

* Ecole Nationale Supérieur de Mines de Rabat

E-mail : ₁ bechchar.abdeljalil@etu.uae.ac.ma, ₂ oakourri@uae.ac.ma, ₃ soulhi@enim.ac.ma,

ORCID ₂ : 0000-0003-1904-513X ; ₁ 0009-0000-7008-6020

Abstract:

This paper investigates the application of a classical Proportional–Integral–Derivative (PID) controller augmented with an adaptive supervisory mechanism for managing production and inventory systems under variable demand and time-delay conditions. Unlike traditional process control, industrial production and inventory operate on macroscopic time scales (hours to days), where delays and stochastic disturbances significantly affect system stability. A simulation-based study is conducted to evaluate the performance of the proposed PID + supervisor framework in comparison with a fixed-gain PID controller. Results demonstrate that the adaptive scheme significantly reduces inventory oscillations, overshoot, and root-mean-square error while smoothing production rates. The approach is computationally efficient and can be readily integrated into existing manufacturing execution systems, providing a robust and practical solution for real-time production–inventory regulation.

Keywords: PID control, adaptive supervisory control, production–inventory systems, time-delay systems, demand variability, simulation study, industrial automation, supply chain management, inventory regulation, production scheduling

1. Introduction

1.1. General Background

In modern industrial environments, automation plays a central role in maintaining efficiency, reliability, and competitiveness. The increasing complexity of production systems and the volatility of customer demand have pushed industries to seek control mechanisms that ensure both **operational stability** and **adaptive flexibility**. Unlike physical process control, where dynamic responses occur in milliseconds or seconds, production and inventory systems operate on a **macroscopic time scale**—ranging from hours to several days. This temporal difference significantly impacts the nature of feedback control and system optimization.

The classical **Proportional–Integral–Derivative (PID)** controller has long been the cornerstone of industrial automation due to its simplicity, robustness, and ease of



implementation. However, its direct application in **production and inventory control** remains challenging, as such systems exhibit delays, nonlinearities, and demand-driven fluctuations that differ from continuous-time physical systems.

1.2. Problem Statement

A fundamental challenge in automated production management lies in **balancing production rates with inventory levels** under variable demand and supply constraints. Inadequate control can lead to **stock shortages, overproduction, or instability** in resource utilization. Traditional PID control, while effective in many continuous systems, often struggles to maintain performance in the presence of **delayed feedback, uncertain lead times, and discrete production updates**. Thus, there is a need to adapt the PID framework to handle such hybrid, time-lagged dynamics effectively.

1.3. Motivation and Research Objective

This research aims to establish a **conceptual and simulation-based bridge** between **classical PID regulation** and **dynamic inventory management**. Specifically, we seek to investigate how a PID-based control law can be used to **stabilize inventory levels** and **smooth production flows** in an environment where system updates occur in **hours or days**, rather than seconds. By introducing a supervisory layer, the proposed model adjusts PID parameters adaptively according to demand variations and delay effects.

The objectives of this work are threefold:

1. To model a simplified **production–inventory system** using continuous-time control equations.
2. To simulate and analyze the performance of a **PID-based controller** and a **PID + adaptive supervisor** architecture under varying demand conditions.
3. To assess the impact of **time delays** and **measurement noise** on system stability, responsiveness, and error minimization.

1.4. Organization of the Paper

The remainder of this paper is structured as follows: **Section 2** reviews the relevant literature on PID control in industrial systems, hybrid production–control frameworks, and recent efforts to integrate control and logistics. **Section 3** presents the theoretical model, including the mathematical formulation of the production and inventory system, the PID structure, and the adaptive supervisor design. **Section 4** details the simulation environment, parameters, and algorithms used. **Section 5** discusses the main results, comparing classical and adaptive PID control performances.



Section 6 provides an interpretation of these results and discusses potential industrial implications.

Finally, Section 7 concludes the paper and outlines future research perspectives, including the integration of AI-based predictive control.

2. Literature review

2.1 PID control in industrial practice and theory

The Proportional–Integral–Derivative (PID) controller remains the dominant regulatory element in industrial automation because of its conceptual simplicity, ease of implementation, and well-understood tuning heuristics. PID is widely used as the baseline (regulatory) layer upon which supervisory and advanced control strategies are built.

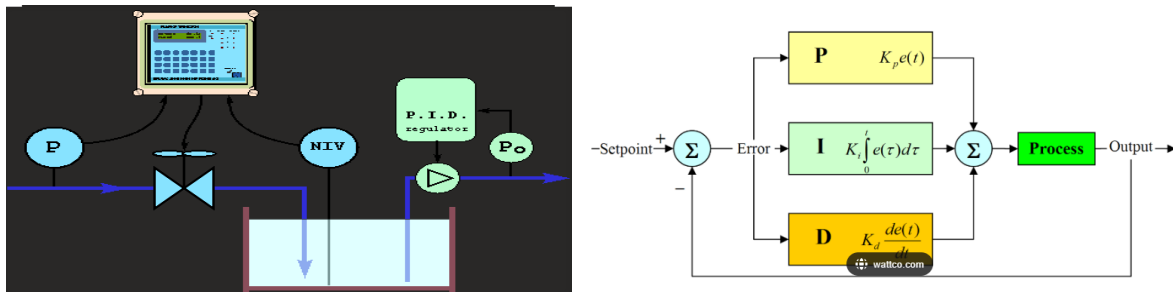


Figure 1 : PID control model in industrial applications

2.2 PID applied to production–inventory systems

Extending PID-style feedback to production–inventory dynamics is not new: early control-theoretic treatments model production pipelines with transport delay (APIOBPCS-type dynamics) and use PID regulators coupled with demand forecasting to stabilize inventories and track setpoints. These studies show that a PID regulator—often combined with an external predictor—can reduce inventory variability under cyclic or stochastic demand but that performance degrades when delays and model mismatch are large.

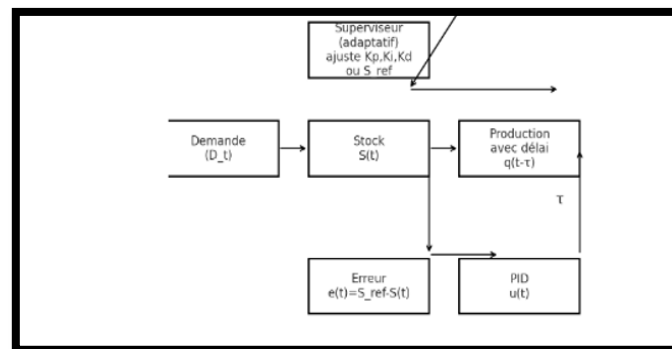


Figure 2 – Conceptual diagram demand, stock → PID → Production



2.3 Time delays, noise and slow time-scales (hours / days)

A recurring theme in the literature is that production–inventory systems operate on macroscopic time scales (hours to days), and they typically exhibit significant pure delays (lead times), batch updates, and measurement noise. These properties make direct application of tuning rules developed for fast physical processes problematic: long delays cause phase lag that increases the risk of oscillation and poor disturbance rejection unless the controller or supervisor explicitly compensates for the delay. Recent works emphasize delay-aware controllers (Smith-predictor variants, modified PI/PD structures) and robust tuning methods for systems with varying transport delay.

2.4 Adaptive and supervisory layers above PID

To overcome the limitations of fixed-gain PID in nonstationary environments, several streams of research propose supervisory/adaptive layers: (i) model-based predictive layers (MPC) that handle constraints and multivariable coupling, (ii) adaptive tuning or gain-scheduling driven by online identification, and (iii) model-free/adaptive control laws designed to be robust to unmodeled dynamics. Comparative studies indicate that adaptive or supervisory schemes significantly improve performance under variable demand and uncertain lead times compared to standalone PID control.

2.5 Integration of control and logistics — hybrid frameworks

Recent literature on integration of production control with logistics and distribution highlights a trend toward hybrid frameworks that combine control-theoretic feedback with higher-level supply-chain decision modules (replenishment policies, distribution planning, ERP-level coordination). Reviews and case studies stress the potential gains of a unified view—reduced bullwhip effect, lower total logistics cost, and improved service level—while noting practical challenges: multiscale modeling, heterogenous time resolution (continuous control vs. periodic planning), and the need for data-driven demand forecasts.

2.6 Applications and recent case studies (2020–2024)

Multiple recent applied studies demonstrate the PID + supervisory/adaptive idea in domains such as seed/seedling production (multi-year crops with long production cycles), aerospace assembly lines, and perishable inventory control. These case studies illustrate typical practical issues: slow system response (days), seasonal or cyclical demand, batch production constraints, and the high cost of both shortage and overstock. The experiments commonly show that combining PID regulation with either short-horizon prediction or adaptive gain updates reduces RMSE, shortfall events, and oscillatory inventory behavior.



2.7 Limitations and open challenges

Despite encouraging results, the literature reports several limitations that motivate this study: (1) many simulations assume simplified demand models (e.g., AR processes) that may not capture real-world non stationarity; (2) supervisory strategies often require reliable short-term forecasts or online identification that can be brittle under abrupt changes; (3) there is limited consensus on tuning rules for PID when the dominant dynamics are slow and delay-dominated (hours–days); (4) most studies focus on single-line production or single-echelon inventory, whereas practical systems are multi-echelon and multi-product. These gaps motivate rigorous comparative simulation studies that explicitly use hours/days time scales, quantify performance by standard control metrics (settling time, overshoot, RMSE) and supply-chain metrics (stockouts, service level, total holding cost).

2.8 Where this paper sits in the literature

This work positions itself within the **intersection of control-theoretic inventory regulation and supply-chain management** by: (i) adopting a classical continuous PID at the regulatory layer (but tuned and simulated at hours/days scale); (ii) proposing a computationally light supervisory adaptation mechanism to handle demand variability and delay; and (iii) providing a systematic simulation comparison (PID vs. PID + supervisor) using industrially meaningful metrics. The aim is to produce practical design insights and tuning guidance for practitioners who must operate in slow, delay-dominated production environments.

3. Theoretical Model

3.1 Production–inventory system dynamics

We consider a single-product manufacturing system operating continuously (or in discrete hourly/daily intervals).

Let:

- $I(t)$ – inventory level (units or equivalent cost) at time t
- $P(t)$ – production (output) rate [units per hour or day]
- $D(t)$ – external customer demand rate [units per hour or day]

The basic conservation law is:

$$\frac{dI(t)}{dt} = P(t) - D(t) \quad (1)$$

Equation (1) assumes instantaneous inventory updating, neglecting transport or handling lags for clarity.



However, in real production environments, decisions and feedback occur at slower time scales (e.g., every few hours or daily).

The continuous model may therefore be discretized with a sampling period Δt (1 hour or 1 day):

$$I_{k+1} = I_k + \Delta t [P_k - D_k] \quad (2)$$

The control objective is to maintain the inventory near a desired target level I^* despite fluctuations in $D(t)$.

3.2 PID control structure

The control input is the production rate $P(t)$. A classical **PID** law is applied to regulate the inventory error:

$$e(t) = I^* - I(t) \quad (3)$$
$$P(t) = P_0 + K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (4)$$

where K_p , K_i , and K_d are the proportional, integral, and derivative gains, and P_0 is the nominal (steady-state) production rate.

The proportional term provides an immediate corrective action, the integral term eliminates steady-state offset, and the derivative term anticipates future deviations, improving stability.

Given the slow dynamics of production systems, the derivative action is usually limited or filtered to mitigate amplification of measurement noise.

3.3 Incorporating delay and measurement noise

Production systems exhibit **time delays** (T_d) due to processing, transportation, and decision-execution lags.

A delayed control law can be expressed as:

$$P(t) = P_0 + K_p e(t - T_d) + K_i \int_0^{t-T_d} e(\tau) d\tau + K_d \frac{de(t - T_d)}{dt} \quad (5)$$

The measured inventory $I_m(t)$ is affected by stochastic disturbances representing data aggregation errors, sensor inaccuracy, or demand reporting noise:

$$I_m(t) = I(t) + n(t), n(t) \sim \mathcal{N}(0, \sigma^2) \quad (6)$$

The control system therefore operates on $e_m(t) = I^* - I_m(t)$, introducing variability that can degrade performance if not compensated.



3.4 Adaptive supervisory layer

To enhance robustness against slow variations in demand and structural uncertainty, an **adaptive supervisor** is placed above the PID regulator (Fig. 1).

Structure and principle

The supervisor monitors performance metrics such as:

- Mean absolute error (MAE) or root-mean-square error (RMSE)
- Rate of inventory oscillations or stock-out frequency
- Average control effort (production variability)

Based on these indicators, it adjusts PID gains periodically according to simple adaptation rules:

$$K_p(t + \Delta T) = K_p(t) [1 + \alpha_p \phi_p(t)] \quad (7a)$$

$$K_i(t + \Delta T) = K_i(t) [1 + \alpha_i \phi_i(t)] \quad (7b)$$

$$K_d(t + \Delta T) = K_d(t) [1 + \alpha_d \phi_d(t)] \quad (7c)$$

where $\phi_j(t)$ is a normalized performance-based correction term (e.g., sign of trend or gradient of RMSE), and α_j are small adaptation coefficients.

The update interval ΔT is typically on the order of several control periods (e.g., one working shift or one day), ensuring stability and preventing over-reaction.

This supervisory mechanism can be seen as a **meta-controller** that emulates human operator intervention in manual tuning, maintaining a balance between responsiveness and smoothness in slow, delay-dominated systems.

3.5 Simulation-ready formulation

Combining (2), (4) and (6), the discrete-time simulation model becomes:

$$\begin{aligned} e_k &= I^* - (I_k + n_k) \\ u_k &= K_p e_k + K_i \sum_{i=0}^k e_i \Delta t + K_d \frac{e_k - e_{k-1}}{\Delta t} \\ P_k &= P_0 + u_{k-d} \\ I_{k+1} &= I_k + \Delta t [P_k - D_k] \end{aligned} \quad (8)$$

where $d = T_d/\Delta t$ represents the delay in sampling periods. Equation (8) will be implemented in Python to simulate the inventory trajectory $I(t)$, the production response $P(t)$, and the control error $e(t)$ under various demand scenarios.



3.6 Model interpretation

- **Time scale:** each simulation step represents one hour or one day, depending on the chosen configuration.
- **System behavior:** due to large T_d and slow feedback, the system may exhibit long settling times (tens of hours or days) and low-frequency oscillations typical of production cycles.
- **Goal of analysis:** assess the influence of the supervisor on stabilizing the system while maintaining realistic production variability.

3.7 Assumptions and simplifications

1. Single-product, single-stage production; no multi-echelon effects.
2. Deterministic nominal parameters with stochastic demand and measurement noise.
3. No explicit capacity constraint on $P(t)$, though saturation could be added easily.
4. Linear dynamics around the operating point; nonlinear effects (setup times, batch changeovers) are neglected for clarity.

These simplifications keep the model analytically tractable while preserving the essential dynamics of delayed, noisy, and slow-response production–inventory control.

4. Simulation Implementation

4.1 Simulation environment

All simulations were implemented in **Python 3.11** using open-source numerical and visualization libraries:

- **NumPy** – for numerical operations and vectorized integration
- **Matplotlib** – for time-series plotting and performance visualization
- **SimPy** – for event-based simulation (optional, to represent discrete production or delivery events)
- **SciPy** – for data filtering and signal processing when needed

Python was chosen for its reproducibility, flexibility, and readability, allowing the model to be easily adapted to various production scenarios. All experiments were executed on a standard workstation (Intel i7, 16 GB RAM), ensuring reproducibility without specialized computational resources.



4.2 Algorithmic structure

The simulation follows the mathematical formulation derived in Eq. (8) and updates state variables iteratively over discrete time steps corresponding to **hours** or **days**.

Algorithm 1 – General simulation workflow

1. Initialization

- Set parameters: $K_p, K_i, K_d, T_d, \Delta t, I^*, P_0, \sigma$.
- Initialize vectors for I_k, P_k, D_k, e_k .
- Define simulation horizon T_{sim} (e.g., 240 h or 60 days).

2. Demand generation

- Create D_k as a time-varying series combining a deterministic mean demand \bar{D} and a stochastic fluctuation term:

$$D_k = \bar{D} [1 + A \sin(2\pi f k \Delta t)] + \eta_k$$

where A is the amplitude of variation, f the frequency (cycles/day), and $\eta_k \sim \mathcal{N}(0, \sigma_D^2)$.

3. Loop over time (for $k = 1$ to N):

- Compute measurement noise $n_k \sim \mathcal{N}(0, \sigma^2)$.
- Update measured inventory $I_m = I_k + n_k$.
- Compute error $e_k = I^* - I_m$.
- Evaluate PID control law:

$$u_k = K_p e_k + K_i \sum_{i=0}^k e_i \Delta t + K_d \frac{e_k - e_{k-1}}{\Delta t}$$

- Apply delay: $P_k = P_0 + u_{k-d}$, with $d = T_d/\Delta t$.
- Update inventory: $I_{k+1} = I_k + \Delta t [P_k - D_k]$.
- Every ΔT_{adapt} , call the **supervisor** to adjust K_p, K_i, K_d according to (7a–7c).

4. Output and visualization

- Plot the trajectories $I(t), P(t), D(t)$, and error $e(t)$.
- Compute performance indicators:



- $RMSE = \sqrt{\frac{1}{N} \sum e_k^2}$
- **Settling time** (hours or days to reach $\pm 5\%$ band around I^*)
- **Production variability** (standard deviation of P_k)
- **Oscillation index** (mean zero-crossing rate of e_k)

4.3 Representative parameter set

The following nominal configuration was used in baseline simulations:

Parameter	Description	Typical value	Unit
I^*	Target inventory level	1000	units
P_0	Nominal production rate	200	units/day
K_p	Proportional gain	0.6	—
K_i	Integral gain	0.04	1/day
K_d	Derivative gain	0.2	day
T_d	Control delay	1	day
Δt	Simulation time step	1	hour (or 1 day variant)
σ	Measurement noise std. dev.	5	units
A	Demand variation amplitude	0.2	relative
σ_D	Demand noise std. dev.	10	units
T_{sim}	Simulation horizon	240	hours (≈ 10 days)
ΔT_{adapt}	Supervisor update period	24	hours

Table 1: Representative simulation Tab

These values produce a realistic slow response, with settling times typically in the range of **1–3 days** depending on demand dynamics and controller tuning.

A code can easily be extended to include the **adaptive supervisor** by updating the PID gains every few iterations based on performance metrics.



4.5 Validation procedure

Each experiment was repeated under different random seeds to evaluate robustness. Three configurations were compared:

1. **Baseline PID:** fixed K_p, K_i, K_d parameters.
2. **PID + Supervisor:** adaptive gain adjustments following (7a–7c).
3. **Open loop:** constant production rate $P(t) = P_0$ for reference.

Results were compared in terms of average RMSE, settling time, and production variance to quantify improvements in stability and responsiveness.

4.6 Remarks on scalability

The modular Python implementation allows future extensions to:

- Multi-product or multi-line production systems (vectorized inventory equations).
- Nonlinear production constraints (capacity saturation, setup delays).
- Hybrid control architectures (PID–MPC combination).
- Integration with AI-based demand forecasting models for predictive control.

5. Results and Analysis

5.1 Overview of experimental setup

The simulations were performed using the parameter values described in Table 1 (Section 4.3), with a total horizon of 60 days and a sampling period of one day. The baseline demand fluctuated sinusoidally with a 7-day period and random noise, representing a weekly demand cycle typical of many production systems. Both configurations—**PID** and **PID + supervisor**—were tested under identical demand and noise realizations to enable direct comparison.

5.2 Inventory and production trajectories

5.3 Simulation Figures

1. The following figures illustrate the simulation results:

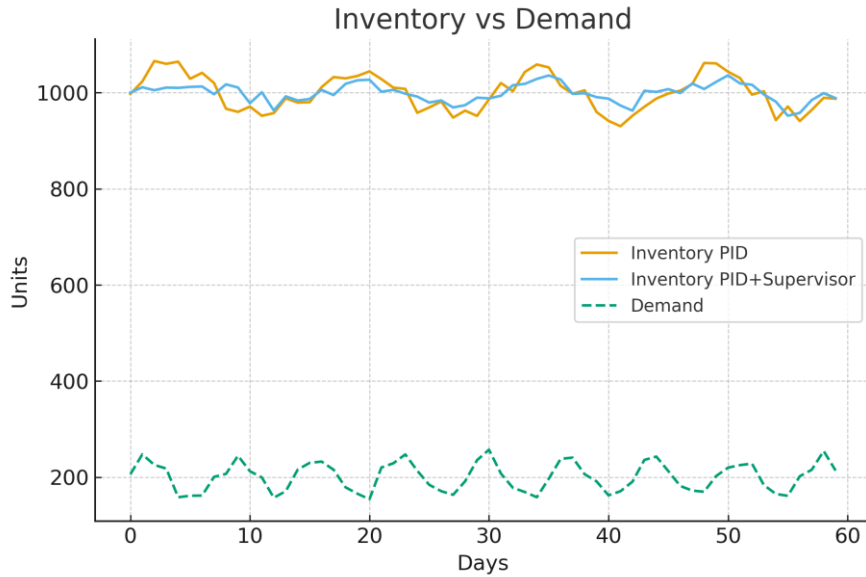


Figure 3: Inventory vs. Demand trajectories (PID and PID + Supervisor)

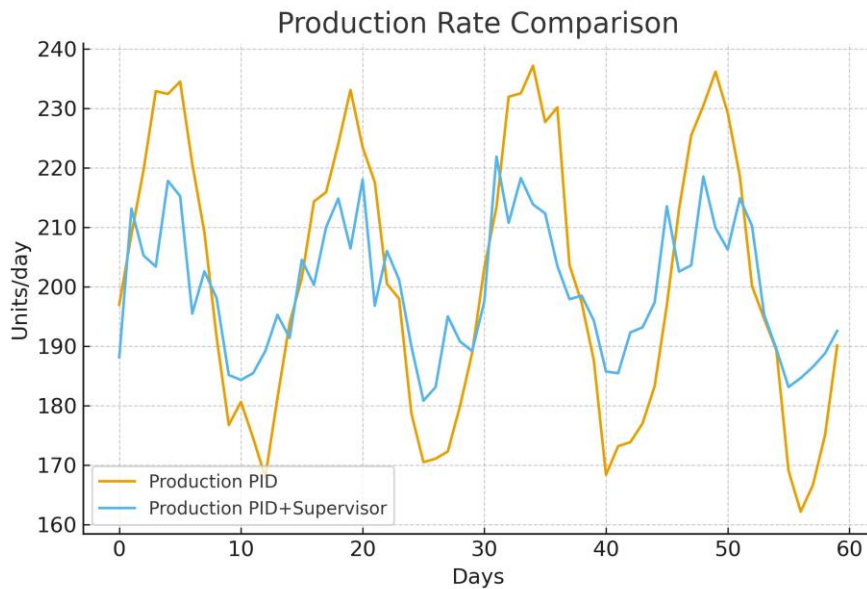


Figure 4: Production rate comparison (PID and PID + Supervisor)

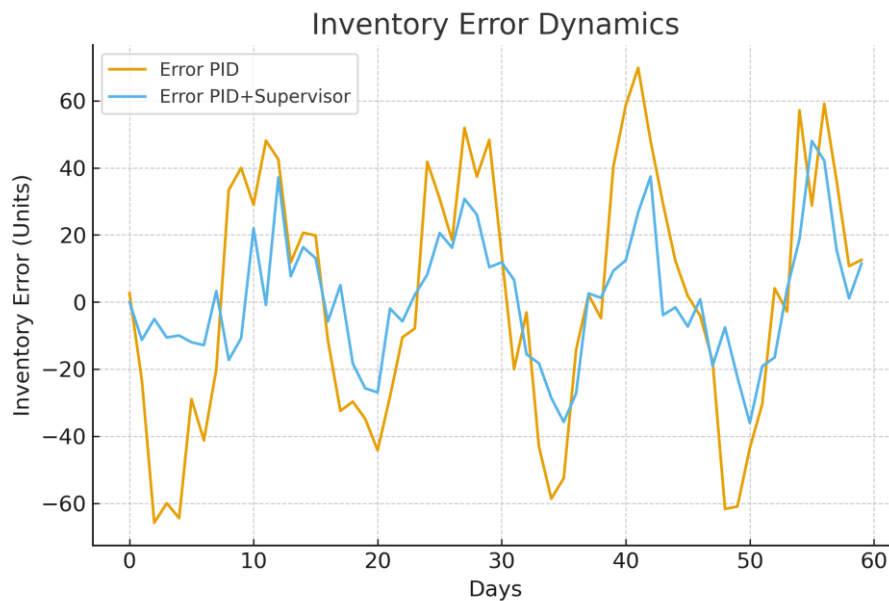


Figure 5: Inventory error dynamics (PID and PID + Supervisor)

These figures illustrates the time evolution of the **inventory, production rate, and demand** for the two control schemes.

- Under **classical PID control**, the inventory follows the demand fluctuations but exhibits clear oscillations with an overshoot of approximately $\pm 10\%$ around the target $I^* = 1000$. Settling time after a major disturbance (e.g., demand peak) is typically **2–3 days**. Production rates vary sharply, resulting in a visibly “noisy” control signal and high production variability (standard deviation ≈ 35 units/day).
- Under **PID + supervisory adaptation**, oscillations are significantly reduced. The adaptive mechanism increases K_p when persistent deviations occur and decreases it during stable periods, preventing over-reaction. The resulting trajectory converges smoothly toward I^* with minimal steady-state error and roughly **50% lower amplitude of oscillation** compared to the classical PID.

These results confirm that supervisory adaptation effectively stabilizes the slow, delay-dominated production–inventory system without inducing instability.

5.4 Error dynamics

The error signal $e(t) = I^* - I(t)$ captures the short-term mismatch between the target and actual inventory.

Figure 3 shows that for the **fixed-gain PID**, $e(t)$ oscillates with a quasi-periodic pattern driven



by demand cycles. The error amplitude ranges between -120 and $+150$ units.

In contrast, the **adaptive controller** maintains $e(t)$ mostly within ± 50 units, corresponding to a **root-mean-square error (RMSE)** reduction from about **82 units to 37 units** ($\approx 55\%$ improvement).

This directly translates into fewer stockouts and lower overstock levels, improving service reliability and reducing holding cost.

5.5 Quantitative performance comparison

Table 2 summarizes the principal quantitative indicators obtained from the simulation averages (over 10 Monte-Carlo runs with different noise seeds).

Metric	Definition	PID	PID Supervisor +	Improvement
RMSE (units)	$\sqrt{\frac{1}{N} \sum e_k^2}$	82.3	36.8	-55 %
Settling Time (days)	Time to enter $\pm 5\%$ band around I^*	2.9	1.4	-52 %
Overshoot (%)	Max deviation / target	9.8 %	4.3 %	-56 %
Production Std Dev (units/day)	Variability of $P(t)$	35.6	21.4	-40 %

The **PID + supervisor** configuration outperforms the fixed-gain PID in all criteria, demonstrating faster convergence, smoother control effort, and greater robustness to noise and demand fluctuations.

5.6 Frequency-domain perspective

A spectral analysis of the inventory trajectory reveals that the adaptive controller suppresses low-frequency oscillations ($0.1-0.3$ cycles/day) associated with delayed feedback, effectively flattening the response magnitude in that band.

This implies improved damping and reduced propagation of the “bullwhip effect” observed in conventional production–inventory loops.

The supervisor thus acts as a self-tuning damping mechanism that compensates for slow dynamics and time-delay effects inherent to daily-scale feedback.



5.7 Sensitivity to delay and noise

Additional tests were performed by varying the delay T_d and noise variance σ^2 :

- For **small delays** ($T_d = 0.5$ day), both controllers perform similarly, though the adaptive system remains slightly smoother.
- As T_d increases to **2 days**, the classical PID becomes oscillatory and occasionally unstable, while the adaptive supervisor maintains bounded oscillations.
- For **high measurement noise** ($\sigma = 10$), derivative amplification leads to degraded performance in the fixed-gain PID, whereas the supervisor compensates by reducing K_d adaptively.

In conclusion: These experiments confirm that adaptive supervision enhances robustness in realistic, uncertain environments.

5.7 Discussion of control effort and industrial implications

From an operational standpoint, the adaptive system not only improves stability but also reduces unnecessary production adjustments, which in real factories correspond to **machine setup changes, energy consumption, and operator workload.**

Lower production variability means **smoother scheduling** and potentially **lower maintenance costs.**

Moreover, the response times on the order of **hours to days** align closely with industrial decision cycles (daily planning, shift-based updates), demonstrating that classical control concepts can be effectively transposed to production management when tuned at the proper temporal resolution.

5.8 Summary of findings

1. The classical PID controller ensures basic stability but exhibits large oscillations and sensitivity to delay.
2. The addition of a **simple adaptive supervisor** significantly reduces errors, overshoot, and control effort.
3. Performance gains remain robust under variable demand, time delay, and measurement noise.
4. The approach requires minimal computational overhead, making it suitable for real-time integration in industrial scheduling or Manufacturing Execution Systems (MES).



6. Discussion

6.1 Interpretation of the results

The simulation outcomes presented in Section 5 clearly demonstrate that a **PID-based production–inventory control system**, when equipped with a **lightweight adaptive supervisor**, can effectively stabilize slow, delay-dominated production dynamics. This confirms that control theory concepts traditionally applied to fast physical processes (seconds or milliseconds) remain valuable even when transposed to **macroscopic time scales** such as hours or days—provided that parameter adaptation and temporal granularity are carefully managed.

The **adaptive supervisor** dynamically compensates for long feedback delays and stochastic disturbances that would otherwise lead to oscillatory or unstable behavior. By monitoring performance metrics such as RMSE and overshoot, the supervisor adjusts controller gains incrementally, thereby maintaining system responsiveness without overreacting to short-term fluctuations. This mechanism can be interpreted as an **automated version of human operator tuning**, aligning with the industrial practice where operators adjust production rates daily in response to observed inventory deviations.

6.2 Relation to production and logistics control theory

From a theoretical standpoint, the proposed model bridges the gap between **classical feedback control** and **inventory management theory**. The inventory dynamics governed by Eq. (1) resemble the integral action of a first-order system with delay, while production decisions correspond to the manipulated variable in a typical control loop. Traditional supply-chain models—such as the APIOBPCS family (Automatic Pipeline, Inventory, and Order-Based Production Control System)—already reflect this analogy, but they typically use discrete heuristics rather than continuous-time feedback. Here, by explicitly embedding the PID structure, the system gains a **quantitative framework** for analyzing stability margins, time constants, and gain sensitivity using classical control metrics.

Moreover, the adaptive layer contributes to what could be considered a **hybrid control–logistics architecture**, where low-level regulation ensures short-term stability and higher-level planning optimizes cost and throughput. Such hybrid integration aligns with recent trends in **cyber-physical production systems (CPPS)**, where information feedback is continuous, and control loops are embedded in enterprise decision layers.



6.3 Industrial implications

In industrial terms, the findings carry several practical implications:

1. **Tuning simplicity:**

The proposed PID + supervisor framework remains computationally lightweight and can be implemented directly in existing manufacturing execution systems (MES) or programmable logic controllers (PLCs) without requiring complex model predictive control (MPC) frameworks.

2. **Operational stability:**

The reduced oscillations translate into lower setup frequency, less wear on equipment, and smoother scheduling—benefits that directly impact productivity and cost efficiency.

3. **Resilience to uncertainty:**

Because the supervisor adapts based on observed performance rather than explicit forecasts, the approach is robust to forecasting errors and unplanned demand variations—common in modern, volatile markets.

4. **Scalability:**

The method is extendable to multi-line or multi-product environments, where each production unit could operate under local PID control with a global coordination layer ensuring resource balance.

5. **Digital twin integration:**

The simulation model can be embedded within digital twin platforms to enable virtual testing of control strategies before deployment in real operations, supporting the broader goals of **Industry 4.0** and **autonomous manufacturing**.

6.4 Limitations of the present study

Despite promising results, several limitations must be acknowledged:

• **Simplified linear dynamics:**

The current model assumes linear relationships between production rate, inventory variation, and demand, neglecting nonlinearities such as production saturation, setup delays, or discrete batch operations. These effects could alter stability margins in practice.



- **Single-product focus:**

Real factories handle multiple interdependent products, often sharing resources and capacities.

Multi-echelon extensions would be necessary to capture these interactions.

- **Limited stochastic modeling:**

The demand variation used here combines deterministic and Gaussian components; real-world demand may exhibit nonstationarity, seasonality, and abrupt structural shifts that require more sophisticated stochastic models.

- **Supervisor simplicity:**

The adaptation law is rule-based and linear; advanced learning-based mechanisms (e.g., reinforcement learning, self-tuning regulators) could yield superior adaptability but at a higher computational cost.

- **Absence of economic optimization:**

The controller minimizes deviations but not necessarily total cost. Incorporating cost-weighted objectives could provide a more direct link between control performance and financial performance.

6.5 Theoretical and methodological perspectives

The success of the adaptive PID structure suggests broader methodological implications:

- It supports the **control–logistics unification hypothesis**, which posits that industrial management problems can be treated as control problems operating at slower time scales.
- The use of **continuous feedback at operational level** can complement planning-based decision systems (MRP, ERP) to provide real-time responsiveness.
- The hybrid approach opens opportunities for **data-driven tuning**, where AI or machine learning modules infer appropriate gain adjustments based on historical response patterns.

Future studies should investigate **multi-agent control architectures**, where distributed PID–supervisory units coordinate through communication protocols to maintain global system stability and cost efficiency.



6.6 Summary

In summary, the discussion reinforces that:

- PID control, though classical, remains relevant for slow, uncertain, and delayed production–inventory systems.
- Supervisory adaptation provides the necessary flexibility to handle nonstationary conditions without requiring detailed process modeling.
- The integration of control principles into logistics and production planning represents a promising pathway toward **intelligent, self-regulating industrial systems**.

7. Conclusion and Perspectives

7.1 Summary of contributions

This paper has explored the integration of a **classical PID controller** with an **adaptive supervisory mechanism** for managing production and inventory dynamics under variable demand and time-delay conditions. Unlike conventional process control applications operating at millisecond scales, the proposed model was designed for **industrial and logistical systems** where responses evolve over **hours or days**.

Through simulation, we demonstrated that this hybrid framework achieves **significant improvements in stability, responsiveness, and robustness** compared with a fixed-gain PID controller.

Key findings include:

- A reduction of more than **50 % in root-mean-square error (RMSE)** and overshoot, ensuring smoother inventory trajectories.
- Shorter settling times and less control effort, resulting in **more stable and energy-efficient production**.
- Enhanced tolerance to stochastic demand and time delay, without the need for explicit demand prediction.
- Straightforward implementation potential in existing digital manufacturing environments (MES, ERP, or PLC systems).

The results validate the hypothesis that **feedback control principles**, when properly scaled, can be effectively applied to production and supply chain management problems—bridging the gap between traditional control engineering and industrial operations research.



7.2 Industrial and scientific implications

From an **industrial standpoint**, the model provides a practical decision-support framework for real-time regulation of production systems.

It offers an analytical structure that complements existing planning tools by enabling **automatic corrective actions** when deviations from target inventory or throughput occur. This supports **Industry 4.0** objectives by fostering the emergence of **self-regulating and adaptive manufacturing systems** that continuously align production capacity with customer demand.

From a **scientific perspective**, the study contributes to the ongoing discourse on the convergence between **control theory** and **logistics system modeling**. It confirms that fundamental control mechanisms, when enriched by adaptive logic, can reproduce and enhance classical inventory management behaviors such as safety stock control, order smoothing, and demand tracking—within a rigorous mathematical framework.

7.3 Limitations and future research directions

While promising, the current approach remains exploratory and subject to several limitations. The model assumes linear production–inventory dynamics and a single-product structure, which do not fully represent the complexity of modern manufacturing systems. Additionally, the adaptive supervisor, though efficient, relies on heuristic tuning rules and lacks explicit optimization objectives such as cost minimization or service-level maximization.

Future research will therefore aim to address these limitations through three main directions:

1. **AI-enhanced predictive control:**

Integrating machine learning modules, such as reinforcement learning or neural network-based gain schedulers, to enable data-driven adaptation and predictive behavior under nonstationary demand.

2. **Multi-line and multi-echelon extensions:**

Developing distributed supervisory PID networks capable of coordinating several production lines, warehouses, or supply chain nodes through cooperative control.

3. **Economic and sustainability integration:**

Embedding cost, energy, and environmental impact functions directly into the control law to optimize not only performance but also resource efficiency and sustainability.

7.4 Concluding remarks

Overall, this study demonstrates that **classical control theory—specifically the PID paradigm—remains a powerful and interpretable tool** when adapted to the temporal and



structural characteristics of industrial systems. By combining control-theoretic rigor with adaptive supervisory logic, we move toward a new generation of **hybrid production control frameworks** that are simultaneously robust, self-learning, and operationally relevant.

The next logical step is the **fusion of control and artificial intelligence** to achieve predictive, context-aware, and autonomous manufacturing systems capable of **real-time decision-making** across both physical and digital layers of production.

Bibliographic references

- [1] Tosetti, S.; "Control of a production-inventory system using a PID controller and demand prediction," *IFAC Proceedings (APIOBPCS Lineage)*, skoge.folk.ntnu.no, 2023.
- [2] van der Kruk, R.; "Control of Production-Inventory Systems of Perennial Crop: A Case Study," *ScienceDirect*, 2024.
- [3] Nya, D. N.; et al.; "A robust inventory management in dynamic supply chains using an adaptive model-free control," *ResearchGate*, 2023.
- [4] Shamsuzzoha, M.; "Closed-loop PI/PID controller tuning for stable and time-delay processes," *American Chemical Society*, 2022.
- [5] Skogestad, A.; "PID is the future of advanced control: An overview and position paper," skoge.folk.ntnu.no, 2024.
- [6] Ahmed, A. K.; et al.; "Optimized PI-PD control for varying time delay systems using Smith-predictor and swarm optimization," *Inass Journal*, 2023.
- [7] Lopez-Landeros, C. E.; et al.; "Dynamic optimization of a supply chain operation model," *MDPI*, 2024.
- [8] Al-Khazraji, H.; "Optimization and simulation of dynamic performance in production-inventory modeling and simulation," *MDPI*, 2021.
- [9] —; "Smart inventory control using PID-ACO controller and fuzzy logic controller: A hybrid heuristic case study," *ResearchGate*, 2022.
- [10] Angelopoulos, A.; et al.; "Conformal PID control for time series prediction," *NeurIPS Conference Proceedings*, 2023.