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A Novel Complex Multi-Fuzzy Extension of N-Soft Expert Sets with Application

Mohammad Asif [1], Hind Y. Saleh [2], Areen A. Salih [3], Muhammad Waheed Rasheed [1,*], Abid Mahboob [1], Dur e Najaf [1]

1. Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan.
2. Department of Mathematics, College of Basic Education, University of Duhok, Duhok 42001, Iraq
3. Department of Mathematics, College of Computer Science and Mathematics, University of Mosul, Mosul 41001, Iraq

* Correspondence: waheedrasheed461@gmail.com

Abstract

In this paper, a novel complex multi-fuzzy N-soft expert set (CM^kFNSES) is introduced that presents a broader spectrum of values and efficiently deals with uncertainties in data. The CM^kFNSES incorporates the opinions and parameterizations of all experts in one mode, making it quite appropriate for use in decision-making problems. The proposed CM^kFNSES can realize more range of values while handling the uncertainty of data that is captured by the amplitude terms and phase terms of the complex numbers, simultaneously. We develop an algorithm in the CM^kFNSES context for decision-making problems, including uncertainty, which is applied in our example. We also define some operations related to this new fuzzy concept, such as complement, union, intersection, AND, and OR operations, and investigate the structural properties of these operations based on this concept. An illustrative example is employed to show that it can be successfully applied to problems that contain uncertainties. In addition, a comparison between the proposed CM^kFNSES approach and other existing approaches are made to reveal the dominance of our research study.

Keywords: CM^kFNSES. decision-making problem. Selection of most performing actor

1. Introduction

Fuzzy set theory [1] is a mathematical framework that has gained widespread recognition for its ability to handle uncertain and vague information in decision-making problems. This theory allows for the representation of imprecise or fuzzy concepts, which are commonly found in real-world scenarios. In 2011, Sebastian and Ramakrishnan [2, 3] introduced the notion of multi-fuzzy sets, which extends the fuzzy set theory to handle multidimensional characterization properties. Multi-fuzzy sets have been applied to various fields, including complete colour characterization of colour images, taste recognition of food items, and decision-making problems with multiple aspects. For example, multi-fuzzy sets can be used to represent the colour information of images, where each pixel can be characterized by multiple colour components. Moreover, the hybridization of fuzzy sets with soft sets has led to the development of various models such as fuzzy soft sets [4], intuitionistic fuzzy soft sets [5], and

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multi-fuzzy soft sets [6]. These models have been applied to decision-making mechanisms, where they can provide a flexible and powerful way to represent and manipulate uncertainty and vagueness in decision-making processes. In essence, these models extend the fuzzy set theory to include additional degrees of uncertainty and vagueness, making them suitable for a wide range of applications. The ability of fuzzy set theory and its extensions to handle uncertain and vague information makes them valuable tools for decision-making in various fields, including engineering, medicine [7], economics, and social sciences.

In today's world, the need for models that can incorporate expert opinions to validate the information provided by observers has been recognized by researchers. To address this issue, Alkhazaleh and Salleh [8] introduced the concept of soft expert sets, which is a single model that combines expert opinions without any operation. As a result, there has been significant progress in the works on soft expert sets in both theories and applications. By combining the advantages of both soft sets and expert opinions, this approach provides a more accurate and reliable approach to decision-making, making it an invaluable asset in a variety of fields. To widen the scope of fuzzy sets from the real field to the complex field, Ramot et al. [9] introduced the concept of complex fuzzy sets. These sets were created to represent information with uncertainty and periodicity. Over time, several extensions of complex fuzzy sets have been developed to further enhance their applicability. These extensions include complex intuitionistic fuzzy sets [10], complex neutrosophic sets [11], complex vague soft sets [12], complex intuitionistic fuzzy soft sets [13], and complex fuzzy soft expert sets [14]. All these models have been developed to represent both the uncertainty and periodicity aspects of an object together, in a single set. With the help of these models, researchers can represent complex data sets and make more accurate and reliable decisions in various fields.

In this paper, a new model called the CM^kFNSES is introduced, which expands upon the complex multi-fuzzy sets originally proposed by Al-Qudah and Hassan [15]. The CM^kFNSES is composed of multi-membership functions, each consisting of an amplitude and a phase term. The phase term introduces wave-like features to the CM^kFNSES, allowing it to explain constructive and destructive interference based on the phase values of an element, which vary over time due to periodicity. This model is particularly helpful in tackling complex problems that involve multidimensional characterization properties. The primary objective of our research is to enhance the decision-making process by incorporating the benefits of soft expert sets into complex multi-fuzzy sets. Our proposed model, the Complex Multi-Fuzzy N-Soft Expert Set (CM^kFNSES) offers a more flexible approach to decision-making that is not limited to discrete programming [16]-[18]. The novelty of our model lies in its ability to provide a succinct, elegant, and comprehensive representation of two-dimensional multi-fuzzy information and expert opinions in a single set. This unique approach can improve the accuracy and reliability of decision-making processes, particularly in complex and uncertain environments. With the use of multi-membership functions and wave-like features, the

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CM^kFNSES can represent constructive and destructive interference that changes over time, making it an exceptionally effective solution for complex problems that require multidimensional characterization properties. In summary, this paper proposes a new model called the CM^kFNSES, which extends the concept of complex multi-fuzzy sets and incorporates the advantages of soft expert sets. The CM^kFNSES model provides a more flexible approach to decision-making, which is useful for handling problems with multidimensional characterization properties. We will demonstrate the effectiveness of our proposed model by applying it to real-world decision-making problems with multidimensional characterization properties. This will help to validate the effectiveness of our proposed model and its applicability in a wide range of decision-making problems.

The remainder of this article is formatted in the following manner. The second section of the paper introduces a key concept known as the Complex Multi-Fuzzy N-Soft Expert Set. This approach involves the use of multiple experts to make decisions based on fuzzy logic, which allows for more nuanced and flexible decision-making. Additionally, it presents the main set-theoretic operations used in this paper, including complement, union, intersection, AND, and OR. It also demonstrates some of the fundamental properties of these operations that are useful in practice. The third section of the paper focuses on a practical application of the Complex Multi-Fuzzy N-Soft Expert Set approach, namely the selection of the most performing actor for a given task, and also proposes an adjustable algorithm that incorporates this approach to enable more accurate and effective decision-making. The fourth section of the paper undertakes a comparative analysis of our proposed method with two other established techniques, to confirm the reliability of our approach. Additionally, the section delves into the merits and demerits of different hybrid structures of fuzzy sets and complex numbers, which were the driving force behind the development of our proposed methodology. The fifth section of the paper presents the conclusions derived from the study as well as the possible future directions for research. This section provides a summary of the key findings and their significance concerning the objectives of the study.

2. Preliminaries

The concept of CM^kFNSES is discussed in this section along with an illustration.

Definition 1: Take $k > 0$ as an integer, $\check{U} \neq \emptyset$ be a universe of elements, a set of parameters is denoted by E, a set of experts (agents) is represented by X. Let $R = \{0,1,2,3,\dots,N-1\}$ be the set of ratings where $N \in \{2,3,\dots,\}$ and $O = \{1 = \text{agree},\ 0 = \text{disagree}\}$ gives a set of opinions about the ratings and multi-fuzzy values. Let $Z = E \times X \times O$ and $A \subseteq Z$. A triple $(\Upsilon, \Re, \wp)$ is known as a complex multi-fuzzy N-soft expert set of dimensions (CM^kFNSES) over $\check{U}$, where Υ is a mapping given by $\Upsilon : \Re \to CM^kF^{\check{U}\times R}$ where $CM^kF^{\check{U}\times R}$ symbolizes the assembly of every complex, multi-fuzzy subset of $\check{U} \times R$.

The $CM^kFNSES(\Upsilon, \Re, \wp)$ is stated as:

$$(\Upsilon, \Re, \wp) = \{(e, \Upsilon(e)) : e \in \Re, \Upsilon(e) \in CM^kF^{\check{U}\times R}\},$$

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Where,

$$\Upsilon(e) = \left\{ \frac{\mu^{i}_{\gamma(e)}(x)}{\langle x, r_e \rangle}; x \in \check{U}, r_e \in \mathfrak{R}, j = 1,2,3, \dots, k \right\}$$

The function $\mu^{i}_{\gamma(e)}(x) = r^{i}_{\gamma(e)}(x).e^{i\omega^{i}_{\gamma(e)}(x)}$ for $j = 1,2,\dots,k$ is known as the complex multi-membership function of $CM^kFNSES(\Upsilon, \mathfrak{R}, \wp)$, k denotes the dimension of $(\Upsilon, \mathfrak{R}, \wp)$. By definition, the values of $\mu^{i}_{\gamma(e)}(x)$ may all be found on the complex plane inside the unit circle, where $(i = \sqrt{-1})$, both of the amplitude terms $r^{i}_{\gamma(e)}(x)$ and the phase terms $\omega^{i}_{\gamma(e)}(x)$ are real-valued, and real-valued, and $r^{i}_{\gamma(e)}(x) \in [0,1]$.

Example 1 A smartphone mobile company is considering launching two new models of a smartphone. Consider $\check{U} = \{x_1, x_2\}$ is the set divided into two distinct smartphone types. Before and after testing them, the company wishes to get some expert viewpoints. The expert's set is given by $X = \{p, q, r\}$. Let $E = \{e_1, e_2, e_3\}$ represent the parameters that the group of experts takes into account, where e_1 denotes camera quality which is classifiable as 120MP, 64MP and 48MP, $e_2$denotes price which consists of three stages: high, medium, low and e_3 stands for charging capacity of 54W,33W and 18W. It should be remembered that after the cell phones are tested, the parameters could potentially alter and be modified. In the first step (before testing the smartphones as claimed by the company), we use CM^kFNSES and take the advice of the experts into account as amplitude terms of the multi-membership function (MMF), and establish the experts' viewpoints in the 2nd procedure (after testing the smartphones) as phase terms of (MMF). Consequently, the 1st and 2nd processes combine to generate a CM^kFNSES. which is shown below. Ratings of these smartphones are given in Table 1.

Table 1: Ratings of smartphones.

$\check{U}$	e_1	e_2	e_3
x_1	◊◊◊	◊◊◊◊	◊
x_2	◊◊	◊◊◊◊◊	◊◊◊

Where five diamonds represent 'outstanding', four diamonds represent 'excellent', three diamonds represent 'pretty good', two diamonds represent 'good', one diamond represents 'ordinary', square represents 'bad'. These diamond ratings are simple to include with the set of scores $R = \{0,1,2,3,4,5\}$ in such a way:

0 acts as ''
1 acts as '◊'
2 acts as '◊◊'
3 acts as '◊◊◊'
4 acts as '◊◊◊◊'

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5 acts as '◊◊◊◊◊'

The criterion used by the experts for the grading of smartphones is shown in Table 2.

Table 2: Criterion for the grade distribution

Grade	Criterion
0	$\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}=0$
1	$0<\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}\leq 0.2$
2	$0.2<\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}\leq 0.4$
3	$0.4<\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}\leq 0.6$
4	$0.6<\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}\leq 0.8$
5	$0.8<\frac{\sum_{j=1}^{k}\Delta\mu_{\Re}^{j}(x)}{k}\leq 1.0$

Where;

$$\Delta\mu_{\Re}^{j}(x)=\frac{r_{\Re}^{j}(x)+\omega_{\Re}^{j}(x)}{2}\ for\ j=1,2,3,\dots,k$$

Assume the business sent surveys to p,q and r to help them decide what smartphones to purchase. Therefore, the following CM^kF6SES by using Definition 1, is defined:

$$F(e_1,p,1)=\left\{\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_1,q,1)=\left\{\frac{(0.5e^{i2\pi(0.54)},0.59e^{i2\pi(0.48)},0.4e^{i2\pi(0.47)})}{\langle x_1,3\rangle},\frac{(0.32e^{i2\pi(0.2)},0.21e^{i2\pi(0.38)},0.33e^{i2\pi(0.25)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_1,r,1)=\left\{\frac{(0.58e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_2,p,1)=\left\{\frac{(0.7e^{i2\pi(0.74)},0.6e^{i2\pi(0.69)},0.6e^{i2\pi(0.66)})}{\langle x_1,4\rangle},\frac{(0.8e^{i2\pi(0.91)},0.94e^{i2\pi(0.99)},0.85e^{i2\pi(0.8)})}{\langle x_2,5\rangle}\right\}$$

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$$F(e_2,q,1) = \left\{\frac{(0.62e^{i2\pi(0.67)},0.78e^{i2\pi(0.6)},0.65e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.9e^{i2\pi(0.98)},0.8e^{i2\pi(0.84)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}$$

$$F(e_2,r,1) = \left\{\frac{(0.63e^{i2\pi(0.61)},0.7^{i2\pi(0.77)},0.68e^{i2\pi(0.64)})}{\langle x_1,4\rangle},\frac{(0.92e^{i2\pi(0.86)},0.82e^{i2\pi(0.93)},0.95e^{i2\pi(0.96)})}{\langle x_2,5\rangle}\right\}$$

$$F(e_3,p,1) = \left\{\frac{(0.1e^{i2\pi(0.19)},0.11^{i2\pi(0.09)},0.1e^{i2\pi(0.18)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.4e^{i2\pi(0.46)},0.5e^{i2\pi(0.53)})}{\langle x_2,4\rangle}\right\}$$

$$F(e_3,q,1) = \left\{\frac{(0.19e^{i2\pi(0.15)},0.12^{i2\pi(0.1)},0.17e^{i2\pi(0.05)})}{\langle x_1,1\rangle},\frac{(0.5e^{i2\pi(0.41)},0.4e^{i2\pi(0.44)},0.51e^{i2\pi(0.55)})}{\langle x_2,4\rangle}\right\}$$

$$F(e_3,r,1) = \left\{\frac{(0.0e^{i2\pi(0.08)},0.1^{i2\pi(0.1)},0.16e^{i2\pi(0.14)})}{\langle x_1,1\rangle},\frac{(0.57e^{i2\pi(0.5)},0.4e^{i2\pi(0.49)},0.45e^{i2\pi(0.49)})}{\langle x_2,4\rangle}\right\}$$

$$F(e_1,p,0) = \left\{\frac{(0.44e^{i2\pi(0.50)},0.42e^{i2\pi(0.52)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.34)},0.3e^{i2\pi(0.37)},0.35e^{i2\pi(0.26)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_1,q,0) = \left\{\frac{(0.55e^{i2\pi(0.5)},0.5e^{i2\pi(0.47)},0.43e^{i2\pi(0.4)})}{\langle x_1,3\rangle},\frac{(0.3e^{i2\pi(0.23)},0.22e^{i2\pi(0.31)},0.33e^{i2\pi(0.29)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_1,r,0) = \left\{\frac{(0.51e^{i2\pi(0.55)},0.54e^{i2\pi(0.49)},0.45e^{i2\pi(0.48)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.27)},0.30e^{i2\pi(0.36)},0.28e^{i2\pi(0.3)})}{\langle x_2,2\rangle}\right\}$$

$$F(e_2,p,0) = \left\{\frac{(0.74e^{i2\pi(0.7)},0.54e^{i2\pi(0.49)},0.69e^{i2\pi(0.68)})}{\langle x_1,4\rangle},\frac{(0.81e^{i2\pi(0.88)},0.94e^{i2\pi(0.97)},0.8e^{i2\pi(0.86)})}{\langle x_2,5\rangle}\right\}$$

$$F(e_2,q,0) = \left\{\frac{(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}$$

$$F(e_2,r,0) = \left\{\frac{(0.61e^{i2\pi(0.62)},0.77e^{i2\pi(0.71)},0.6e^{i2\pi(0.65)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.87)},0.83e^{i2\pi(0.9)},0.91e^{i2\pi(0.99)})}{\langle x_2,5\rangle}\right\}$$

$$F(e_3,p,0) = \left\{\frac{(0.12e^{i2\pi(0.11)},0.19^{i2\pi(0.1)},0.18e^{i2\pi(0.11)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.41e^{i2\pi(0.4)},0.56e^{i2\pi(0.57)})}{\langle x_2,3\rangle}\right\}$$

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$$F(e_3,q,0) = \left\{ \frac{(0.14e^{i2\pi(0.19)}, 0.12^{i2\pi(0.19)}, 0.05e^{i2\pi(0.17)})}{\langle x_1, 1\rangle}, \frac{(0.51e^{i2\pi(0.49)}, 0.44e^{i2\pi(0.54)}, 0.41e^{i2\pi(0.49)})}{\langle x_2, 3\rangle} \right\}$$

$$F(e_3,r,0) = \left\{ \frac{(0.1e^{i2\pi(0.08)}, 0.12^{i2\pi(0.17)}, 0.13e^{i2\pi(0.19)})}{\langle x_1, 1\rangle}, \frac{(0.52e^{i2\pi(0.5)}, 0.59e^{i2\pi(0.49)}, 0.55e^{i2\pi(0.58)})}{\langle x_2, 3\rangle} \right\}$$

The amplitude terms in the CM^kFNSES above indicates the expert opinions in procedure 1st (beforeatesting the smart-˘ phones), whereas the phase terms represent the expert opinions in procedure 2nd (after smartphone testing). To further understand what we meant, have a look at the estimation.

$$F(e_1,p,1) = \left\{ \frac{(0.4e^{i2\pi(0.51)}, 0.5e^{i2\pi(0.42)}, 0.57e^{i2\pi(0.43)})}{\langle x_1, 3\rangle} \right\}$$

For the parameter $(e_1, p, 1)$, the first membership values $[0.4e^{i2\pi(0.51)}]$ for the first smartphone kind (x_1) shows that before testing the smartphone, expert p thinks that the camera quality of a smartphone of type one is 129MP with a degree of 0.4, but after testing the same smartphone he modified his former viewpoint, and now concluded that the camera quality being 120MP is only 0.51 degree. For the same parameter and smartphone, the second membership values $[0.5e^{i2\pi(0.42)}]$ shows that before testing the smartphone, expert p believed that the camera quality of a smartphone of type one is 64MP with degree 0.5, however, after testing the same smartphone, he changed his mind about it and now stated that the camera quality being 64MP is only of degree 0.42. For the third membership value, $[0.57e^{i2\pi(0.43)}]$ for the same parameter and smartphone demonstrates that before testing the smartphone, expert p believed that the camera quality of smartphone of type one is 48MP with degree 0.57, but after testing the same smartphone he changed his mind, and now believes that the camera quality being 48MP is only of degree 0.43. In the end, the expert p gave a grade of '3' using the criteria defined in Table 2. The following definitions will introduce the idea of the subset and equality operations on twoă CM^kFNSES.

Definition 2. Consider $\mathfrak{R}, B \in E$. Take two CM^kFNSES $(\Upsilon, \mathfrak{R}, \wp)$ and $(G, \mathcal{B}, \wp)$ on the universe Ǔ. $(\Upsilon, \mathfrak{R}, \wp)$ is considered CM^kFNSE subset of $(G, \mathcal{B}, \wp)$ when we have,

1. $\mathfrak{R} \subseteq \mathcal{B}$.
2. $\forall e \in \mathfrak{R}, \Upsilon(e) \subseteq G(e)$

The subset relation is represented by $(\Upsilon, \mathfrak{R}, \wp) \subseteq (G, \mathcal{B}, \wp)$.

$$\mathfrak{R} = \{(e_1,p,1),(e_2,q,1),(e_3,p,1),(e_1,r,0),(e_2,q,0)\}$$

and

$$\mathcal{B} = \{(e_1,r,1),(e_1,p,1),(e_2,q,1),(e_3,q,1),(e_3,p,0),(e_1,r,0),(e_2,q,0)\}$$

Clearly $\mathfrak{R} \subseteq \mathcal{B}$. Take two $CM^kF6NSES$ $(\Upsilon, \mathfrak{R}, \wp)$ and $(G, B, \wp)$ over the universe Ǔ as follows:

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$$(\Upsilon,\Re,6)=\left\{\begin{array}{l}\left((e_1,p,1),\left\{\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_2,2\rangle}\right\}\right),\\ \left((e_2,q,1),\left\{\frac{(0.62e^{i2\pi(0.67)},0.78e^{i2\pi(0.6)},0.65e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.9e^{i2\pi(0.98)},0.8e^{i2\pi(0.84)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\ \left((e_3,p,1),\left\{\frac{(0.1e^{i2\pi(0.19)},0.11^{i2\pi(0.09)},0.1e^{i2\pi(0.18)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.4e^{i2\pi(0.46)},0.5e^{i2\pi(0.53)})}{\langle x_2,4\rangle}\right\}\right),\\ \left((e_1,r,0),\left\{\frac{(0.51e^{i2\pi(0.55)},0.54e^{i2\pi(0.49)},0.45e^{i2\pi(0.48)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.27)},0.30e^{i2\pi(0.36)},0.28e^{i2\pi(0.3)})}{\langle x_2,2\rangle}\right\}\right),\\ \left((e_2,q,0),\left\{\frac{(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right)\end{array}\right\}$$

$$(\mathcal{G},\mathcal{B},6)=\left\{\begin{array}{l}\left((e_1,p,1),\left\{\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_2,2\rangle}\right\}\right),\\ \left((e_2,q,1),\left\{\frac{(0.62e^{i2\pi(0.67)},0.78e^{i2\pi(0.6)},0.65e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.9e^{i2\pi(0.98)},0.8e^{i2\pi(0.84)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\ \left((e_2,q,0),\left\{\frac{(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\ \left((e_3,p,1),\left\{\frac{(0.1e^{i2\pi(0.19)},0.11^{i2\pi(0.09)},0.1e^{i2\pi(0.18)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.4e^{i2\pi(0.46)},0.5e^{i2\pi(0.53)})}{\langle x_2,4\rangle}\right\}\right),\\ \left((e_3,p,1),\left\{\frac{(0.1e^{i2\pi(0.19)},0.11^{i2\pi(0.09)},0.1e^{i2\pi(0.18)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.4e^{i2\pi(0.46)},0.5e^{i2\pi(0.53)})}{\langle x_2,4\rangle}\right\}\right),\\ \left((e_1,r,0),\left\{\frac{(0.51e^{i2\pi(0.55)},0.54e^{i2\pi(0.49)},0.45e^{i2\pi(0.48)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.27)},0.30e^{i2\pi(0.36)},0.28e^{i2\pi(0.3)})}{\langle x_2,2\rangle}\right\}\right),\\ \left((e_2,q,0),\left\{\frac{(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\ \left((e_3,p,0),\left\{\frac{(0.12e^{i2\pi(0.11)},0.19^{i2\pi(0.1)},0.18e^{i2\pi(0.11)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.41e^{i2\pi(0.4)},0.56e^{i2\pi(0.57)})}{\langle x_2,3\rangle}\right\}\right)\end{array}\right\}$$

Thus $(\Upsilon,\Re,6)\subseteq(\mathcal{G},\mathcal{B},6)$.

Definition 3. Suppose that $\Re,\mathcal{B}\in E$. Take two $CM^kFNSESs$ $(\Upsilon,\Re,\wp)$ and $(\mathcal{G},\mathcal{B},\wp)$ over the universe $(\Upsilon,\Re,\wp)$ *and* $(\mathcal{G},\mathcal{B},\wp)$are considered equal sets, when we have $(\Upsilon,\Re,\wp)$ as a CM^kFNSE subset of $(\mathcal{G},\mathcal{B},\wp)$ and $(\mathcal{G},\mathcal{B},\wp)$ as a CM^kFNSE subset of $(\Upsilon,\Re,\wp)$. $(\Upsilon,\Re,\wp)$. The descriptions of an agree-$CM^kFNSESs$ and a disagree-$CM^kFNSESs$ are now offered.

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Definition 4. A CM^kFNSE subset of $(\Upsilon, \Re, \wp)$ is considered as an agree-$CM^kFNSESs(\Upsilon, \Re, \wp)_1$ on the universe, Ǔ and is represented as

$$(\Upsilon, \Re, \wp)_1 = \{\Upsilon_1(e) : e \in E \times X \times \{1\}\}.$$

Example 3 Rethink about the Example 1. The $CM^kF6SES\,(\Upsilon, \Re, \wp)$ is an agree-$CM^kFNSESs(\Upsilon, \Re, 6)_1$ on the universe Ǔ and can be given as

$$(\Upsilon, \Re, 6)_1 = \left\{\begin{array}{l}
\left((e_2,q,1),\left\{\frac{(0.62e^{i2\pi(0.67)},0.78e^{i2\pi(0.6)},0.65e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.9e^{i2\pi(0.98)},0.8e^{i2\pi(0.84)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_1,p,1),\left\{\frac{(0.5e^{i2\pi(0.54)},0.59e^{i2\pi(0.48)},0.4e^{i2\pi(0.47)})}{\langle x_1,3\rangle},\frac{(0.32e^{i2\pi(0.2)},0.21e^{i2\pi(0.38)},0.33e^{i2\pi(0.25)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_1,q,1),\left\{\frac{(0.5e^{i2\pi(0.54)},0.59e^{i2\pi(0.48)},0.4e^{i2\pi(0.47)})}{\langle x_1,3\rangle},\frac{(0.32e^{i2\pi(0.2)},0.21e^{i2\pi(0.38)},0.33e^{i2\pi(0.25)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_1,r,1),\left\{\frac{(0.58e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_2,p,1),\left\{\frac{(0.7e^{i2\pi(0.74)},0.6e^{i2\pi(0.69)},0.6e^{i2\pi(0.66)})}{\langle x_1,4\rangle},\frac{(0.8e^{i2\pi(0.91)},0.94e^{i2\pi(0.99)},0.85e^{i2\pi(0.8)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_2,r,1),\left\{\frac{(0.63e^{i2\pi(0.61)},0.7^{i2\pi(0.77)},0.68e^{i2\pi(0.64)})}{\langle x_1,4\rangle},\frac{(0.92e^{i2\pi(0.86)},0.82e^{i2\pi(0.93)},0.95e^{i2\pi(0.96)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_3,p,1),\left\{\frac{(0.1e^{i2\pi(0.19)},0.11^{i2\pi(0.09)},0.1e^{i2\pi(0.18)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.4e^{i2\pi(0.46)},0.5e^{i2\pi(0.53)})}{\langle x_2,4\rangle}\right\}\right),\\
\left((e_3,q,1),\left\{\frac{(0.19e^{i2\pi(0.15)},0.12^{i2\pi(0.1)},0.17e^{i2\pi(0.05)})}{\langle x_1,1\rangle},\frac{(0.5e^{i2\pi(0.41)},0.4e^{i2\pi(0.44)},0.51e^{i2\pi(0.55)})}{\langle x_2,4\rangle}\right\}\right)\\
,\left((e_3,r,1),\left\{\frac{(0.0e^{i2\pi(0.08)},0.1^{i2\pi(0.1)},0.16e^{i2\pi(0.14)})}{\langle x_1,1\rangle},\frac{(0.57e^{i2\pi(0.5)},0.4e^{i2\pi(0.49)},0.45e^{i2\pi(0.49)})}{\langle x_2,4\rangle}\right\}\right)
\end{array}\right\}$$

Definition 5. A disagree- $CM^kFNSES(\Upsilon, \Re, \wp)_0$ over Ǔ is a CM^kFNSES subset of $(\Upsilon, \Re, \wp)$ which is formed this way:

$$(\Upsilon, \Re, \wp)_0 = \{\Upsilon_0(e) : e \in E \times X \times \{0\}\}.$$

Example 4 Rethink about the Example 1. Then a disagree- $CM^kF6SES(\Upsilon, \Re, \wp)_0$ of the -$CM^kF6SES(\Upsilon, \Re, \wp)$ over Ǔ is written as:

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$$(\Upsilon,\Re,6)_0 = \left\{\begin{array}{l}
\left((e_1,p,0),\left\{\frac{(0.44e^{i2\pi(0.50)},0.42e^{i2\pi(0.52)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.34)},0.3e^{i2\pi(0.37)},0.35e^{i2\pi(0.26)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_1,q,0),\left\{\frac{(0.55e^{i2\pi(0.5)},0.5e^{i2\pi(0.47)},0.43e^{i2\pi(0.4)})}{\langle x_1,3\rangle},\frac{(0.3e^{i2\pi(0.23)},0.22e^{i2\pi(0.31)},0.33e^{i2\pi(0.29)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_1,r,0),\left\{\frac{(0.51e^{i2\pi(0.55)},0.54e^{i2\pi(0.49)},0.45e^{i2\pi(0.48)})}{\langle x_1,3\rangle},\frac{(0.21e^{i2\pi(0.27)},0.30e^{i2\pi(0.36)},0.28e^{i2\pi(0.3)})}{\langle x_2,2\rangle}\right\}\right),\\
\left((e_2,p,0),\left\{\frac{(0.74e^{i2\pi(0.7)},0.54e^{i2\pi(0.49)},0.69e^{i2\pi(0.68)})}{\langle x_1,4\rangle},\frac{(0.81e^{i2\pi(0.88)},0.94e^{i2\pi(0.97)},0.8e^{i2\pi(0.86)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_2,q,0),\left\{\frac{(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_2,r,0),\left\{\frac{(0.61e^{i2\pi(0.62)},0.77e^{i2\pi(0.71)},0.6e^{i2\pi(0.65)})}{\langle x_1,4\rangle},\frac{(0.91e^{i2\pi(0.87)},0.83e^{i2\pi(0.9)},0.91e^{i2\pi(0.99)})}{\langle x_2,5\rangle}\right\}\right),\\
\left((e_3,p,0),\left\{\frac{(0.12e^{i2\pi(0.11)},0.19^{i2\pi(0.1)},0.18e^{i2\pi(0.11)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.56)},0.41e^{i2\pi(0.4)},0.56e^{i2\pi(0.57)})}{\langle x_2,3\rangle}\right\}\right),\\
\left((e_3,q,0),\left\{\frac{(0.14e^{i2\pi(0.19)},0.12^{i2\pi(0.19)},0.05e^{i2\pi(0.17)})}{\langle x_1,1\rangle},\frac{(0.51e^{i2\pi(0.49)},0.44e^{i2\pi(0.54)},0.41e^{i2\pi(0.49)})}{\langle x_2,3\rangle}\right\}\right),\\
\left((e_3,r,0),\left\{\frac{(0.1e^{i2\pi(0.08)},0.12^{i2\pi(0.17)},0.13e^{i2\pi(0.19)})}{\langle x_1,1\rangle},\frac{(0.52e^{i2\pi(0.5)},0.59e^{i2\pi(0.49)},0.55e^{i2\pi(0.58)})}{\langle x_2,3\rangle}\right\}\right)
\end{array}\right\}$$

Here are several fundamental algebraic properties and operations of CM^kFNSESs, such as complement, union, intersection, AND, and OR, along with examples to illustrate each.

Definition 6. Take $(\Upsilon,\Re,\wp)$ as a CM^kFNSES from the universe Ŭ consisting of n elements. The complement is denoted by $(\Upsilon,\Re,\wp)^c$ and is given by $(\Upsilon,\Re,\wp)^c = (\Upsilon^c,\sim\Re,\wp)$ while a function $\Upsilon^c :\sim A \rightarrow CM^kF^{\breve{U}\times R}$ which can be defined as follows:

$$\Upsilon^c(e) = \left\{\frac{\mu^j_{\Upsilon^c(e)}(x_i) = r^j_{\Upsilon^c(e)}(x_i).\, e^{i\omega^i_{\Upsilon^c(e)}(x_i)}}{\langle x_i, N - r_e\rangle}; x \in \breve{U}, r_e \in \Re, i = 1,2,3,\dots,n\right\}$$

where the amplitude term's complement is $r^i_{\Upsilon^c(e)}(x) = 1 - r^i_{\Upsilon(e)}(x)$) and the complement of the phase term is $\omega^j_{\Upsilon^c(e)}(x) = 2x - i\omega^j_{\Upsilon(e)}(x)\ for\ j = 1,2,\dots,k$.

Example 5 Consider the approximations of CM^kF6SES as taken in the Example 1, while

$$F(e_1,p,1) = \left\{\frac{(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)})}{\langle x_1,3\rangle},\frac{(0.2e^{i2\pi(0.35)},0.3e^{i2\pi(0.39)},0.34e^{i2\pi(0.29)})}{\langle x_2,2\rangle}\right\}$$

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By using Definition 6, We acquire the approximation's complement as

$$F^c(e_1,p,1) = \left\{\frac{(0.6e^{i2\pi(0.49)}, 0.5e^{i2\pi(0.58)}, 0.43e^{i2\pi(0.57)})}{\langle x_1,3\rangle}, \frac{(0.8e^{i2\pi(0.65)}, 0.7e^{i2\pi(0.61)}, 0.66e^{i2\pi(0.71)})}{\langle x_2,2\rangle}\right\}$$

Proposition 1. If $(\Upsilon,\Re,\wp)$ is a CM^kFNSES over Ǔ then $((\Upsilon,\Re,\wp)^c)^c = (\Upsilon,\Re,\wp)$.

Proof. By using Definition 6, we can write $(\Upsilon,\Re,\wp)^c = (\Upsilon^c,\sim\Re,\wp)$ while

$$(\Upsilon,\Re,\wp)^c = \left\{\frac{r^j_{\Upsilon^c(e)}(x).e^{i\omega^i_{\Upsilon^c(e)}(x)}}{\langle x, N-r_e\rangle}; x\in Ǔ, r_e\in\Re, j=1,2,3,\dots,k\right\}$$

$$= \left\{\frac{[1-r^i_{\Upsilon(e)}(x)].e^{i[2x-\omega^i_{\Upsilon^c(e)}(x)]}}{\langle x, N-r_e\rangle}; x\in Ǔ, r_e\in\Re, j=1,2,3,\dots,k\right\}$$

We can suppose now $(\Upsilon,\Re,\wp)^c = (\mathcal{G},\mathcal{B},\wp) = (\Upsilon^c,\sim\Re,\wp)$ Next, we discover the following:

$$(\mathcal{G},\mathcal{B},\wp)^c = \left\{\frac{[1-r^j_{\Upsilon^c(e)}(x)].e^{i[2\pi-\omega^i_{\Upsilon^c(e)}(x)]}}{\langle x, N-(N-r_e)\rangle}; x\in Ǔ, r_e\in\Re, j=1,2,3,\dots,k\right\}$$

$$= \left\{\frac{[1-(1-r^j_{\Upsilon^c(e)}(x))].e^{i[2\pi-\omega^i_{\Upsilon^c(e)}(x)]}}{\langle x, N-(N-r_e)\rangle}; x\in Ǔ, r_e\in\Re, j=1,2,3,\dots,k\right\}$$

$$= \left\{\frac{r^i_{\Upsilon(e)}(x).e^{i\omega^i_{\Upsilon(e)}(x)}}{\langle x, r_e\rangle}; x\in Ǔ, r_e\in\Re, j=1,2,3,\dots,k\right\}$$

$= (\Upsilon,\Re,\wp)$.

We will now define the intersection and union operations of two $CM^kFNSESs$ and provide a suitable illustration. The intersection operation uses the minimum ǎ operators, whereas the union operation uses the maximum.

Definition 7. A CM^kFNSES $(\mathcal{H},C,\wp)$, is referred to as the union of two $CM^kFNSESs(\Upsilon,\Re,\wp)$ and $(\mathcal{G},\mathcal{B},\wp)$ on the universe Ǔ, taking $C = \Re\cup\mathcal{B}$, $\forall e \in C$ and $x \in$ Ǔ,

$$\mathcal{H}(e) = \begin{cases} \Upsilon(e) = r^i_{\Upsilon(e)}(x).e^{i\omega^i_{\Upsilon(e)}(x)} & if\ e\in\Re-\mathcal{B} \\ \mathcal{G}(e) = r^i_{\mathcal{G}(e)}(x).e^{i\omega^i_{\mathcal{G}(e)}(x)} & if\ e\in\mathcal{B}-\Re \\ \Upsilon(e)\sqcup\mathcal{G}(e) = max\left[r^i_{\Upsilon(e)}(x), r^i_{\mathcal{G}(e)}(x)\right].e^{i\,max\left[\omega^i_{\Upsilon(e)}(x),\omega^i_{\mathcal{G}(e)}(x)\right]} & ,if\ e\in\Re\cap\mathcal{B} \end{cases}$$

It is denoted by $(\Upsilon,\Re,\wp)\ \bar{\cup}\ (\mathcal{G},\mathcal{B},\wp)$ and we can write $(\mathcal{H},C,\wp) = (\Upsilon,\Re,\wp)\ \bar{\cup}\ (\mathcal{G},\mathcal{B},\wp)$.

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Example 6 Take a set Ǔ of the universe consisting of two objects, $E = \{e_1, e_2, e_3\}$ represents the set of attributes, and $X = p, q$ denotes the set of agents for giving opinions about the objects under consideration. The experts assign a grade to each object from the set of grades i.e. $R = \{1,2,\dots,N-1\}$ with $N \in \{2,3,\dots\}$. Let $\mathfrak{R} = \{(e_1,p,1),(e_1,q,1),(e_3,p,0)\}$ and $\mathcal{B} = \{(e_1,q,1),(e_2,r,1),(e_3,p,0),(e_1,q,0)\}$. Let us say that $(\Upsilon,\mathfrak{R},\wp)$ *and* $(\mathcal{G},\mathcal{B},\wp)$ are two $CM^kFNSESs$ with dimension three on the universe Ǔ written as below:

$$(\Upsilon,\mathfrak{R},\wp) = \left\{\begin{array}{l} \left((e_1,p,1),\left\{\frac{\left(0.6e^{i2\pi(0.49)},0.5e^{i2\pi(0.58)},0.43e^{i2\pi(0.57)}\right)}{\langle x_1,3\rangle},\frac{\left(0.8e^{i2\pi(0.65)},0.7e^{i2\pi(0.61)},0.66e^{i2\pi(0.71)}\right)}{\langle x_2,2\rangle}\right\}\right), \\ \left((e_1,p,1),\left\{\frac{\left(0.4e^{i2\pi(0.51)},0.5e^{i2\pi(0.42)},0.57e^{i2\pi(0.43)}\right)}{\langle x_1,3\rangle},\frac{\left(0.2e^{i2\pi(0.35)},0.3e^{i2\pi(0.39)},0.34e^{i2\pi(0.29)}\right)}{\langle x_2,2\rangle}\right\}\right), \\ \left((e_3,p,0),\left\{\frac{\left(0.4e^{i2\pi(0.50)},0.5e^{i2\pi(0.41)},0.49e^{i2\pi(0.42)}\right)}{\langle x_1,3\rangle},\frac{\left(0.2e^{i2\pi(0.34)},0.22e^{i2\pi(0.35)},0.32e^{i2\pi(0.26)}\right)}{\langle x_2,2\rangle}\right\}\right) \end{array}\right\}$$

And

$$(\mathcal{G},\mathcal{B},N) = \left\{\begin{array}{l} \left((e_1,q,1),\left\{\frac{\left(0.44e^{i2\pi(0.50)},0.42e^{i2\pi(0.52)},0.57e^{i2\pi(0.43)}\right)}{\langle x_1,3\rangle},\frac{\left(0.21e^{i2\pi(0.34)},0.3e^{i2\pi(0.17)},0.35e^{i2\pi(0.26)}\right)}{\langle x_2,2\rangle}\right\}\right), \\ \left((e_2,r,1),\left\{\frac{\left(0.61e^{i2\pi(0.62)},0.77e^{i2\pi(0.71)},0.6e^{i2\pi(0.65)}\right)}{\langle x_1,4\rangle},\frac{\left(0.91e^{i2\pi(0.87)},0.83e^{i2\pi(0.9)},0.91e^{i2\pi(0.99)}\right)}{\langle x_2,5\rangle}\right\}\right), \\ \left((e_3,p,0),\left\{\frac{\left(0.12e^{i2\pi(0.11)},0.19e^{i2\pi(0.1)},0.18e^{i2\pi(0.11)}\right)}{\langle x_1,1\rangle},\frac{\left(0.52e^{i2\pi(0.56)},0.41e^{i2\pi(0.4)},0.56e^{i2\pi(0.57)}\right)}{\langle x_2,3\rangle}\right\}\right), \\ \left((e_1,q,0),\left\{\frac{\left(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)}\right)}{\langle x_1,4\rangle},\frac{\left(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)}\right)}{\langle x_2,5\rangle}\right\}\right) \end{array}\right\}$$

By using Definition 7, we obtain the union $(H, C, \wp) = (\Upsilon,\mathfrak{R},\wp)\ \bar{\cup}\ (\mathcal{G},\mathcal{B},\wp)$ as follows:

$$(\mathcal{H}, C, N) = \left\{\begin{array}{l} \left((e_1,p,1),\left\{\frac{\left(0.6e^{i2\pi(0.63)},0.71e^{i2\pi(0.6)},0.62e^{i2\pi(0.78)}\right)}{\langle x_1,3\rangle},\frac{\left(0.9e^{i2\pi(0.94)},0.8e^{i2\pi(0.92)},0.96e^{i2\pi(0.56)}\right)}{\langle x_2,2\rangle}\right\}\right), \\ \left((e_1,q,1),\left\{\frac{\left(0.61e^{i2\pi(0.60)},0.73e^{i2\pi(0.6)},0.64e^{i2\pi(0.78)}\right)}{\langle x_1,4\rangle},\frac{\left(0.9e^{i2\pi(0.95)},0.8e^{i2\pi(0.83)},0.91e^{i2\pi(0.88)}\right)}{\langle x_2,5\rangle}\right\}\right), \\ \left((e_2,r,1),\left\{\frac{\left(0.61e^{i2\pi(0.62)},0.77e^{i2\pi(0.71)},0.6e^{i2\pi(0.65)}\right)}{\langle x_1,4\rangle},\frac{\left(0.91e^{i2\pi(0.87)},0.83e^{i2\pi(0.9)},0.91e^{i2\pi(0.99)}\right)}{\langle x_2,5\rangle}\right\}\right), \\ \left((e_3,p,0),\left\{\frac{\left(0.4e^{i2\pi(0.50)},0.5e^{i2\pi(0.41)},0.49e^{i2\pi(0.42)}\right)}{\langle x_1,3\rangle},\frac{\left(0.2e^{i2\pi(0.34)},0.22e^{i2\pi(0.35)},0.32e^{i2\pi(0.26)}\right)}{\langle x_2,2\rangle}\right\}\right), \\ \left((e_1,q,0),\left\{\frac{\left(0.6e^{i2\pi(0.67)},0.72e^{i2\pi(0.62)},0.62e^{i2\pi(0.79)}\right)}{\langle x_1,4\rangle},\frac{\left(0.91e^{i2\pi(0.94)},0.8e^{i2\pi(0.94)},0.97e^{i2\pi(0.89)}\right)}{\langle x_2,5\rangle}\right\}\right) \end{array}\right\}$$

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Definition 8. A CM^kFNSES $(\mathcal{H}, C, \wp)$, is referred to as the union of two $CM^kFNSESs(\Upsilon, \mathfrak{R}, \wp)$ and $(\mathcal{G}, \mathcal{B}, \wp)$ on the universe Ǔ, taking $C = \mathfrak{R} \cup \mathcal{B}, \forall e \in C$ and $x \in$ Ǔ,

$$\mathcal{H}(e) = \begin{cases} \Upsilon(e) = r^i_{\Upsilon(e)}(x).e^{i\omega^i_{\Upsilon(e)}(x)} & if\ e \in \mathfrak{R} - \mathcal{B} \\ \mathcal{G}(e) = r^i_{\mathcal{G}(e)}(x).e^{i\omega^i_{\mathcal{G}(e)}(x)} & if\ e \in \mathcal{B} - \mathfrak{R} \\ \Upsilon(e) \sqcap \mathcal{G}(e) = min\left[r^i_{\Upsilon(e)}(x), r^i_{\mathcal{G}(e)}(x)\right].e^{i\,min\left[\omega^i_{\Upsilon(e)}(x), \omega^i_{\mathcal{G}(e)}(x)\right]} & , if\ e \in \mathfrak{R} \cap \mathcal{B} \end{cases}$$

It is denoted by $(\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcap}\ (\mathcal{G}, \mathcal{B}, \wp)$ and we can write $(\mathcal{H}, C, \wp) = (\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcap}\ (\mathcal{G}, \mathcal{B}, \wp)$.

Example 7 Rethink about the Example 6. By employing the Definition 8, it can be written $(\mathcal{H}, C, \wp) = (\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcap}\ (\mathcal{G}, \mathcal{B}, \wp)$, and

$$(\mathcal{H}, C, N) = \left\{ \begin{array}{l} \left((e_1, p, 1), \left\{\frac{(0.6e^{i2\pi(0.63)}, 0.71e^{i2\pi(0.6)}, 0.62e^{i2\pi(0.78)})}{\langle x_1, 3\rangle}, \frac{(0.9e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.92)}, 0.96e^{i2\pi(0.56)})}{\langle x_2, 2\rangle}\right\}\right), \\ \left((e_1, q, 1), \left\{\frac{(0.44e^{i2\pi(0.50)}, 0.42e^{i2\pi(0.52)}, 0.57e^{i2\pi(0.43)})}{\langle x_1, 3\rangle}, \frac{(0.21e^{i2\pi(0.34)}, 0.3e^{i2\pi(0.17)}, 0.35e^{i2\pi(0.26)})}{\langle x_2, 2\rangle}\right\}\right), \\ \left((e_2, r, 1), \left\{\frac{(0.61e^{i2\pi(0.62)}, 0.77e^{i2\pi(0.71)}, 0.6e^{i2\pi(0.65)})}{\langle x_1, 4\rangle}, \frac{(0.91e^{i2\pi(0.87)}, 0.83e^{i2\pi(0.9)}, 0.91e^{i2\pi(0.99)})}{\langle x_2, 5\rangle}\right\}\right), \\ \left((e_3, p, 0), \left\{\frac{(0.12e^{i2\pi(0.11)}, 0.19e^{i2\pi(0.1)}, 0.18e^{i2\pi(0.11)})}{\langle x_1, 1\rangle}, \frac{(0.52e^{i2\pi(0.56)}, 0.41e^{i2\pi(0.4)}, 0.56e^{i2\pi(0.57)})}{\langle x_2, 3\rangle}\right\}\right), \\ \left((e_1, q, 0), \left\{\frac{(0.6e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.62)}, 0.62e^{i2\pi(0.79)})}{\langle x_1, 4\rangle}, \frac{(0.91e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.94)}, 0.97e^{i2\pi(0.89)})}{\langle x_2, 5\rangle}\right\}\right) \end{array} \right\}$$

We will now present several theorems on the complement, intersection, and union of $CM^kFNSESs$. These theorems describe the connections between the set-theoretic operations covered above.

Theorem 1. Take two $CM^kFNSESs(\Upsilon, \mathfrak{R}, \wp)$ and $(\mathcal{G}, \mathcal{B}, \wp)$ on the universe Ǔ. Therefore, the following commutative laws are valid:

1. $(\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcup}\ (\mathcal{G}, \mathcal{B}, \wp) = (\mathcal{G}, \mathcal{B}, \wp)\ \bar{\sqcup}\ (\Upsilon, \mathfrak{R}, \wp)$,
2. $(\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcap}\ (\mathcal{G}, \mathcal{B}, \wp) = (\mathcal{G}, \mathcal{B}, \wp)\ \bar{\sqcap}\ (\Upsilon, \mathfrak{R}, \wp)$,

Proof. Assume that $(\mathcal{H}, C, \wp) = (\Upsilon, \mathfrak{R}, \wp)\ \bar{\sqcup}\ (\mathcal{G}, \mathcal{B}, \wp)$ where $C = \mathfrak{R} \cup \mathcal{B}$. By Definition 7, we have $CM^kFNSESs(\mathcal{H}, C, \wp)$ where $C = \mathfrak{R} \cup \mathcal{B}$, and $\forall e \in C$

$$\mathcal{H}(e) = \begin{cases} \Upsilon(e) = r^i_{\Upsilon(e)}(x).e^{i\omega^i_{\Upsilon(e)}(x)} & if\ e \in \mathfrak{R} - \mathcal{B} \\ \mathcal{G}(e) = r^i_{\mathcal{G}(e)}(x).e^{i\omega^i_{\mathcal{G}(e)}(x)} & if\ e \in \mathcal{B} - \mathfrak{R} \\ \Upsilon(e) \sqcup \mathcal{G}(e) = max\left[r^i_{\Upsilon(e)}(x), r^i_{\mathcal{G}(e)}(x)\right].e^{i\,max\left[\omega^i_{\Upsilon(e)}(x), \omega^i_{\mathcal{G}(e)}(x)\right]} & , if\ e \in \mathfrak{R} \cap \mathcal{B} \end{cases}$$

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Because the other cases are trivial. So, we only take the case when $e \in \Re \cap \mathcal{B}$. Hence we can write

$$(\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp) = \Upsilon(e) \sqcup \mathcal{G}(e)$$
$$= max\left[r^{i}_{\Upsilon(e)}(x), r^{i}_{\mathcal{G}(e)}(x)\right].e^{i\,max\left[\omega^{i}_{\Upsilon(e)}(x), \omega^{i}_{\mathcal{G}(e)}(x)\right]}$$
$$= max\left[r^{i}_{\mathcal{G}(e)}(x), r^{i}_{\Upsilon(e)}(x)\right].e^{i\,max\left[\omega^{i}_{\Upsilon(e)}(x), \omega^{i}_{\mathcal{G}(e)}(x)\right]}$$
$$= \Upsilon(e) \sqcup \mathcal{G}(e)$$
$$= (\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp)$$

Consequently, we have got $(\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp) = (\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cup}\, (\Upsilon, \Re, \wp)$. Hence the first assertion of Theorem 1 has been proven. Similarly, we can confirm the second assertion.

Theorem 2. Take three $CM^{k}FNSESs(\Upsilon, \Re, \wp), (\mathcal{G}, \mathcal{B}, \wp)\ and\ (\mathcal{Q}, \mathcal{D}, \wp)$ on the universe Ǔ. Therefore, the following distributive laws are valid:

1. $(\Upsilon, \Re, \wp) \,\bar{\cup}\, ((\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp)) =$
$((\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp)) \,\bar{\cap}\, ((\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{Q}, \mathcal{D}, \wp))$
2. $(\Upsilon, \Re, \wp) \,\bar{\cap}\, ((\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cup}\, (\mathcal{Q}, \mathcal{D}, \wp)) =$
$((\Upsilon, \Re, \wp) \,\bar{\cap}\, (\mathcal{G}, \mathcal{B}, \wp)) \,\bar{\cup}\, ((\Upsilon, \Re, \wp) \,\bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp))$

Proof. We will provide the proof of assertion 1 since the proof of assertion 2 is straightforward from Definition 7 and Definition 8. Suppose that $(\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp) = (\mathcal{T}, \mathcal{P}, \wp)$ with $\mathcal{P} = \mathcal{B} \cap \mathcal{D}$. By Definition 8 we get $(\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp)$ becomes a $CM^{k}FNSES(\mathcal{T}, \mathcal{P}, \wp)$ where $\mathcal{P} = \mathcal{B} \cap \mathcal{D}$ and $\forall e \in \mathcal{P}$ in such a way

$$\mathcal{T}(e) = \mathcal{G}(e) \sqcap \mathcal{Q}(e) = min\left[r^{i}_{\mathcal{Q}(e)}(x), r^{i}_{\mathcal{G}(e)}(x)\right].e^{i\,min\left[\omega^{i}_{\mathcal{Q}(e)}(x), \omega^{i}_{\mathcal{G}(e)}(x)\right]}$$

Let $(\mathcal{S}, \mathcal{R}, \wp) = (\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{T}, \mathcal{P}, \wp)$ where $\mathcal{R} = \Re \cup \mathcal{P}$. By Definition 7, then we have $(\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{T}, \mathcal{P}, \wp)$ to be $CM^{k}FNSES(\mathcal{S}, \mathcal{R}, \wp)$ where $\mathcal{R} = \Re \cup \mathcal{P}$ and $\forall e \in \mathcal{R}$ in such a way

$$\mathcal{S}(e) = \begin{cases} \Upsilon(e) = r^{i}_{\Upsilon(e)}(x).e^{i\omega^{i}_{\Upsilon(e)}(x)} & if\ e \in \Re - \mathcal{P} \\ \mathcal{T}(e) = r^{i}_{\mathcal{T}(e)}(x).e^{i\omega^{i}_{\mathcal{T}(e)}(x)} & if\ e \in \mathcal{P} - \Re \\ \Upsilon(e) \sqcup \mathcal{T}(e) = max\left[r^{i}_{\Upsilon(e)}(x), r^{i}_{\mathcal{T}(e)}(x)\right].e^{i\,max\left[\omega^{i}_{\Upsilon(e)}(x), \omega^{i}_{\mathcal{T}(e)}(x)\right]} & ,if\ e \in \Re \cap \mathcal{P} \end{cases}$$

Now let $(\Upsilon, \Re, \wp) \,\bar{\cup}\, \big((\mathcal{G}, \mathcal{B}, \wp) \,\bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp)\big) = \big((\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{T}, \mathcal{P}, \wp)\big)$. We consider the case when $e \in \Re \cap \mathcal{P}$ as the other cases are trivial. Therefore,

$$(\Upsilon, \Re, \wp) \,\bar{\cup}\, (\mathcal{T}, \mathcal{P}, \wp) = \Upsilon(e) \sqcup \mathcal{T}(e)$$
$$= max\left[r^{i}_{\Upsilon(e)}(x), r^{i}_{\mathcal{T}(e)}(x)\right].e^{i\,max\left[\omega^{i}_{\Upsilon(e)}(x), \omega^{i}_{\mathcal{T}(e)}(x)\right]}$$
$$= max\left[r^{i}_{\Upsilon(e)}(x), r^{i}_{\mathcal{G}(e) \sqcap \mathcal{Q}(e)}(x),\right].e^{i\,max\left[\omega^{i}_{\Upsilon(e)}(x), \omega^{i}_{\mathcal{G}(e) \sqcap \mathcal{Q}(e)}(x)\right]}$$

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$= max\left[r^i_{\Upsilon(e)}(x), min\left[r^i_{\mathcal{Q}(e)}(x), r^i_{\mathcal{G}(e)}(x)\right]\right].e^{i\, max\left[i\omega^i_{\Upsilon(e)}(x), min\left[r^i_{\mathcal{Q}(e)}(x), r^i_{\mathcal{G}(e)}(x)\right]\right]}$

$= min(max\left[r^i_{\Upsilon(e)}(x), r^i_{\mathcal{G}(e)}(x)\right], max\left[r^i_{\Upsilon(e)}(x), r^i_{\mathcal{Q}(e)}(x)\right]).e^{i\, min(max\left[\omega^i_{\Upsilon(e)}(x), \omega^i_{\mathcal{G}(e)}(x)\right], max\left[\omega^i_{\Upsilon(e)}(x), \omega^i_{\mathcal{Q}(e)}(x)\right]}$

$= (\Upsilon(e) \sqcup \mathcal{G}(e)) \sqcap (\Upsilon(e) \sqcup \mathcal{Q}(e))$

$= ((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp))\, \bar{\cap}\, ((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, (\mathcal{Q}, \mathcal{D}, \wp))$

Hence, we get $(\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, ((\mathcal{G}, \mathcal{B}, \wp)\, \bar{\cap}\, (\mathcal{Q}, \mathcal{D}, \wp)) = ((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp))\, \bar{\cap}\, ((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, (\mathcal{Q}, \mathcal{D}, \wp))$.

Theorem 3. Take two $CM^kFNSESs(\Upsilon, \mathfrak{R}, \wp)$ and $(\mathcal{G}, \mathcal{B}, \wp)$ on the universe Ǔ. Therefore, the following De Morgan's laws are valid:

1. $\left((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cup}\, (\mathcal{G}, \mathcal{B}, \wp)\right)^c = (\mathcal{G}, \mathcal{B}, \wp)^c\, \bar{\cup}\, (\Upsilon, \mathfrak{R}, \wp)^c$,
2. $\left((\Upsilon, \mathfrak{R}, \wp)\, \bar{\cap}\, (\mathcal{G}, \mathcal{B}, \wp)\right)^c = (\mathcal{G}, \mathcal{B}, \wp)^c\, \bar{\cap}\, (\Upsilon, \mathfrak{R}, \wp)^c$,

Proof: Using Definitions 6, 7 and 8 *ÂăÂăthen* proofs are simple.

With a proposition of these two operations, we shall present the concepts of AND and OR operations in two $CM^kFNSESs$.

Definition 9. Take two $CM^kFNSESs(\Upsilon, \mathfrak{R}, \wp)$ and $(\mathcal{G}, \mathcal{B}, \wp)$ on the universe Ǔ. Then $(\Upsilon, \mathfrak{R}, \wp)$ AND $(\mathcal{G}, \mathcal{B}, \wp)$ is defined by

$$(\Upsilon, \mathfrak{R}, \wp)\, \bar{\wedge}\, (\mathcal{G}, \mathcal{B}, \wp) = (\mathcal{O}, \mathfrak{R} \times \mathcal{B}, \wp)$$

Where $(\Upsilon, \mathfrak{R}, \wp)\, \bar{\wedge}\, (\mathcal{G}, \mathcal{B}, \wp)$ denotes the AND operation and $(\mathcal{O}, \mathfrak{R} \times \mathcal{B}, \wp) = \mathcal{O}(\alpha, \beta)$ such that $\mathcal{O}(\alpha, \beta) = \Upsilon(\alpha) \cap \mathcal{G}(\beta)$, $\forall\, (\alpha, \beta) \in \mathfrak{R} \times \mathcal{B}$, and $\cap$ represents the complex multi-fuzzy intersection.

Example 8 Reconsider the Example 1. Take

$A = \{(e_2, p, 1), (e_1, q, 1), (e_3, r, 0)\}$

$$B = \{(e_2, p, 1), (e_1, q, 0)\}$$

Consider $(\Upsilon, \mathfrak{R}, \wp)$ *and* $(\mathcal{G}, \mathcal{B}, \wp)$ as two $CM^kFNSESs$ over Ǔ, *such that*

$$(\Upsilon, \mathfrak{R}, \wp) = \left\{ \begin{array}{l} \left((e_1, q, 1), \left\{\frac{(0.5e^{i2\pi(0.54)}, 0.59e^{i2\pi(0.48)}, 0.4e^{i2\pi(0.47)})}{\langle x_1, 3\rangle}, \frac{(0.32e^{i2\pi(0.2)}, 0.21e^{i2\pi(0.38)}, 0.33e^{i2\pi(0.25)})}{\langle x_2, 2\rangle}\right\}\right), \\ \left((e_2, p, 1), \left\{\frac{(0.7e^{i2\pi(0.74)}, 0.6e^{i2\pi(0.69)}, 0.6e^{i2\pi(0.66)})}{\langle x_1, 4\rangle}, \frac{(0.8e^{i2\pi(0.91)}, 0.94e^{i2\pi(0.99)}, 0.85e^{i2\pi(0.8)})}{\langle x_2, 5\rangle}\right\}\right), \\ \left((e_3, r, 0), \left\{\frac{(0.1e^{i2\pi(0.08)}, 0.12^{i2\pi(0.17)}, 0.13e^{i2\pi(0.19)})}{\langle x_1, 1\rangle}, \frac{(0.52e^{i2\pi(0.5)}, 0.59e^{i2\pi(0.49)}, 0.55e^{i2\pi(0.58)})}{\langle x_2, 3\rangle}\right\}\right) \end{array} \right\}$$

and

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$$(\mathcal{G},\mathcal{B},\wp) = \left\{ \begin{array}{l} \left((e_1,p,1), \left\{ \frac{(0.4e^{i2\pi(0.51)}, 0.5e^{i2\pi(0.42)}, 0.57e^{i2\pi(0.43)})}{\langle x_1, 3\rangle}, \frac{(0.2e^{i2\pi(0.35)}, 0.3e^{i2\pi(0.39)}, 0.34e^{i2\pi(0.29)})}{\langle x_2, 2\rangle} \right\} \right), \\ \left((e_2,q,0), \left\{ \frac{(0.6e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.62)}, 0.62e^{i2\pi(0.79)})}{\langle x_1, 4\rangle}, \frac{(0.91e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.94)}, 0.97e^{i2\pi(0.89)})}{\langle x_2, 5\rangle} \right\} \right) \end{array} \right\}$$

By the intersection of two $CM^kFNSESs$ we have $(\Upsilon,\Re,\wp)\,\overline{\wedge}\,(\mathcal{G},\mathcal{B},\wp) = (\mathcal{O},\Re\times\mathcal{B},\wp)$,

$$(\mathcal{O},\Re\times\mathcal{B},\wp) = \left\{ \begin{array}{l} \left((e_1,p,1),(e_1,q,1), \left\{ \frac{(0.4e^{i2\pi(0.51)}, 0.5e^{i2\pi(0.42)}, 0.4e^{i2\pi(0.43)})}{\langle x_1, 3\rangle}, \frac{(0.2e^{i2\pi(0.2)}, 0.21e^{i2\pi(0.38)}, 0.33e^{i2\pi(0.25)})}{\langle x_2, 2\rangle} \right\} \right), \\ \left((e_1,q,1),(e_2,q,0), \left\{ \frac{(0.5e^{i2\pi(0.54)}, 0.59e^{i2\pi(0.48)}, 0.4e^{i2\pi(0.47)})}{\langle x_1, 3\rangle}, \frac{(0.32e^{i2\pi(0.2)}, 0.21e^{i2\pi(0.38)}, 0.33e^{i2\pi(0.25)})}{\langle x_2, 2\rangle} \right\} \right), \\ \left((e_2,p,1),(e_1,p,1), \left\{ \frac{(0.4e^{i2\pi(0.51)}, 0.5e^{i2\pi(0.42)}, 0.57e^{i2\pi(0.43)})}{\langle x_1, 3\rangle}, \frac{(0.2e^{i2\pi(0.35)}, 0.3e^{i2\pi(0.39)}, 0.34e^{i2\pi(0.29)})}{\langle x_2, 2\rangle} \right\} \right), \\ \left((e_2,p,1),(e_2,q,0), \left\{ \frac{(0.6e^{i2\pi(0.67)}, 0.6e^{i2\pi(0.62)}, 0.6e^{i2\pi(0.66)})}{\langle x_1, 4\rangle}, \frac{(0.8e^{i2\pi(0.91)}, 0.8e^{i2\pi(0.94)}, 0.85e^{i2\pi(0.8)})}{\langle x_2, 5\rangle} \right\} \right), \\ \left((e_3,r,0),(e_1,p,1), \left\{ \frac{(0.1e^{i2\pi(0.08)}, 0.12^{i2\pi(0.17)}, 0.13e^{i2\pi(0.19)})}{\langle x_1, 1\rangle}, \frac{(0.2e^{i2\pi(0.2)}, 0.21e^{i2\pi(0.38)}, 0.33e^{i2\pi(0.25)})}{\langle x_2, 2\rangle} \right\} \right), \\ \left((e_3,r,0),(e_2,q,0), \left\{ \frac{(0.1e^{i2\pi(0.08)}, 0.12^{i2\pi(0.17)}, 0.13e^{i2\pi(0.19)})}{\langle x_1, 1\rangle}, \frac{(0.52e^{i2\pi(0.5)}, 0.59e^{i2\pi(0.49)}, 0.55e^{i2\pi(0.58)})}{\langle x_2, 3\rangle} \right\} \right), \end{array} \right\}$$

Definition 10. Take two $CM^kFNSESs(\Upsilon,\Re,\wp)$ and $(\mathcal{G},\mathcal{B},\wp)$ on the universe Ǔ. Then $(\Upsilon,\Re,\wp)$ OR $(\mathcal{G},\mathcal{B},\wp)$ is defined by

$$(\Upsilon,\Re,\wp)\,\overline{\vee}\,(\mathcal{G},\mathcal{B},\wp) = (\mathcal{Q},\Re\times\mathcal{B},\wp)$$

Where $(\Upsilon,\Re,\wp)\,\overline{\vee}\,(\mathcal{G},\mathcal{B},\wp)$ denotes the AND operation and $(\mathcal{Q},\Re\times\mathcal{B},\wp) = \mathcal{Q}(\alpha,\beta)$ such that $\mathcal{Q}(\alpha,\beta) = \Upsilon(\alpha)\cup\mathcal{G}(\beta)$, $\forall\,(\alpha,\beta)\in\ \Re\times\mathcal{B}$, and $\cup$ represents the complex multi-fuzzy.

Example 9 Rethink about the Example 8. Then by employing the definition of the union of two $CM^kFNSESs$ we get $(\Upsilon,\Re,\wp)\,\overline{\vee}\,(\mathcal{G},\mathcal{B},\wp) = (\mathcal{Q},\Re\times\mathcal{B},\wp)$

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$$(\mathcal{Q}, \mathfrak{R} \times \mathcal{B}, \wp) = \left\{ \begin{array}{l} \left((e_1, p, 1), (e_1, q, 1), \left\{ \frac{(0.5e^{i2\pi(0.54)}, 0.59e^{i2\pi(0.48)}, 0.57e^{i2\pi(0.47)})}{\langle x_1, 3 \rangle}, \frac{(0.32e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.38)}, 0.35e^{i2\pi(0.29)})}{\langle x_2, 2 \rangle} \right\} \right), \\ \left((e_1, q, 1), (e_2, q, 0), \left\{ \frac{(0.6e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.62)}, 0.62e^{i2\pi(0.79)})}{\langle x_1, 4 \rangle}, \frac{(0.91e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.94)}, 0.97e^{i2\pi(0.89)})}{\langle x_2, 5 \rangle} \right\} \right), \\ \left((e_2, p, 1), (e_1, p, 1), \left\{ \frac{(0.7e^{i2\pi(0.74)}, 0.6e^{i2\pi(0.69)}, 0.6e^{i2\pi(0.66)})}{\langle x_1, 4 \rangle}, \frac{(0.8e^{i2\pi(0.91)}, 0.94e^{i2\pi(0.99)}, 0.85e^{i2\pi(0.8)})}{\langle x_2, 5 \rangle} \right\} \right), \\ \left((e_2, p, 1), (e_2, q, 0), \left\{ \frac{(0.6e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.62)}, 0.62e^{i2\pi(0.79)})}{\langle x_1, 4 \rangle}, \frac{(0.91e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.94)}, 0.97e^{i2\pi(0.89)})}{\langle x_2, 5 \rangle} \right\} \right), \\ \left((e_3, r, 0), (e_1, p, 1), \left\{ \frac{(0.4e^{i2\pi(0.51)}, 0.5e^{i2\pi(0.42)}, 0.57e^{i2\pi(0.43)})}{\langle x_1, 3 \rangle}, \frac{(0.52e^{i2\pi(0.5)}, 0.59e^{i2\pi(0.49)}, 0.55e^{i2\pi(0.58)})}{\langle x_2, 3 \rangle} \right\} \right), \\ \left((e_3, r, 0), (e_2, q, 0), \left\{ \frac{(0.6e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.62)}, 0.62e^{i2\pi(0.79)})}{\langle x_1, 4 \rangle}, \frac{(0.91e^{i2\pi(0.94)}, 0.8e^{i2\pi(0.94)}, 0.97e^{i2\pi(0.89)})}{\langle x_2, 5 \rangle} \right\} \right), \end{array} \right\}$$

Proposition 2. Take two $CM^kFNSESs(\Upsilon, \mathfrak{R}, \wp)$ and $(\mathcal{G}, \mathcal{B}, \wp)$ on the universe Ǔ, we possess these properties.

1. $\left((\Upsilon, \mathfrak{R}, \wp) \,\overline{\vee}\, (\mathcal{G}, \mathcal{B}, \wp)\right)^c = (\Upsilon, \mathfrak{R}, \wp)^c \,\overline{\wedge}\, (\mathcal{G}, \mathcal{B}, \wp)^c$
2. $\left((\Upsilon, \mathfrak{R}, \wp) \,\overline{\wedge}\, (\mathcal{G}, \mathcal{B}, \wp)\right)^c = (\Upsilon, \mathfrak{R}, \wp)^c \,\overline{\vee}\, (\mathcal{G}, \mathcal{B}, \wp)^c$

The proof is straightforward.

3. Application of CM^kFNSES

By utilizing the mean of each $MFNSES$, we build applications of CM^kFNSES theory in DM situations in this part.

3.1 Casting of an Actor

Suppose that a film production company is interested in casting an actor for a particular role in a film. The company considers eight actors which are shown in the set of universe $\check{U} = \{x_1, x_2, \ldots, x_8\}$ where x_1=Humayun Saeed, x_2=Fahad Mustafa, x_3=Fawad Khan, x_4=Hamza Ali Abbasi, x_5=Feroze Khan, x_6=Shehryar Munawar, x_7= Bilal Abbas and x_8= Ahad Raza Mir. The film production company is interested in casting such an actor who best describes the role written for the leading character. Suppose $E = \{e_1, e_2, e_3\}$ is the set of qualities that the company is looking for in the actor. Where e_1 stands for speaking Punjabi which has two levels; fluent and average. e_2 stands for fitness which also has two levels; excellent and Good. Finally, e_3 stands for acting performance which has two levels too; outstanding and average. Take $X = \{p, q, r\}$ as the set of film casting experts who have been tasked with examining these eight actors depending upon the number of awards, they won because of their previous films and the number of audience (fans) who want to see them in further movies and projects. The film production business received a graded evaluation report of these actors based on the attributes

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indicated above (see Table 3). In Table 3, stars can be easily correlated with 1,2,3,4,5 as discussed in the previous example. On the basis of results obtained from the experts, we have the following CM^kFNSES $(\Upsilon, \Re, \wp)$ of dimension 2:

$$\Upsilon(e_1, p, 1) = \left\{ \begin{array}{l} \frac{(0.41e^{i2\pi(0.49)}, 0.5e^{i2\pi(0.4)})}{\langle x_1, 3\rangle}, \frac{0.81e^{i2\pi(0.9)}, 0.87e^{i2\pi(0.99)})}{\langle x_2, 5\rangle} \\ \frac{(0.7e^{i2\pi(0.75)}, 0.69e^{i2\pi(0.7)})}{\langle x_1, 4\rangle}, \frac{0.8e^{i2\pi(0.82)}, 0.9e^{i2\pi(0.88)})}{\langle x_2, 5\rangle} \\ \frac{(0.32e^{i2\pi(0.35)}, 0.3e^{i2\pi(0.25)})}{\langle x_1, 2\rangle}, \frac{0.82e^{i2\pi(0.92)}, 0.99e^{i2\pi(0.87)})}{\langle x_2, 5\rangle} \\ \frac{(0.11e^{i2\pi(0.19)}, 0.12e^{i2\pi(0.17)})}{\langle x_1, 1\rangle}, \frac{0.33e^{i2\pi(0.3)}, 0.27e^{i2\pi(0.26)})}{\langle x_2, 2\rangle} \end{array} \right\}$$

$$\Upsilon(e_1, q, 1) = \left\{ \begin{array}{l} \frac{(0.4e^{i2\pi(0.59)}, 0.43e^{i2\pi(0.46)})}{\langle x_1, 3\rangle}, \frac{0.82e^{i2\pi(0.86)}, 0.82e^{i2\pi(0.9)})}{\langle x_2, 5\rangle} \\ \frac{(0.71e^{i2\pi(0.77)}, 0.73e^{i2\pi(0.67)})}{\langle x_1, 4\rangle}, \frac{0.8e^{i2\pi(0.81)}, 0.85e^{i2\pi(0.89)})}{\langle x_2, 5\rangle} \\ \frac{(0.35e^{i2\pi(0.31)}, 0.22e^{i2\pi(0.29)})}{\langle x_1, 2\rangle}, \frac{0.94e^{i2\pi(0.99)}, 0.9e^{i2\pi(0.86)})}{\langle x_2, 5\rangle} \\ \frac{(0.19e^{i2\pi(0.14)}, 0.09e^{i2\pi(0.16)})}{\langle x_1, 1\rangle}, \frac{0.26e^{i2\pi(0.24)}, 0.28e^{i2\pi(0.36)})}{\langle x_2, 2\rangle} \end{array} \right\}$$

$$\Upsilon(e_1, r, 1) = \left\{ \begin{array}{l} \frac{(0.59e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.55)})}{\langle x_1, 3\rangle}, \frac{0.83e^{i2\pi(0.8)}, 0.9e^{i2\pi(0.93)})}{\langle x_2, 5\rangle} \\ \frac{(0.73e^{i2\pi(0.79)}, 0.66e^{i2\pi(0.7)})}{\langle x_1, 4\rangle}, \frac{0.92e^{i2\pi(0.85)}, 0.89e^{i2\pi(0.98)})}{\langle x_2, 5\rangle} \\ \frac{(0.38e^{i2\pi(0.39)}, 0.22e^{i2\pi(0.3)})}{\langle x_1, 2\rangle}, \frac{0.89e^{i2\pi(0.98)}, 0.8e^{i2\pi(0.85)})}{\langle x_2, 5\rangle} \\ \frac{(0.11e^{i2\pi(0.17)}, 0.15e^{i2\pi(0.08)})}{\langle x_1, 1\rangle}, \frac{0.35e^{i2\pi(0.37)}, 0.24e^{i2\pi(0.36)})}{\langle x_2, 2\rangle} \end{array} \right\}$$

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$$\Upsilon(e_2,q,1)=\left\{\begin{matrix}\frac{\left(0.3e^{i2\pi(0.32)},0.32e^{i2\pi(0.39)}\right)}{\langle x_1,2\rangle},\frac{0.61e^{i2\pi(0.66)},0.68e^{i2\pi(0.71)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.45e^{i2\pi(0.50)},0.53e^{i2\pi(0.41)}\right)}{\langle x_1,3\rangle},\frac{0.23e^{i2\pi(0.28)},0.25e^{i2\pi(0.33)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.45e^{i2\pi(0.41)},0.53e^{i2\pi(0.55)}\right)}{\langle x_1,3\rangle},\frac{0.78e^{i2\pi(0.74)},0.76e^{i2\pi(0.67)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.67e^{i2\pi(0.72)},0.6e^{i2\pi(0.67)}\right)}{\langle x_1,4\rangle},\frac{0.44e^{i2\pi(0.4)},0.5e^{i2\pi(0.41)}\,)}{\langle x_2,3\rangle}\end{matrix}\right\}$$

$$\Upsilon(e_2,q,1)=\left\{\begin{matrix}\frac{\left(0.3e^{i2\pi(0.32)},0.22e^{i2\pi(0.24)}\right)}{\langle x_1,2\rangle},\frac{0.78e^{i2\pi(0.68)},0.6e^{i2\pi(0.69)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.47e^{i2\pi(0.5)},0.43e^{i2\pi(0.41)}\right)}{\langle x_1,3\rangle},\frac{0.3e^{i2\pi(0.33)},0.3e^{i2\pi(0.37)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.59e^{i2\pi(0.5)},0.56e^{i2\pi(0.48)}\right)}{\langle x_1,3\rangle},\frac{0.69e^{i2\pi(0.6)},0.78e^{i2\pi(0.75)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.67e^{i2\pi(0.72)},0.76e^{i2\pi(0.67)}\right)}{\langle x_1,4\rangle},\frac{0.45e^{i2\pi(0.55)},0.51e^{i2\pi(0.47)}\,)}{\langle x_2,3\rangle}\end{matrix}\right\}$$

$$\Upsilon(e_2,r,1)=\left\{\begin{matrix}\frac{\left(0.34e^{i2\pi(0.39)},0.36e^{i2\pi(0.27)}\right)}{\langle x_1,2\rangle},\frac{0.66e^{i2\pi(0.72)},0.78e^{i2\pi(0.75)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.55e^{i2\pi(0.5)},0.41e^{i2\pi(0.46)}\right)}{\langle x_1,3\rangle},\frac{0.25e^{i2\pi(0.37)},0.29e^{i2\pi(0.31)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.45e^{i2\pi(0.5)},0.54e^{i2\pi(0.59)}\right)}{\langle x_1,3\rangle},\frac{0.63e^{i2\pi(0.66)},0.72e^{i2\pi(0.71)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.66e^{i2\pi(0.69)},0.73e^{i2\pi(0.65)}\right)}{\langle x_1,4\rangle},\frac{0.49e^{i2\pi(0.51)},0.52e^{i2\pi(0.54)}\,)}{\langle x_2,3\rangle}\end{matrix}\right\}$$

$$\Upsilon(e_3,p,1)=\left\{\begin{matrix}\frac{\left(0.2e^{i2\pi(0.25)},0.3e^{i2\pi(0.39)}\right)}{\langle x_1,2\rangle},\frac{0.42e^{i2\pi(0.48)},0.5e^{i2\pi(0.55)}\,)}{\langle x_2,3\rangle}\\ \frac{\left(0.27e^{i2\pi(0.24)},0.26e^{i2\pi(0.31)}\right)}{\langle x_1,2\rangle},\frac{0.12e^{i2\pi(0.15)},0.15e^{i2\pi(0.19)}\,)}{\langle x_2,1\rangle}\\ \frac{\left(0.13e^{i2\pi(0.11)},0.06e^{i2\pi(0.17)}\right)}{\langle x_1,1\rangle},\frac{0.77e^{i2\pi(0.7)},0.72e^{i2\pi(0.68)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.88e^{i2\pi(0.94)},0.9e^{i2\pi(0.88)}\right)}{\langle x_1,5\rangle},\frac{0.17e^{i2\pi(0.12)},0.11e^{i2\pi(0.18)}\,)}{\langle x_2,1\rangle}\end{matrix}\right\}$$

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$$\Upsilon(e_3,p,1)=\left\{\begin{array}{c}\frac{\left(0.33e^{i2\pi(0.37)},0.39e^{i2\pi(0.3)}\right)}{\langle x_1,2\rangle},\frac{0.43e^{i2\pi(0.53)},0.52e^{i2\pi(0.47)})}{\langle x_2,3\rangle}\\ \frac{\left(0.33e^{i2\pi(0.39)},0.34e^{i2\pi(0.39)}\right)}{\langle x_1,2\rangle},\frac{0.18e^{i2\pi(0.13)},0.14e^{i2\pi(0.09)})}{\langle x_2,1\rangle}\\ \frac{\left(0.13e^{i2\pi(0.19)},0.18e^{i2\pi(0.07)}\right)}{\langle x_1,1\rangle},\frac{0.71e^{i2\pi(0.73)},0.66e^{i2\pi(0.65)})}{\langle x_2,4\rangle}\\ \frac{\left(0.8e^{i2\pi(0.87)},0.9e^{i2\pi(0.95)}\right)}{\langle x_1,5\rangle},\frac{0.07e^{i2\pi(0.16)},0.11e^{i2\pi(0.14)})}{\langle x_2,1\rangle}\end{array}\right\}$$

$$\Upsilon(e_3,r,1)=\left\{\begin{array}{c}\frac{\left(0.31e^{i2\pi(0.37)},0.38e^{i2\pi(0.31)}\right)}{\langle x_1,2\rangle},\frac{0.42e^{i2\pi(0.53)},0.51e^{i2\pi(0.41)})}{\langle x_2,3\rangle}\\ \frac{\left(0.36e^{i2\pi(0.32)},0.29e^{i2\pi(0.3)}\right)}{\langle x_1,2\rangle},\frac{0.13e^{i2\pi(0.17)},0.19e^{i2\pi(0.11)})}{\langle x_2,1\rangle}\\ \frac{\left(0.13e^{i2\pi(0.18)},0.14e^{i2\pi(0.11)}\right)}{\langle x_1,1\rangle},\frac{0.67e^{i2\pi(0.7)},0.73e^{i2\pi(0.75)})}{\langle x_2,4\rangle}\\ \frac{\left(0.81e^{i2\pi(0.89)},0.86e^{i2\pi(0.92)}\right)}{\langle x_1,5\rangle},\frac{0.12e^{i2\pi(0.09)},0.13e^{i2\pi(0.15)})}{\langle x_2,1\rangle}\end{array}\right\}$$

$$\Upsilon(e_1,p,0)=\left\{\begin{array}{c}\frac{\left(0.42e^{i2\pi(0.48)},0.51e^{i2\pi(0.41)}\right)}{\langle x_1,3\rangle},\frac{0.82e^{i2\pi(0.53)},0.85e^{i2\pi(0.92)})}{\langle x_2,5\rangle}\\ \frac{\left(0.72e^{i2\pi(0.78)},0.7e^{i2\pi(0.75)}\right)}{\langle x_1,4\rangle},\frac{0.81e^{i2\pi(0.82)},0.94e^{i2\pi(0.89)})}{\langle x_2,5\rangle}\\ \frac{\left(0.3e^{i2\pi(0.35)},0.36e^{i2\pi(0.24)}\right)}{\langle x_1,2\rangle},\frac{0.8e^{i2\pi(0.9)},0.93e^{i2\pi(0.86)})}{\langle x_2,5\rangle}\\ \frac{\left(0.12e^{i2\pi(0.15)},0.13e^{i2\pi(0.19)}\right)}{\langle x_1,1\rangle},\frac{0.35e^{i2\pi(0.3)},0.26e^{i2\pi(0.29)})}{\langle x_2,2\rangle}\end{array}\right\}$$

$$\Upsilon(e_1,q,0)=\left\{\begin{array}{c}\frac{\left(0.42e^{i2\pi(0.6)},0.42e^{i2\pi(0.47)}\right)}{\langle x_1,3\rangle},\frac{0.83e^{i2\pi(0.53)},0.87e^{i2\pi(0.9)})}{\langle x_2,5\rangle}\\ \frac{\left(0.71e^{i2\pi(0.74)},0.73e^{i2\pi(0.61)}\right)}{\langle x_1,4\rangle},\frac{0.83e^{i2\pi(0.81)},0.85e^{i2\pi(0.8)})}{\langle x_2,5\rangle}\\ \frac{\left(0.36e^{i2\pi(0.31)},0.22e^{i2\pi(0.3)}\right)}{\langle x_1,2\rangle},\frac{0.92e^{i2\pi(0.99)},0.9e^{i2\pi(0.87)})}{\langle x_2,5\rangle}\\ \frac{\left(0.19e^{i2\pi(0.13)},0.1e^{i2\pi(0.16)}\right)}{\langle x_1,1\rangle},\frac{0.27e^{i2\pi(0.24)},0.28e^{i2\pi(0.32)})}{\langle x_2,2\rangle}\end{array}\right\}$$

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$$\Upsilon(e_1,r,0)=\left\{\begin{array}{c}\frac{\left(0.59e^{i2\pi(0.51)},0.59e^{i2\pi(0.55)}\right)}{\langle x_1,3\rangle},\frac{0.82e^{i2\pi(0.8)},0.9e^{i2\pi(0.93)}\,)}{\langle x_2,5\rangle}\\ \frac{\left(0.71e^{i2\pi(0.79)},0.66e^{i2\pi(0.71)}\right)}{\langle x_1,4\rangle},\frac{0.95e^{i2\pi(0.85)},0.88e^{i2\pi(0.98)}\,)}{\langle x_2,5\rangle}\\ \frac{\left(0.34e^{i2\pi(0.39)},0.22e^{i2\pi(0.31)}\right)}{\langle x_1,2\rangle},\frac{0.87e^{i2\pi(0.98)},0.8e^{i2\pi(0.83)}\,)}{\langle x_2,5\rangle}\\ \frac{\left(0.1e^{i2\pi(0.17)},0.15e^{i2\pi(0.11)}\right)}{\langle x_1,1\rangle},\frac{0.33e^{i2\pi(0.37)},0.24e^{i2\pi(0.33)}\,)}{\langle x_2,2\rangle}\end{array}\right\}$$

$$\Upsilon(e_2,p,0)=\left\{\begin{array}{c}\frac{\left(0.33e^{i2\pi(0.32)},0.32e^{i2\pi(0.35)}\right)}{\langle x_1,2\rangle},\frac{0.6e^{i2\pi(0.66)},0.68e^{i2\pi(0.77)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.45e^{i2\pi(0.50)},0.53e^{i2\pi(0.41)}\right)}{\langle x_1,3\rangle},\frac{0.22e^{i2\pi(0.28)},0.25e^{i2\pi(0.37)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.44e^{i2\pi(0.41)},0.53e^{i2\pi(0.5)}\right)}{\langle x_1,3\rangle},\frac{0.71e^{i2\pi(0.75)},0.76e^{i2\pi(0.68)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.66e^{i2\pi(0.6)},0.6e^{i2\pi(0.62)}\right)}{\langle x_1,4\rangle},\frac{0.42e^{i2\pi(0.4)},0.5e^{i2\pi(0.46)}\,)}{\langle x_2,3\rangle}\end{array}\right\}$$

$$(e_2,q,0)=\left\{\begin{array}{c}\frac{\left(0.3e^{i2\pi(0.36)},0.22e^{i2\pi(0.29)}\right)}{\langle x_1,2\rangle},\frac{0.79e^{i2\pi(0.68)},0.6e^{i2\pi(0.7)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.48e^{i2\pi(0.5)},0.43e^{i2\pi(0.4)}\right)}{\langle x_1,3\rangle},\frac{0.34e^{i2\pi(0.33)},0.3e^{i2\pi(0.33)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.58e^{i2\pi(0.5)},0.56e^{i2\pi(0.46)}\right)}{\langle x_1,3\rangle},\frac{0.6e^{i2\pi(0.69)},0.78e^{i2\pi(0.35)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.72e^{i2\pi(0.7)},0.76e^{i2\pi(0.70)}\right)}{\langle x_1,4\rangle},\frac{0.46e^{i2\pi(0.51)},0.51e^{i2\pi(0.46)}\,)}{\langle x_2,3\rangle}\end{array}\right\}$$

$$\Upsilon(e_2,r,0)=\left\{\begin{array}{c}\frac{\left(0.35e^{i2\pi(0.39)},0.36e^{i2\pi(0.29)}\right)}{\langle x_1,2\rangle},\frac{0.61e^{i2\pi(0.72)},0.78e^{i2\pi(0.71)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.55e^{i2\pi(0.5)},0.41e^{i2\pi(0.46)}\right)}{\langle x_1,3\rangle},\frac{0.23e^{i2\pi(0.37)},0.29e^{i2\pi(0.33)}\,)}{\langle x_2,2\rangle}\\ \frac{\left(0.44e^{i2\pi(0.5)},0.54e^{i2\pi(0.46)}\right)}{\langle x_1,3\rangle},\frac{0.63e^{i2\pi(0.69)},0.72e^{i2\pi(0.35)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.63e^{i2\pi(0.59)},0.73e^{i2\pi(0.66)}\right)}{\langle x_1,4\rangle},\frac{0.45e^{i2\pi(0.51)},0.52e^{i2\pi(0.53)}\,)}{\langle x_2,3\rangle}\end{array}\right\}$$

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$$\Upsilon(e_3,p,0)=\left\{\begin{matrix}\frac{\left(0.21e^{i2\pi(0.25)},0.3e^{i2\pi(0.34)}\right)}{\langle x_1,2\rangle},\frac{0.41e^{i2\pi(0.48)},0.5e^{i2\pi(0.54)}\,)}{\langle x_2,3\rangle}\\ \frac{\left(0.24e^{i2\pi(0.23)},0.26e^{i2\pi(0.3)}\right)}{\langle x_1,2\rangle},\frac{0.11e^{i2\pi(0.15)},0.15e^{i2\pi(0.13)}\,)}{\langle x_2,1\rangle}\\ \frac{\left(0.13e^{i2\pi(0.12)},0.06e^{i2\pi(0.18)}\right)}{\langle x_1,1\rangle},\frac{0.77e^{i2\pi(0.7)},0.72e^{i2\pi(0.68)}\,)}{\langle x_2,4\rangle}\\ \frac{\left(0.88e^{i2\pi(0.91)},0.9e^{i2\pi(0.88)}\right)}{\langle x_1,5\rangle},\frac{0.17e^{i2\pi(0.13)},0.11e^{i2\pi(0.13)}\,)}{\langle x_2,1\rangle}\end{matrix}\right\}$$

$$\Upsilon(e_3,q,0)=\left\{\begin{matrix}\frac{\left(0.34e^{i2\pi(0.37)},0.39e^{i2\pi(0.31)}\right)}{\langle x_1,2\rangle},\frac{0.42e^{i2\pi(0.53)},0.52e^{i2\pi(0.44)}\,)}{\langle x_2,3\rangle}\\ \frac{\left(0.33e^{i2\pi(0.39)},0.34e^{i2\pi(0.39)}\right)}{\langle x_3,2\rangle},\frac{0.11e^{i2\pi(0.12)},0.14e^{i2\pi(0.11)}\,)}{\langle x_4,1\rangle}\\ \frac{\left(0.13e^{i2\pi(0.14)},0.18e^{i2\pi(0.1)}\right)}{\langle x_5,1\rangle},\frac{0.71e^{i2\pi(0.73)},0.66e^{i2\pi(0.65)}\,)}{\langle x_6,4\rangle}\\ \frac{\left(0.81e^{i2\pi(0.84)},0.9e^{i2\pi(0.91)}\right)}{\langle x_7,5\rangle},\frac{0.07e^{i2\pi(0.13)},0.11e^{i2\pi(0.13)}\,)}{\langle x_8,1\rangle}\end{matrix}\right\}$$

$$\Upsilon(e_3,r,0)=\left\{\begin{matrix}\frac{\left(0.31e^{i2\pi(0.33)},0.38e^{i2\pi(0.3)}\right)}{\langle x_1,2\rangle},\frac{0.42e^{i2\pi(0.54)},0.52e^{i2\pi(0.41)}\,)}{\langle x_2,3\rangle}\\ \frac{\left(0.32e^{i2\pi(0.32)},0.25e^{i2\pi(0.3)}\right)}{\langle x_3,2\rangle},\frac{0.12e^{i2\pi(0.17)},0.14e^{i2\pi(0.11)}\,)}{\langle x_4,1\rangle}\\ \frac{\left(0.12e^{i2\pi(0.18)},0.16e^{i2\pi(0.11)}\right)}{\langle x_5,1\rangle},\frac{0.67e^{i2\pi(0.7)},0.73e^{i2\pi(0.75)}\,)}{\langle x_6,4\rangle}\\ \frac{\left(0.82e^{i2\pi(0.89)},0.86e^{i2\pi(0.91)}\right)}{\langle x_7,5\rangle},\frac{0.14e^{i2\pi(0.09)},0.1e^{i2\pi(0.14)}\,)}{\langle x_8,1\rangle}\end{matrix}\right\}$$

$\mathbb{U}$	e_1	e_2	e_3
x_1	★★★	★★	★★
x_2	★★★★★	★★★★	★★★
x_3	★★★★	★★★	★★
x_4	★★★★★	★★	★
x_5	★★	★★★	★
x_6	★★★★★	★★★★	★★★★
x_7	★	★★★★	★★★★★
x_8	★★	★★★	★

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The amount of awards the performers have received as a result of their prior work in the movies is displayed in this example's amplitude term of the membership values, while the phase term shows the number of audiences who want to see these actors in upcoming films. The values for both of the terms can be measured effectively by film casting experts. The interval [0,1]contains the terms for both the amplitude and phase. The value of an amplitude term near to 0 shows that the above-discussed actors have won a very low number of awards because of their skills in previous films which means they are not good enough to win that award. Similarly, an amplitude term having a value near to 1 shows that the above-discussed actors have won enough awards to show their skills in previous films which means they dare to win an award. The value of a phase term near to 0 shows that the audience has the least interest in seeing these actors in upcoming films and a phase term having a value near to 1 shows that the audience is very eager to see these actors in upcoming films and projects. To explain what we want to say, consider the approximation,

$$\gamma(e_1,p,1)=\left\{\begin{array}{c}\frac{\left(0.41e^{i2\pi(0.49)},0.5e^{i2\pi(0.4)}\right)}{\langle x_1,3\rangle},\frac{\left(0.81e^{i2\pi(0.9)},0.87e^{i2\pi(0.99)}\right)}{\langle x_2,5\rangle},\\ \frac{\left(0.7e^{i2\pi(0.75)},0.69e^{i2\pi(0.7)}\right)}{\langle x_3,4\rangle},\frac{\left(0.8e^{i2\pi(0.82)},0.9e^{i2\pi(0.88)}\right)}{\langle x_4,5\rangle},\\ \frac{\left(0.32e^{i2\pi(0.35)},0.3e^{i2\pi(0.25)}\right)}{\langle x_5,2\rangle},\frac{\left(0.82e^{i2\pi(0.92)},0.99e^{i2\pi(0.87)}\right)}{\langle x_6,5\rangle},\\ \frac{\left(0.11e^{i2\pi(0.19)},0.12e^{i2\pi(0.17)}\right)}{\langle x_7,1\rangle},\frac{\left(0.33e^{i2\pi(0.3)},0.27e^{i2\pi(0.26)}\right)}{\langle x_8,2\rangle}\end{array}\right\}$$

The term $\frac{\left(0.11e^{i2\pi(0.19)},0.12e^{i2\pi(0.17)}\right)}{\langle x_7,1\rangle}$reveals that the actor Bilal Abbas hasn't won any awards due to his previous performance in the films. In other words, the CMFNSE values

$$\left(0.11e^{i2\pi(0.19)},0.12e^{i2\pi(0.17)}\right)$$

points out that the expert p agrees that Bilal Abbas hasn't won award because of speaking good punjabi language with degrees $(0.11,0.12)$. The audience who want to see him in upcoming films is $(0.19,0.17)$ which is very close to 0.

Similarly, the term $\frac{\left(0.81e^{i2\pi(0.9)},0.87e^{i2\pi(0.99)}\right)}{\langle x_2,5\rangle}$ implies that the actor Fahad Mustafa has won a plenty number of awards as a prize of his good performance in the previous films. The membership values$\left(0.81e^{i2\pi(0.9)},0.87e^{i2\pi(0.99)}\right)$

indicate that the expert p agrees that Fahad Mustafa has won many awards with degrees $(0.81,0.87)$ and the audience who want to see him in upcoming films has degrees $(0.9,0.99)$ which is very close to 1. In order to tackle the DM problem, we will now combine the CM^kNSES with a generalized algorithm in casting the most appropriate actor for the particular role. The algorithm described below converts the CMK^FNSES to $MFNSES$ using weighted

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aggregate values and moves forward inăfiguring out the optimal decisionăbased on the mean and score of each $MENSES$ MFNSES element. Following are the steps of the algorithm:

Algorithm

1. Enter the $CMK^F NSES(\gamma, \mathfrak{R}, \wp)$.
2. Transform the $CMK^F NSES(\gamma, \mathfrak{R}, \wp)$ to $MFNSES(\tilde{F}, A, N)$ by figuring out the weighted aggregate values of $\mu^{\jmath}\gamma_{(e)}(x), \forall e \in \mathfrak{R}, \forall x \in \breve{\mathbb{U}}$ and $\jmath = 1,2,3, \dots, k$ as in the equation below:

$$\mu^{\jmath}{}_{\breve{F}(e)}(x) = v_1 \gamma^{\jmath}{}_{\gamma(e)}(x) + v_2 \left(1/_{2\pi}\right) \omega^{\jmath}{}_{\gamma(e)}(x)$$

Here $\gamma^{\jmath}{}_{\gamma(e)}(x)$ represents the amplitude terms and $\omega^{\jmath}{}_{\gamma(e)}(x)$ represents the phase terms in the $CMK^F NSES(\gamma, \mathfrak{R}, \wp)$. $\mu^{\jmath}{}_{\breve{F}(e)}(x)$ denotes the function having multi-membership in the $MFNSES(\tilde{F}, A, N)$. The weight for the amplitude terms (number of awards) is given by v_1, while the weight for phase terms (demand by audience) is given by v_2 with v_1 and $v_2 \in [0,1]$ also

$v_1 + v_2 = 1$.

3. Using the equation below, find the mean of each $MFNSES, \forall e \in \mathfrak{R}$ and $\forall \in \breve{\mathbb{U}}$

$$\bar{M}_{\tilde{F}(e)}(x) = \frac{\sum_{j=0}^{n} \mu^{\jmath}{}_{\breve{F}(e)}(x)}{n}$$

4. For agree-$MK^F NSES$, calculate the score for every member $x_\ell \in \breve{\mathbb{U}}$ employing the equation

$C_\ell = \sum_{j=1}^{n} x_{\jmath\,\ell}$

5. For agree-$MK^F NSES$, calculate the score for every member $x_\ell \in \breve{\mathbb{U}}$ employing the equation

$K_\ell = \sum_{j=1}^{n} x_{\jmath\,\ell}$

6. For every member $x_\ell \in \breve{\mathbb{U}}$, evaluate the score $R_\ell = C_\ell - K_\ell$.
7. Identify the highest score's value as $s = max_{x\,\ell \in \breve{\mathbb{U}}}\{\mathcal{R}_\ell\}$. Choosing element x_ℓ as the perfect or optimal solution to the problem is the best way to take the$\hat{A}\breve{a}$decision. Any element might have been picked if there were multiple elements$\hat{A}\breve{a}$with the highest $\mathcal{R}_\ell$ score.

To explain the concept, we will now transform the $CMK^F NSES(\gamma, \mathfrak{R}, \wp)$to $MFNSES(\tilde{F}, A, N)$. To implement this step and to find the weighted aggregation values of $\mu^{\jmath}{}_{\breve{F}(e)}(x)$, let $v_1 = 0.7$ and $v_2 = 0.3 \forall x \in \breve{\mathbb{U}}$ with $\jmath = 1,2,3, \dots, k$ as depicted next

$$\mu^1_{\breve{F}(e_1,p,1)}(x_1) = v_1 \gamma^1_{\gamma(e_1,p,1)}(x_1) + v_2 \left(1/_{2\pi}\right) \omega^1_{\breve{\gamma}(e_1,p,1)}(x_1)$$
$$= (0.7)(0.33) + (0.3)\left(1/_{2\pi}\right)(2\pi)(0.37) = 0.342$$
$$\mu^2_{\breve{F}(e_1,p,1)}(x_1) = v_1 \gamma^2_{\gamma(e_1,p,1)}(x_1) + v_2 \left(1/_{2\pi}\right) \omega^2_{\breve{\gamma}(e_1,p,1)}(x_1)$$
$$= (0.7)(0.39) + (0.3)\left(1/_{2\pi}\right)(2\pi)(0.3) = 0.363$$

Then for $e = (e_1, p, 1)$ and $x = x_1$. the multi-fuzzy N-soft expert values

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$$\left(\mu^{1}_{\tilde{F}(e_1,p,1)}(x_1), \mu^{2}_{\tilde{F}(e_1,p,1)}(x_1)\right) = (0.342, 0.363).$$

We also determine the other $MFNSE$ values in the same manner $\forall e \in \mathfrak{R}$ and $\forall e \in \breve{\mathbb{U}}$ and Table 4 and 5 presents the which gives values of the mean of each $MFNSE$ values $\forall e \in \mathfrak{R}$ and $\forall e \in \breve{\mathbb{U}}$. Notice that, in Table 4 and 5 the upper and lower terms of every member shows the $MFNSE$ values $\forall e \in \mathfrak{R}$ and $\forall e \in \breve{\mathbb{U}}$ and in Tables 6 and 7 the values of the mean of each $MFNSE$ values $\forall e \in \mathfrak{R}$ and $\forall e \in \breve{\mathbb{U}}$, are represented.

Table 4: Values of $\left(\tilde{F}, A, N\right)$ of each $MFNSE$

$\breve{\mathbb{U}}$	x_1	x_2	x_3	x_4
$(e_1, p, 1)$	(0.432,0.47)	(0.837,0.906)	(0.715,0.693)	(0.806,0.894)
$(e_1, q, 1)$	(0.457,0.439)	(0.832,0.844)	(0.728,0.695)	(0.803,0.862)
$(e_1, r, 1)$	(0.563,0.515)	(0.821,0.909)	(0.708,0.672)	(0.911,0.917)
$(e_2, p, 1)$	(0.306,0.341)	(0.625,0.689)	(0.465,0.494)	(0.245,0.274)
$(e_2, q, 1)$	(0.309,0.226)	(0.75,0.624)	(0479,0.451)	(0.309,0.321)
$(e_2, r, 1)$	(0.355,0.333)	(0.678,0.771)	(0.535,0.425)	(0.286,0.296)
$(e_3, p, 1)$	(0.215,0.327)	(0.438,0.515)	(0.261,0.275)	(0.129,0.162)
$(e_3, q, 1)$	(0.342,0.363)	(0.460,0.505)	(0.333,0.355)	(0165,0.125)
$(e_3, r, 1)$	(0.328,0.359)	(0453,0.480)	(0.348,0.293)	(0142,0.166)
$(e_1, p, 0)$	(0.438,0.48)	(0.829,0.871)	(0.738,0.715)	(0.567,0.925)
$(e_1, q, 0)$	(0.474,0.835)	(0.345,0.244)	(0.719,0.694)	(0.824,0.835)
$(e_1, r, 0)$	(0.566,0.578)	(0.814,0.909)	(0.734,0.675)	(0.92,0.91)
$(e_2, p, 0)$	(0.327,0.329)	(0.618,0.707)	(0.465,0.496)	(0.238,0.286)
$(e_2, q, 0)$	(0.318,0.241)	(0.757,0.63)	(0.486,0.451)	(0.337,0.309)
$(e_2, r, 0)$	(0.362,0.339)	(0.643,0.759)	(0.535,0.425)	(0.272,0.302)
$(e_3, p, 0)$	(0.222,0.312)	(0.431,0.512)	(0.24,0.272)	(0.122,0.144)
$(e_3, q, 0)$	(0.349,0.366)	(0.453,0.496)	(0.348,0.355)	(0.113,0.131)
$(e_3, r, 0)$	(0.316,0.356)	(0.456,0.487)	(0.32,0.625)	(0.135,0.131)

Table 5: Values of $\left(\tilde{F}, A, N\right)$ of each $MFNSE$

$\breve{\mathbb{U}}$	x_5	x_6	x_7	x_8
$(e_1, p, 1)$	(0.329,0.285)	(0.85,0.954)	(0.134,0.135)	(0.331,0.267)
$(e_1, q, 1)$	(0.338,0.241)	(0.955,0.888)	(0.175,0.111)	(0.254,0.304)
$(e_1, r, 1)$	(0.383,0.244)	(0.917,0.815)	(0.128,0.129)	(0.356,0.276)
$(e_2, p, 1)$	(0.563,0.536)	(0.768,0.733)	(0.649,0.621)	(0.428,0.473)
$(e_2, q, 1)$	(0.438,0.536)	(0.663,0.771)	(0.685,0.769)	(0.48,0.498)
$(e_2, r, 1)$	(0.465,0.555)	(0.639,0.717)	(0.669,0.706)	(0.496,0.526)
$(e_3, p, 1)$	(0.124,0.093)	(0.749,0.708)	(0.898,0.894)	(0.115,0.131)
$(e_3, q, 1)$	(0.148,0.147)	(0.716,0.657)	(0.821,0.915)	(0.097,0.915)
$(e_3, r, 1)$	(0.145,0.131)	(0.679,0.736)	(0.834,0.878)	(0.111,0.136)
$(e_1, p, 0)$	(0.315,0.324)	(0.83,0.909)	(0.129,0.148)	(0.335,0.269)
$(e_1, q, 0)$	(0.345,0.349)	(0.941,0.89)	(0.172,0.118)	(0.24,0.257)
$(e_1, r, 0)$	(0.355,0.247)	(0.903,0.809)	(0.121,0.138)	(0.342,0.267)
$(e_2, p, 0)$	(0.431,0.521)	(0.722,0.736)	(0.642,0.606)	(0.414,0.488)
$(e_2, q, 0)$	(0.556,0.53)	(0.627,0.651)	(0.714,0.742)	(0.475,0.495)
$(e_2, r, 0)$	(0.456,0.594)	(0.639,0.717)	(0.641,0.709)	(0.468,0.523)
$(e_3, p, 0)$	(0.127,0.096)	(0.749,0.708)	(0.889,0.894)	(0.158,0.116)
$(e_3, q, 0)$	(0.113,0.156)	(0.716,0.657)	(0.819,0.903)	(0.088,0.116)
$(e_3, r, 0)$	(0.138,0.145)	(0.679,0.736)	(0.841,0.875)	(0.125,0.133)

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Tables 8 and 9 represents the agree $MFNSE$ and disagree $MFNSE$ respectively by using the mean of each $MFNSE$. (See Tables 6 and 7)

Table 6 : Mean of each $MFNSE$ in Table 4.

$\breve{\mathbb{U}}$	x_1	x_2	x_3	x_4
$(e_1, p, 1)$	0.451	0.8715	0.704	0.85
$(e_1, q, 1)$	0.448	0.838	0.7115	0.8325
$(e_1, r, 1)$	0.539	0.865	0.69	0.914
$(e_2, p, 1)$	0.3235	0.657	0.4795	0.2595
$(e_2, q, 1)$	0.2875	0.6875	0.465	0.315
$(e_2, r, 1)$	0.344	0.7245	0.48	0.291
$(e_3, p, 1)$	0.271	0.4765	0.268	0.1455
$(e_3, q, 1)$	0.3525	0.4825	0.344	0.145
$(e_3, r, 1)$	0.3435	0.4665	0.3205	0.154
$(e_1, p, 0)$	0.459	0.85	0.7265	0.746
$(e_1, q, 0)$	0.6545	0.2945	0.7065	0.8295
$(e_1, r, 0)$	0.572	0.8615	0.7045	0.915
$(e_2, p, 0)$	0.328	0.6625	0.4795	0.262
$(e_2, q, 0)$	0.2795	0.6935	0.4685	0.323
$(e_2, r, 0)$	0.3505	0.701	0.48	0.287
$(e_3, p, 0)$	0.267	0.472	0.256	0.133
$(e_3, q, 0)$	0.3575	0.4745	0.3515	0.122
$(e_3, r, 0)$	0.336	0.4715	0.2925	0.133

Table 7 : Mean of each $MFNSE$ in Table 5.

$\breve{\mathbb{U}}$	x_5	x_6	x_7	x_8
$(e_1, p, 1)$	0.307	0.425	0.1345	0.299
$(e_1, q, 1)$	0.2895	0.9215	0.143	0.279
$(e_1, r, 1)$	0.312	0.866	0.1285	0.266
$(e_2, p, 1)$	0.487	0.7505	0.635	0.4505
$(e_2, q, 1)$	0.5495	0.717	0.727	0.489
$(e_2, r, 1)$	0.51	0.678	0.6875	0.511
$(e_3, p, 1)$	0.1085	0.7285	0.896	0.123
$(e_3, q, 1)$	0.1475	0.6865	0.868	0.506
$(e_3, r, 1)$	0.138	0.7075	0.856	0.1235
$(e_1, p, 0)$	0.3195	0.8695	0.1385	0.302
$(e_1, q, 0)$	0.3354	0.9155	0.145	0.2485
$(e_1, r, 0)$	0.301	0.856	0.1295	0.3045
$(e_2, p, 0)$	0.476	0.729	0.624	0.451
$(e_2, q, 0)$	0.543	0.639	0.728	0.485
$(e_2, r, 0)$	0.525	0.678	0.675	0.4955
$(e_3, p, 0)$	0.1115	0.7285	0.8115	0.137
$(e_3, q, 0)$	0.1345	0.358	0.861	0.102
$(e_3, r, 0)$	0.1415	0.7075	0.858	0.129

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Let the score of each numerical grade for the agree $MK^{F}NSES$ and dis-agree-$MK^{F}NSES$, is represented by C_{ℓ} and K_{ℓ} , respectively. The Table 10 displays the calculated values while the values are calculated in the Tables 8 and 9.

Table 9: Tabular representation of agree $MFNSE$

$\breve{\mathbb{U}}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$(e_1, p, 1)$	0.451	0.8715	0.704	0.85	0.307	0.425	0.1345	0.299
$(e_1, q, 1)$	0.448	0.838	0.7115	0.8325	0.2895	0.9215	0.143	0.279
$(e_1, r, 1)$	0.539	0.865	0.69	0.914	0.312	0.866	0.1285	0.266
$(e_2, p, 1)$	0.328	0.6625	0.4795	0.262	0.328	0.6625	0.4795	0.262
$(e_2, q, 1)$	0.2875	0.6875	0.465	0.315	0.5495	0.717	0.727	0.489
$(e_2, r, 1)$	0.344	0.7245	0.48	0.291	0.51	0.678	0.6875	0.511
$(e_3, p, 1)$	0.271	0.4765	0.268	0.1455	0.1085	0.7285	0.896	0.123
$(e_3, q, 1)$	0.3525	0.4825	0.344	0.145	0.1475	0.6865	0.868	0.506
$(e_3, r, 1)$	0.3435	0.4665	0.3205	0.154	0.138	0.7075	0.856	0.1235
$C_{\ell} = \sum_{j=1}^{n} x_{j\ell}$	$C_1 = 3.0075$	$C_2 = 5.5865$	$C_3 = 4.1185$	$C_4 = 3.7615$	$C_5 = 2.7015$	$C_6 = 5.794$	$C_7 = 4.2075$	$C_8 = 2.541$

Table 9: Tabular representation of dis-agree $MFNSE$

$\breve{\mathbb{U}}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$(e_1, p, 0)$	0.459	0.85	0.7265	0.746	0.3195	0.8695	0.1385	0.302

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$(e_1,q,0)$	0.6545	0.2945	0.7065	0.8295	0.3354	0.9155	0.145	0.2485
$(e_1,r,0)$	0.572	0.8615	0.7045	0.915	0.301	0.856	0.1295	0.3045
$(e_2,p,0)$	0.3235	0.657	0.4795	0.2595	0.487	0.7505	0.635	0.4505
$(e_2,q,0)$	0.2795	0.6935	0.4685	0.323	0.543	0.639	0.728	0.485
$(e_2,r,0)$	0.3505	0.701	0.48	0.287	0.525	0.678	0.675	0.4955
$(e_3,p,0)$	0.267	0.472	0.256	0.133	0.1115	0.7285	0.8115	0.137
$(e_3,q,0)$	0.3575	0.4745	0.3515	0.122	0.1345	0.358	0.861	0.102
$(e_3,r,0)$	0.336	0.4715	0.2925	0.133	0.1415	0.7075	0.858	0.129
$k_\ell=\sum_{i=1}^{n} x_{j\,\ell}$	$K_1=3.2465$	$K_2=5.0065$	$K_3=4.114$	$K_4=3.6285$	$K_5=2.6429$	$K_6=6.223$	$K_7=4.1095$	$K_8=2.5525$

Table 10: The score $R_\ell = C_\ell - k_\ell$

ℓ	$\mho$	C_ℓ	k_ℓ	R_ℓ
1	x_1	3.0075	3.2465	-0.239
2	x_2	5.5865	5.0065	0.58
3	x_3	4.1185	4.114	0.0045
4	x_4	3.7615	3.6285	0.133
5	x_5	2.7015	2.6429	0.0586
6	x_6	5.794	6.223	-0.429
7	x_7	4.2075	4.1095	0.098
8	x_8	2.541	2.5525	-0.0115

Clearly, we can see from Table 10 the maximum choice value, is 0.58. Therefore, choosingx_2 is the wisest move. Consequently, the expert should choose Fahad Mustafa as the best actor for the particular role according to the specified weights for the parameters given.

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4 Comparative Analysis

This section will evaluate the usefulness and efficacy of the $CMK^{F}NSES$ model that has been created by analyzing its advantages, limitations, and comparing it to both methods of MFNSE and CFSES.

a. Comparison

In recent times, extensive research has been conducted to extend existing models such as MFS_s and $MFSS_s$. This includes the exploration of $MFSES_s$and $MFNSS_s$[19], which are soft set's natural expansions.Additionally, $MFSES_s$ are a concept that extends fuzzy soft expert sets ($FSES_s$)[20], enabling their use in a variety of decision-making scenarios. Our proposed methods serve as extensions of various models, namely soft set [21] [21], N-soft set [22], soft expert set, $MFSES_s$, and CMFSES[23] models. Notably, the CMFSES model is generalized in the $CMK^{F}NSES$ model. The MFNSS and the complex FSES are the two ways that we will now compare with the complicated CMFSES. Upon examining Example 3, it becomes apparent that the MFNSES is insufficient for solving the presented DM problem, which includes two-dimensional data, specifically the total time of influence and degree. This inadequacy stems from the absence of a phase term in the multi-fuzzy N-soft set (MFNSS), which is crucial for representing the time frame of the problem. Furthermore, the MFNSS cannot handle multiple experts, adding to its limitations. On the other hand, the complex fuzzy soft expert set is capable of dealing with problems involving two-dimensional information. However, it fails to solve decision-making problems that entail multiple agents, attributes, objects, indices, and uncertainties due to its reliance on a single complex membership function. However, in capturing two-dimensional multi-fuzzy information, our model surpasses both the MFNSS and the CFSES in terms of computational effectiveness, applicability, usability, and accuracy. It achieves this superiority by effectively representing two-dimensional information through the use of multi-membership functions. Hence, the proposed method offers several advantages. Firstly, it utilizes the $CMK^{F}NSES$ to effectively represent decision-making problems involving multiple agents, attributes, objects, indices, and uncertainties by employing multi-membership functions. Secondly, the $CMK^{F}NSES$ incorporates evaluation information that is absent in the MFSES model, such as the inclusion of a phase term to represent the time frame. Additionally, It eliminates the need for time-consuming additional operations by enabling users to access the opinions of any expert within a single model. Thirdly, by simultaneously handling the amplitude and phase terms of complex numbers, the $CMK^{F}NSES$ used in our method can handle uncertainty information. Fourth, the $CMK^{F}NSES$ is transformed from a complex state to a real state using a workable formula, which makes the decision-making process easier by eliminating the need for complex number operations. Finally, by including both the amplitude term and the phase term, our suggested model offers a more accurate representation of two-dimensional multi-fuzzy information. In the decision process, the phase term captures the time

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factor, which can either enhance or diminish the impact of the amplitude term associated with it. Consequently, for decision-making issues that call for choosing the best option, our model is quite effective.

b. **Advantages**

Researchers have observed significant advancements while creating soft set models for dealing with imprecise data based on an analysis of recent decades. However, these models, for the most part, only deal with binary data involving one or more experts. This limitation poses problems for users, particularly in comparison to soft sets and soft expert sets, they performed data sorting utilizing numerous grading values for their investigations. Two innovative hybrid models, called $MFSES_s$ and $CMK^{F}NSES_s$, have been presented in light of this expanding tendency. In multinary categorization, our proposed ideas allow users to consider the opinions of all experts. These suggested models are quite effective in dealing with complex, uncertain, and vague information that involves two-dimensional data. Additionally, the model can represent two-dimensional information using multi-membership functions. Specifically, when dealing with decision-making that involves existing data such as grades, ratings, fuzziness, and input from various experts, the MFNSES model is utilized. In this model, the initial step involves constructing agree and disagree tables, which are multi-fuzzy N-soft expert tables containing entries based on grades. Subsequently, a decision is made by computing the final score table and identifying the maximum score value within it. On the other hand, the $CMK^{F}NSES$ model follows a different approach. It involves constructing agree and disagree tables, which are CMFNSE tables, where the fuzzy values, are determined based on ratings provided by the experts.

c. **Limitations**

The developed model exhibits certain limitations, particularly in cases where the computational process becomes slow due to an abundance of parameters, extensive data encompassing grades and fuzziness, and complex information. Managing such large datasets is challenging due to the intricate computations involved, making it difficult to perform efficiently. However, this limitation can be addressed by implementing a well-coded algorithm of the proposed model within various mathematical software platforms such as MATLAB. By leveraging the capabilities of these software tools, the model becomes capable of swiftly and effectively handling large datasets, overcoming the challenges associated with complex computations.

5 Conclusion and future plans

Numerous practical applications of the principle of soft expert sets have been discovered in numerous domains, including computer science and artificial intelligence. To handle data with uncertainties, a promising mathematical method called the N-soft set model has emerged. However, traditional soft expert sets face limitations when confronted with data that requires multinary categorizations. With the help of new mathematical techniques, information is represented in this model in a way that takes time into account and makes it possible to grasp

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the perspectives of several experts within a single framework. By combining the characteristics of complex multi-fuzzy sets and N-soft expert sets, the $CMK^F NSES$ is created. We introduce the concept of the MFNSES and $CMK^F NSES$, defining basic operations such as complement, union, intersection, AND, and OR operations. Numerous properties are examined. As a result, we suggest a novel general framework for uncertainty decision-making called the $CMK^F NSES$. An associated algorithm is constructed to facilitate its implementation. Comparisons with two existing models highlight the robustness of our proposed model, demonstrating its ability to give information that is two-dimensionally multi-fuzzy a more accurate representation. To illustrate its practical application, we showcase a case study in the film industry. The steps of the algorithm are exemplified, showcasing its feasibility and ease of implementation for generating the required decision. This study introduces the $CMK^F NSES_S$ model as a robust tool for addressing uncertainty in two-dimensional fuzzy soft information, incorporating the perspectives of multiple experts. The developed method has the potential to enhance existing frameworks by forming powerful combinations with fewer limitations, thus enabling the resolution of highly complex problems. Looking ahead, it is hoped that this work will continue to evolve and expand in the future,

1. Intuitionistic complex $MFNSES_S$,
2. m-polar complex $MFNSES_S$,
3. Spherical complex $MFNSES_S$,
4. Bipolar complex $MFNSES_S$,
5. Picture complex $MFNSES_S$,
6. n-rung orthpair complex $MFNSES_S$,

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