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Theoretical Investigation of Entropy Generation in a Two-Dimensional Rotating Disc Fin

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Abstract

In this paper, the entropy generation in a rotating disc fin is studied analytically. In order to obtain the fin temperature distribution, the energy balance is written for a two-dimensional differential element of the fin. The governing equation of fin temperature is a linear partial differential equation that has been solved by the analytical methods. Also, analytical expressions for the fin efficiency and fin entropy generation are obtained. The results of temperature of the present study fin in zero angular velocity has been validated with the temperature distribution of the stationary fin of the previous literature. The results show that the fin temperature is a weak function of the tangential location due to the symmetry of geometry and physical conditions of the fin. Further, to have maximum efficiency and minimum entropy generation simultaneously, a moderate angular velocity should be selected.

Keywords: Two-dimensional disc fin, Rotating motion, Analytical calculation, Entropy generation.

Introduction

Thermal fins are devices used for dissipating heat generated by electronic components or machinery. They consist of a series of thin metal or plastic elements with high conductivity that are attached to the components or machinery to increase the surface area for heat dissipation. As the heat is transferred to the fins, it is then dispersed into the surrounding air, helping to cool down the components or machinery. Thermal fins are commonly used in computers, machinery, and other industrial devices to prevent overheating and improve performance. Rotating thermal fins refer to a type of heat sink or cooling solution that utilizes rotation to dissipate heat more effectively. These fins are strategically placed and rotated to maximize heat transfer rates, improve thermal performance, and reduce energy consumption. This technology offers a compact and cost-effective solution for cooling or heating systems in industrial processes, refrigeration, electronics, and more. Car brake disc can be mentioned as a practical example of a rotating disc fin. Figure 1 shows a picture of a brake disc.

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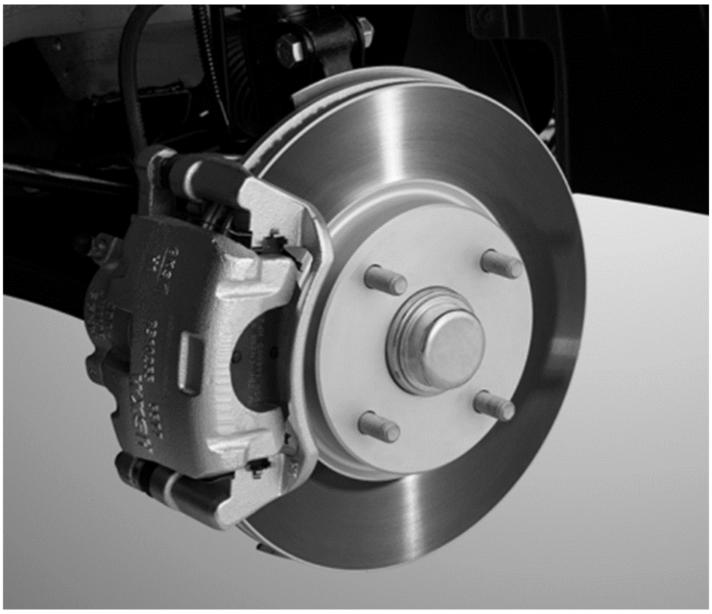

Figure 1. Picture of a brake disc [1]

Entropy generation is a concept that is fundamental to the second law of thermodynamics. It refers to the natural tendency for systems to move towards a state of disorder or randomness, also known as entropy. In any process, there is always some amount of entropy generated as energy is transformed and transferred within a system. This is due to the irreversible nature of most physical processes, resulting in a loss of energy that increases the overall entropy of the system. Entropy generation plays a crucial role in determining the efficiency and feasibility of various engineering processes and systems. By minimizing the amount of entropy generated, engineers can optimize the performance and energy efficiency of their designs. Overall, entropy generation serves as a key factor in understanding the behavior of physical systems and plays a critical role in the study of thermodynamics and energy transfer. Investigating the entropy generation in a rotating fin can provide suitable ideas for its better design and efficiency.

Numerous theoretical and numerical researches have been conducted regarding the performance of fins. In the following, some of these studies have been reviewed.

Poulikakos and Bejan (1982) established a theoretical framework for the minimization of entropy generation in longitudinal moving fins [2]. They obtained the optimum dimensions of longitudinal fins based on entropy generation minimization.

Culham and Muzychka (2001) obtained the optimum design parameters of an array of plate fins based on entropy generation minimization [3]. Their results include the correlation for the pressure drop in terms of volumetric flow rate, optimal fins number and optimum fins length.

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Alam and Ghoshdastidar (2002) carried out the numerical study on a circular tube with internal longitudinal fins using finite difference method (FDM) [4]. Fins profile was tapered lateral shape. It was observed that there is a significant improvement in the net heat transfer rate with the fins installation.

Kim et al. (2008) proposed a fan-integrated heat sink (scroll heat sink) [5]. In the scroll heat sink, the moving fins, which rotate with two eccentric shafts, are inserted between the fixed (cooling) fins. They developed a theoretical model to estimate the required pumping power and the thermal resistance. They obtained the optimized thermal resistance and plate-fin heat sinks for low pumping power condition.

Sahiti et al. (2008) derived the heat transfer and pressure drop characteristics of a double-pipe pin fin heat exchanger based on entropy generation minimization [6]. Their results showed that from thermodynamic view of point, larger number of passages with smaller pin fin height have better performance than less heat exchanger passages with larger pin height.

Aziz and Khani (2010) developed an analytical solution for the thermal performance investigation of a radial rotating fin with various convex parabolic profiles by homotopy analysis method (HAM) [7]. They studied the effect of profile thickness, the ratio of outer to inner radius, and the angular speed on the temperature distribution, heat transfer rate, and the fin efficiency.

Aziz and Khan (2012) studies the heat transfer and entropy generation in a stationary pin fin [8]. Their results include the investigation of various parameters effect on the entropy generation rate and the determination of the optimum fin length.

Jeng et al. (2014) investigated the oscillation motion effect on the heat transfer enhancement of pin-fin heat sinks with circular impinging jets, experimentally [9]. Their experimental results showed that stroke of the platform moving up and down is too long to enhance the overall heat transfer of the pin-fin heat sinks.

Sun and Xu (2015) used the spectral collocation method (SCM) to find the temperature distribution of a moving fin with temperature dependent thermal conductivity, heat transfer coefficient and surface emissivity [10]. They analyzed the effects of various physical parameters, such as Peclet number, thermal conductivity parameter, emissivity parameter, parameter of heat transfer coefficient, on the temperature distribution and the volume adjusted fin efficiency.

Dogonchi and Ganji (2016) studied the convection-radiation heat transfer of a moving fin by differential transformation method (DTM) [11]. Their results showed that the fin tip temperature increases with an increase in the heat generation gradient and a decrease in the Peclet number and the radiation-conduction parameter. Also, they pointed out that the optimal temperature distribution occurs in the case of heat generation without radiation.

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Ndlovu and Moitsheki (2019) carried out thermal analysis of radial moving fins of hyperbolic and rectangular profiles [12]. They solved the nonlinear differential equation modeling heat transfer of radial moving fin by differential transform method (DTM). Their results showed that the fin rapidly dissipates heat to the surrounding fluid with the increase of the moving fin speed. Also, the heat transfer through a rectangular fin is more efficient than hyperbolic fin.

Krishnayatra et al. (2020) investigated the thermal performance of fins in axial finned-tube heat exchangers by machine learning regression technique [13]. They studied the effect of variation in the fin spacing, fin thickness, material, and the convective heat transfer coefficient on the overall fin efficiency. Their results showed that the fin efficiency increases with fin thickness decrease.

Najafabadi et al. (2021) analyzed the heat transfer and temperature distribution of a moving fin by radial basis function (RBF) method [14]. They examined several parameters such as thermal conductivity parameter, sink temperature parameter, and Peclet number. Their results illustrated that the increase of the thermal conductivity increases the dimensionless temperature. Moreover, the increase of the sink temperature leads to a slow rise in the fin temperature and increase of the Pe number by 100% results in a rise in the temperature distribution by about 7%.

Zhang et al. (2022) investigated the rotation effect on the heat transfer characteristics of a lateral outflow trapezoidal channel connected to pin fins [15]. In their study, the effect of high Reynolds number and Coriolis force and centrifugal force induced by rotation on the heat transfer characteristics was investigated. Their results showed that the connection of wedge-shaped pin fins to trapezoidal channel could be improved the heat transfer characteristics, extraordinary.

Din et al. (2022) investigated the performance of moving porous longitudinal fins based on entropy generation concept [16]. They observed that the entropy generation depends on parameters such as porosity, temperature ratio, temperature distribution, thermal conductivity and fins structure. Also, their results showed that entropy generation is maximum at the base of fin and the average entropy generation depends on porosity parameter and temperature ratio.

Sharma and Kumar (2023) investigated the hydrothermal aspects of ferrofluid flow over an upward/downward moving rotating disk under a low-oscillating magnetic field [17]. Their study showed that the presence of a low oscillating magnetic field with the downward motion of the rotating disk enhances the heat transfer rate more efficiently than the stationary disk.

Mountrichas et al. (2023) investigated the entropy generation and heat transfer of a nanofluid in a confined cavity with a moving top wall and a rectangular fin at the bottom [18]. They studied the effect of various parameters such as fin geometry, Rayleigh number, Reynolds number, and nanofluid concentration on the entropy generation. Their results indicated that

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with the usage of nanofluids instead of conventional heat transfer fluids, the heat transfer rate can be improved, while entropy generation can be minimized.

Zhang et al. (2024) studied the effect of rotational radius ratio on heat transfer of a channel equipped with water droplet-shaped pin-fins [19]. Their results showed that the rotation could be improved the heat transfer rate more than 20%.

Puspanathan et al. (2024) investigated the swirling flow over a shrinking rotating disk in Reiner-Rivlin fluid [20]. They calculated some key parameters such as local skin friction, local Nusselt number, velocity profile and temperature distribution. Their results showed that the increase of shrinking parameter decreases heat transfer and boundary layer thickness.

Pavithra and Gireesha (2024) studied thermal analysis of a non-integer-ordered wet porous fin, moving at a constant speed with a hybrid nanofluid of diverse-shaped nanoparticles by adomian decomposition method (ADM) and Sumudu transformation [21]. Their results showed that the highest heat flux occurs for nanoparticles with spherical-platelet shape.

From the study of the previous literature [1-21], it was found that the study of entropy generation in rotating disc fins has not been done. In this paper, we investigate the performance of a two-dimensional rotating disc fin from the perspective of entropy generation.

Governing equations

The schematic view of a disc fin is shown in Figure 2.

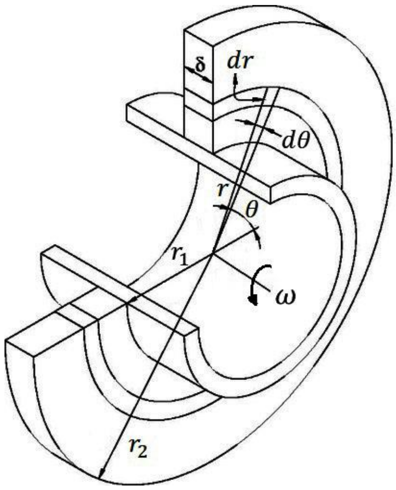

Figure 2. Schematic view of a rotating disc fin [22]

Where r_1, r_2, δ, and ω are inner radius of disc, outer radius of disc, thickness of disc and uniform angular velocity, respectively. Unlike the previous literature [1-21], due to the presence of rotating motion in the tangential direction, there is a possibility of temperature changes in the tangential direction, therefore, a two-dimensional analysis is used in the heat

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transfer analysis of the fin. In order to perform the thermal analysis of rotating disc fin a sectoral differential element according to Figure 3 is considered.

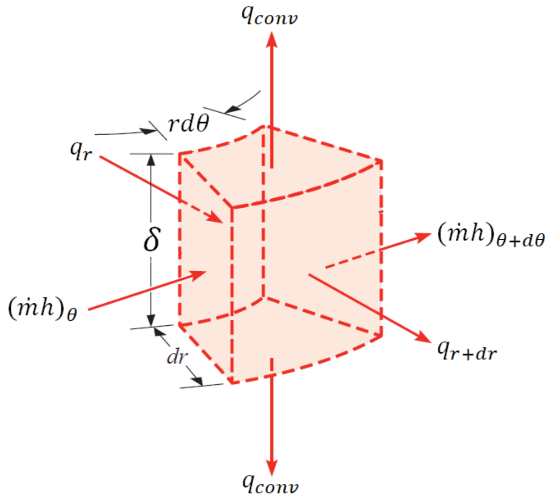

Figure 3. Sectoral differential element of a rotating disc fin

The general form of the energy balance for the mentioned differential element is defined as follows

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad (1)$$

Here, $\dot{E}_{in}$, $\dot{E}_{out}$, $\dot{E}_{gen}$, and $\dot{E}_{st}$ are the rate of input energy to the control volume, the rate of output energy from the control volume, the rate of energy generation in the control volume, and the time changes of energy in the control volume, respectively. There is no energy generation rate in the fin. Also, for steady state conditions, the time changes of energy is zero. The energy balance for the differential element of Figure 3 is simplified as follows

$$q_r + (\dot{m}h)_{\theta} - q_{r+dr} - (\dot{m}h)_{\theta+d\theta} - 2q_{conv} = 0 \quad (2)$$

Different energy rates and geometric parameters are defined as follows

$$q_r = -kA_r \frac{\partial T}{\partial r} \quad (3)$$

$$q_{r+dr} = q_r + \frac{\partial q_r}{\partial r} dr \quad (4)$$

$$(\dot{m}h)_{\theta} = \rho V A_{\theta} c T \quad (5)$$

$$(\dot{m}h)_{\theta+d\theta} = (\dot{m}h)_{\theta} + \frac{\partial (\dot{m}h)_{\theta}}{r\partial\theta} rd\theta \quad (6)$$

$$q_{conv} = h_{conv} A_{conv} (T - T_{\infty}) \quad (7)$$

$$A_r = \delta r d\theta \quad (8)$$

$$A_{\theta} = \delta dr \quad (9)$$

$$A_{conv} = r dr d\theta \quad (10)$$

$$V = r\omega \quad (11)$$

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Where k, ρ, c, h, V and T_∞ are fin conductivity, fin density, fin heat capacity, enthalpy, linear velocity and ambient temperature, respectively. The convective heat transfer coefficient (h_{conv}) on vertical rotating disc is calculated as follows [23]

$$h_{conv} = \frac{Nu_m k_a}{2(r_2 - r_1)} \tag{12}$$

$$\begin{cases} Nu_m = 20 & Re_\omega < 1000 \\ Nu_m = 0.982 Re_\omega^{0.443} & 1000 \le Re_\omega \le 100000 \end{cases} \tag{13}$$

$$Re_\omega = \frac{r_2^2 \omega}{\nu_a} \tag{14}$$

Where Nu_m, Re_ω, k_a, and ν_a are Nusselt number, rotational Reynolds number, air conductivity and air viscosity, respectively. Substituting Eqs. (3)-(11) in Eq. (2), a linear partial differential equation is obtained as follows

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{\rho \omega c}{k}\frac{\partial T}{\partial \theta} - \frac{2h_{conv}}{k\delta}(T - T_\infty) = 0 \tag{15}$$

Boundary conditions of the problem are defined as follows

$$T(r_1, \theta) = T_0 \tag{16}$$

$$\frac{\partial T(r_2, \theta)}{\partial r} = 0 \tag{17}$$

$$T(r, 0) = T_\infty \tag{18}$$

With the definition of $\tilde{T} = T - T_\infty$, the Eq. (15) and its boundary conditions are rewritten as follows

$$\frac{\partial^2 \tilde{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{T}}{\partial r} - \frac{\rho \omega c}{k}\frac{\partial \tilde{T}}{\partial \theta} - \frac{2h_{conv}}{k\delta}\tilde{T} = 0 \tag{19}$$

$$\tilde{T}(r_1, \theta) = T_0 - T_\infty = \tilde{T}_0 \tag{20}$$

$$\frac{\partial \tilde{T}(r_2, \theta)}{\partial r} = 0 \tag{21}$$

$$\tilde{T}(r, 0) = 0 \tag{22}$$

In order to obtain an analytical solution for the Eq. (19) by the method of separation of variables, a general solution is considered as follows [24]

$$\tilde{T}(r, \theta) = F(r) + G(\theta) \tag{23}$$

By substituting Eq. (23) in Eq. (19), the following separation expression is obtained

$$\frac{d^2 F}{dr^2} + \frac{1}{r}\frac{dF}{dr} + i^2 \xi^2 F = \gamma^2 \frac{dG}{d\theta} + \xi^2 G = \lambda^2 \tag{24}$$

Here $\xi^2 = 2h_{conv}/k\delta$, $\gamma^2 = \rho\omega c/k$ and $i^2 = -1$. From Eq. (24), two ordinary differential equations for $F(r)$ and $G(\theta)$ are obtained. The ordinary differential equation for $F(r)$ is as follows

$$\frac{d^2 F}{dr^2} + \frac{1}{r}\frac{dF}{dr} + i^2 \xi^2 F = \lambda^2 \tag{25}$$

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$$F(r_1) = \tilde{T}_0 \tag{26}$$
$$\frac{dF(r_2)}{dr} = 0 \tag{27}$$
By solving Eq. (25) under its boundary conditions, the following analytical solution is obtained
$$F(r) = \left(\tilde{T}_0 + \frac{\lambda^2}{\xi^2}\right)\left(\frac{J_0(i\xi r)Y_1(i\xi r_2) - Y_0(i\xi r)J_1(i\xi r_2)}{J_0(i\xi r_1)Y_1(i\xi r_2) - Y_0(i\xi r_1)J_1(i\xi r_2)}\right) - \frac{\lambda^2}{\xi^2} \tag{28}$$
Where J and Y are the first and second type Bessel functions of zero or one order. The ordinary differential equation for $G(\theta)$ is defined as follows
$$\frac{dG}{d\theta} + \frac{\xi^2}{\gamma^2}G = \frac{\lambda^2}{\gamma^2} \tag{29}$$
$$G(0) = 0 \tag{30}$$
The analytical solution of Eq. (29) is as follows
$$G(\theta) = \frac{\lambda^2}{\xi^2}\left\{1 - exp\left(-\frac{\xi^2}{\gamma^2}\theta\right)\right\} \tag{31}$$
The final solution of Eq. (15) is given as follows
$$T(r,\theta) = T_\infty + \left(T_0 - T_\infty + \frac{\lambda^2}{\xi^2}\right)\left(\frac{J_0(i\xi r)Y_1(i\xi r_2) - Y_0(i\xi r)J_1(i\xi r_2)}{J_0(i\xi r_1)Y_1(i\xi r_2) - Y_0(i\xi r_1)J_1(i\xi r_2)}\right) - \frac{\lambda^2}{\xi^2} exp\left(-\frac{\xi^2}{\gamma^2}\theta\right) \tag{32}$$
The additional condition is needed to obtain the separation parameter of λ^2. In radius of r_1, the temperature of fin is constant and does not change with the θ direction. Therefore, the fin governing equation should satisfy this condition. The condition obtained based on this fact is presented as follows
$$\left(\frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr} + i^2\xi^2F\right)_{r=r_1} = 0 \tag{33}$$
From the Eq. (33), the parameter of λ^2 is obtained as follows
$$\lambda^2 = \frac{(T_\infty - T_0)\left(k_1 + \frac{k_2}{r_1} - \xi^2 k_3\right)}{k_3 + \frac{1}{\xi^2}\left(k_1 + \frac{k_2}{r_1} - \xi^2 k_3\right)} \tag{34}$$
The coefficients of k_1, k_2 and k_3 are defined as follows
$$k_1 = -i\xi\left(-i\xi J_2(i\xi r_1) + \frac{1}{r_1}J_1(i\xi r_1)\right)Y_1(i\xi r_2) + i\xi\left(-i\xi Y_2(i\xi r_1) + \frac{1}{r_1}Y_1(i\xi r_1)\right)J_1(i\xi r_2) \tag{35}$$
$$k_2 = -i\xi J_1(i\xi r_1)Y_1(i\xi r_2) + i\xi Y_1(i\xi r_1)J_1(i\xi r_2) \tag{36}$$

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$$k_3 = J_0(i\xi r_1)Y_1(i\xi r_2) - Y_0(i\xi r_1)J_1(i\xi r_2) \quad (37)$$

The fin efficiency is defined as the ratio of actual heat transfer from the fin to the heat transfer from the fin at the base temperature.

$$\eta = \frac{q_{conv}}{q_b} = \frac{\int_{r_1}^{r_2}\int_0^{2\pi} 2h_{conv}\,[T(r,\theta) - T_\infty]\delta r dr d\theta}{2h_{conv}(T_0 - T_\infty)\pi\delta(r_2^2 - r_1^2)} \quad (38)$$

Substituting the temperature distribution of $T(r,\theta)$ in Eq. 38 and solving the double integral analytically, the following expression for fin efficiency is obtained

$$\eta = \frac{2\left(T_0 - T_\infty + \frac{\lambda^2}{\xi^2}\right)\left([r_2J_1(i\xi r_2) - r_1J_1(i\xi r_1)]Y_1(i\xi r_2) - [r_2Y_1(i\xi r_2) - r_1Y_1(i\xi r_1)]J_1(i\xi r_2)\right)}{i\xi(r_2^2 - r_1^2)(T_0 - T_\infty)[J_0(i\xi r_1)Y_1(i\xi r_2) - Y_0(i\xi r_1)J_1(i\xi r_2)]} + \frac{\lambda^2\gamma^2}{2\pi\xi^4(T_0 - T_\infty)}\left[exp\left(-2\pi\frac{\xi^2}{\gamma^2}\right) - 1\right] \quad (39)$$

Entropy generation analysis is a useful tool in various engineering applications where energy efficiency and system optimization are important. In rotating systems such as rotating fins, entropy generation analysis can help identify ways to increase the heat dissipation from the fin and improve its performance. The rate of entropy generation per unit volume is given as follows [25]

$$s_{gen}''' = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial T}{\partial \theta}\right)^2\right] \quad (40)$$

$$\frac{\partial T}{\partial r} = \left(T_0 - T_\infty + \frac{\lambda^2}{\xi^2}\right)\left(\frac{-i\xi J_1(i\xi r)Y_1(i\xi r_2) + i\xi Y_1(i\xi r)J_1(i\xi r_2)}{J_0(i\xi r_1)Y_1(i\xi r_2) - Y_0(i\xi r_1)J_1(i\xi r_2)}\right) \quad (41)$$

$$\frac{\partial T}{\partial \theta} = \frac{\lambda^2}{\gamma^2} exp\left(-\frac{\xi^2}{\gamma^2}\theta\right) \quad (42)$$

Finally, the entropy generation rate in the fin is obtained as follows

$$\dot{S}_{gen} = \iiint_{\forall} s_{gen}''' d\forall = \int_{r_1}^{r_2}\int_0^{2\pi} \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial T}{\partial \theta}\right)^2\right]\delta r dr d\theta \quad (43)$$

Due to the presence of Bessel functions with power of two in the temperature distribution and its derivations, the double integral of the entropy generation rate does not have an analytical solution. Therefore, it should be solved by numerical methods.

Validation

In the previous literatures [1-21], two-dimensional disc fin has not been studied, so there is no experimental data or analytical solution results for it. However, for the simple case of stationary disc fin, one-dimensional analytical solution is available. If the angular velocity is set to zero

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for the disc fin of the present research, then its results can be compared with the analytical solution of stationary disc fin. The temperature distribution of a stationary disc fin is given as follows [26]

$$\frac{T(r)-T_\infty}{T_0-T_\infty}=\frac{I_0(\xi r)K_1(\xi r_2)+K_0(\xi r)I_1(\xi r_2)}{I_0(\xi r_1)K_1(\xi r_2)+K_0(\xi r_1)I_1(\xi r_2)} \tag{44}$$

Where I and K are the first and second type modified Bessel functions of zero and one order. In Table 1 the design parameters of disc fin are given.

Table 1. Design parameters of disc fin [27]

Parameter	**Value**
Fin material	Cast iron
Fin inner radius	$r_1 = 63.5\ mm$
Fin outer radius	$r_2 = 113.5\ mm$
Fin thickness	$\delta = 11\ mm$
Fin density	$\rho = 7150\ kg/m^3$
Fin conductivity	$k = 53.3\ W/m.K$
Fin heat capacity	$c = 0.46\ kJ/kg.K$
Fin base temperature	$T_0 = 50$ ℃
Ambient temperature	$T_\infty = 25$ ℃
Air conductivity	$k_a = 0.026\ W/m.K$
Air viscosity	$\nu_a = 1.6 \times 10^{-5}\ m^2/s$

The temperature distribution of the present study fin and the stationary disc fin is compared in Figure 4.

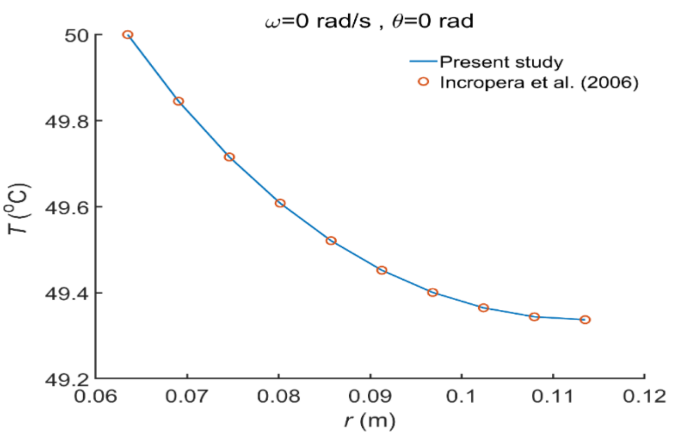

Figure 4. Temperature distribution of the present study fin and the stationary disc fin

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It is concluded from Figure 4 that the trend of the temperature distribution of the present study fin (zero angular velocity) and the stationary disc fin is completely similar. Also, the temperature values obtained for the two mentioned fins are the same.

Results and Discussion

In Figure 5, the fin temperature distribution is plotted in three dimensions in terms of radial location and tangential location.

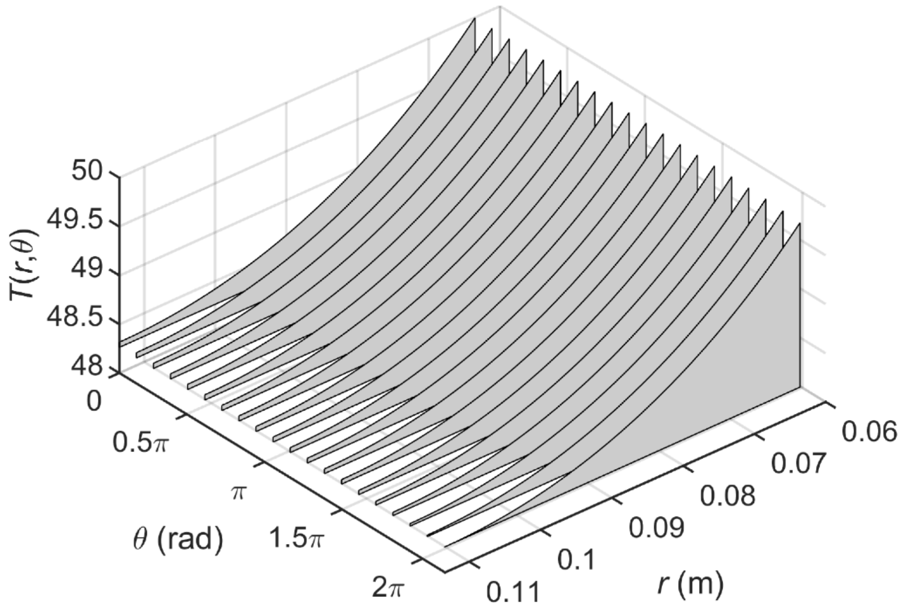

Figure 5. 3D temperature distribution of fin in terms of radial location and tangential location $(\omega = 10\,rad/s)$

According to Figure 5, as the radial distance from the fin base increases, the fin temperature decreases. The points far from the fin base have higher tangential speed, therefore, more convection heat transfer takes place from them to the environment and their temperature is lower. On the other hand, with the increase of the tangential location, the fin temperature decreases due to the presence of angular velocity. Although this reduction is not noticeable.

In order to further study, the fin temperature contour in terms of radial location and tangential location is drawn in Figure 6.

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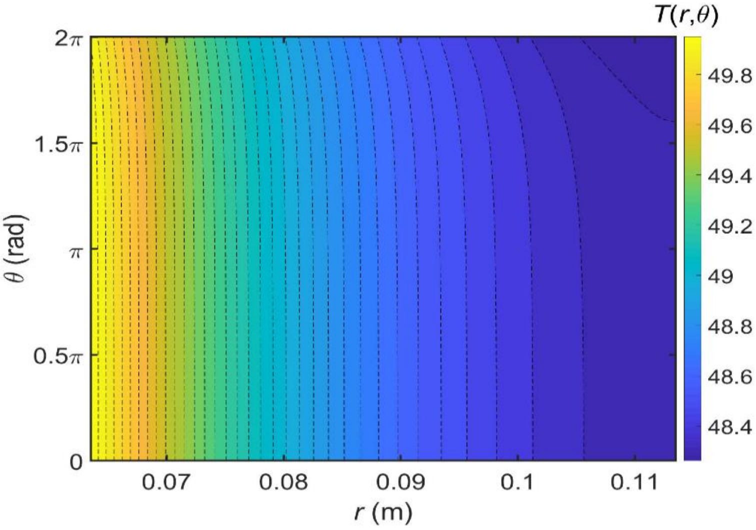

Figure 6. Fin temperature contour in terms of radial location and tangential location
$(\omega = 10\,rad/s)$

It is observed from Figure 6 that the fin temperature changes are small compared to tangential location changes. The annular fin has the symmetry of geometry and physical conditions in the tangential direction. This makes the temperature changes in the tangent direction not significant.

In Figure 7 the contour of fin temperature in terms of radial location and angular velocity is plotted.

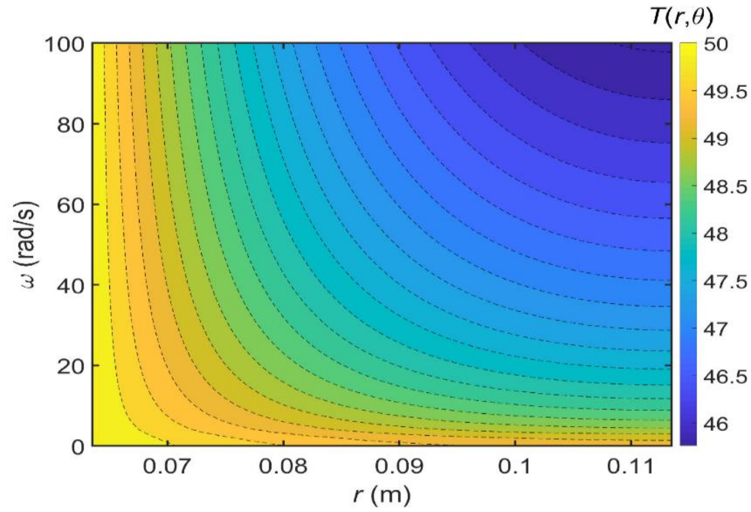

Figure 7. Contour of fin temperature in terms of radial location and angular velocity
$(\theta = 0\,rad)$

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It is observed from Figure 7 that the fin temperature decreases as the angular velocity increases. Especially, this temperature decrease is greater at distances far from the fin base. The tangential velocity component in the fin is calculated from the product of the angular velocity at the radial distance from the fin base. As the radial distance from the fin base increases, the tangential velocity increases and this increase is significant in larger values of the angular velocity. As a rule, an increase in tangential velocity leads to an increase in convection transfer, and heat dissipation from the fin increases and its temperature decreases.

In order to better understand, the fin temperature changes in terms of radial location at different values of angular velocity are plotted in Figure 8.

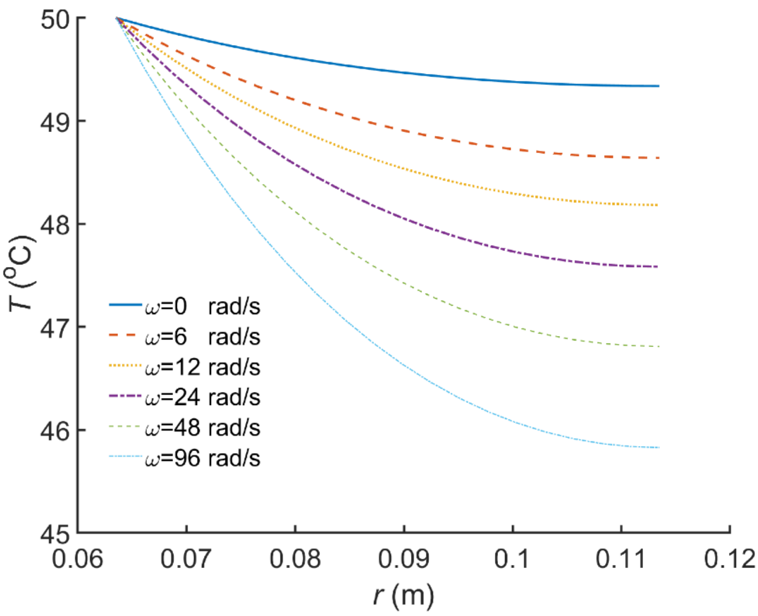

Figure 8. Fin temperature changes in terms of radial location at different values of angular velocity ($\theta = 0\, rad$)

As can be seen in Figure 8, the fin temperature decreases with the increase of radial location and angular velocity. An increase in radial location and angular velocity leads to an increase in the tangential velocity component in the fin. By increasing the tangential velocity, the convection heat transfer from the fin increases and it causes the fin temperature to decrease.

In Figure 9, the contour of fin efficiency is plotted in terms of the dimensionless wing length of fin and the angular velocity.

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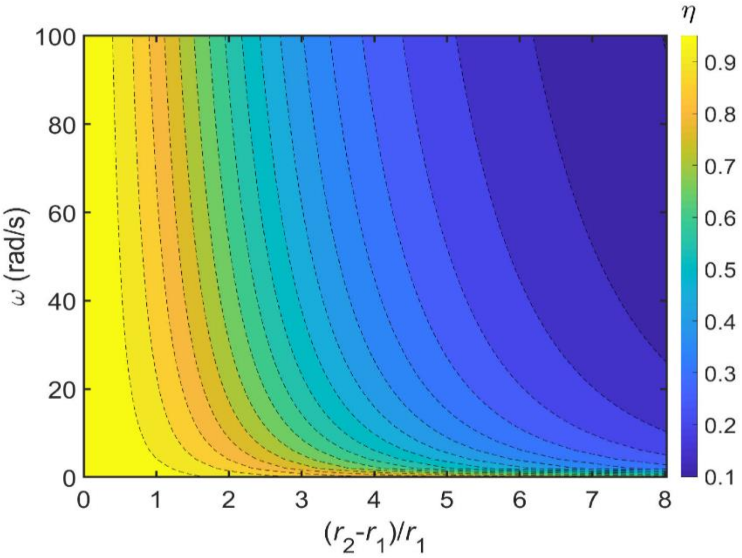

Figure 9. Contour of fin efficiency in terms of the dimensionless wing length of fin and the angular velocity

The dimensionless wing length of fin is defined as follows

$$r^* = \frac{r_2 - r_1}{r_1} \tag{45}$$

In Figure 10, the fin efficiency is plotted in terms of the dimensionless wing length of fin at different values of the angular velocity.

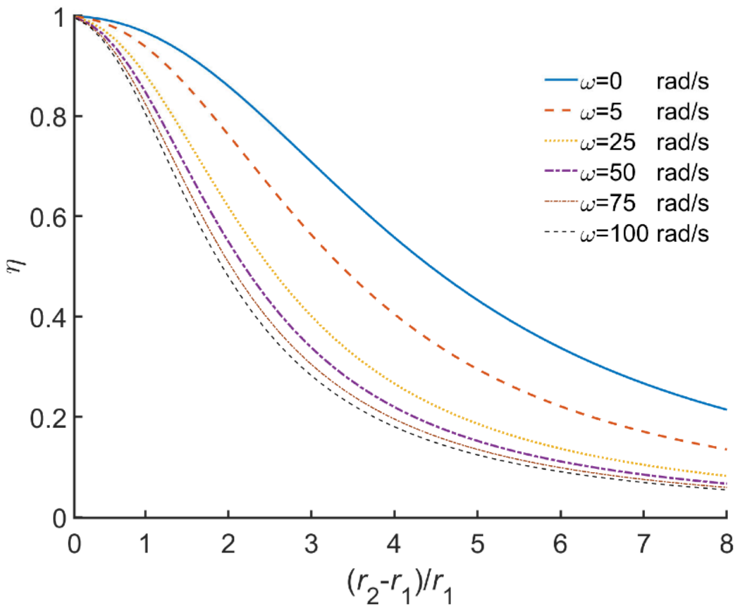

Figure 10. Fin efficiency in terms of the dimensionless wing length of fin at different values of the angular velocity

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According to Figure 9 and Figure 10, with the increase of the dimensionless wing length and the angular velocity, the fin efficiency decreases. The increase of the dimensionless wing length and the angular velocity causes the increase of convection heat transfer (heat dissipation) from fin to ambient (q_{conv}). Also, the increase of these two parameters leads to the increase of convection heat transfer at the base temperature of the fin (q_b). However, the increase in q_b is far more than the increase in q_{conv}. Fin efficiency is calculated from the result of dividing q_{conv} by q_b. A greater increase in q_b compared to an increase in q_{conv} decreases the fin efficiency. Similar to the result obtained in the present research, it has also been observed in previous references [28]. On the other hand, when the fin wing length approaches zero, the fin temperature is equaled the fin base temperature and the consequence of it the fin efficiency is maximized.

In order to better understand this fact, the rate of convective heat transfer from the fin to the environment in terms of the dimensionless wing length of fin at different values of angular velocity is plotted in Figure 11.

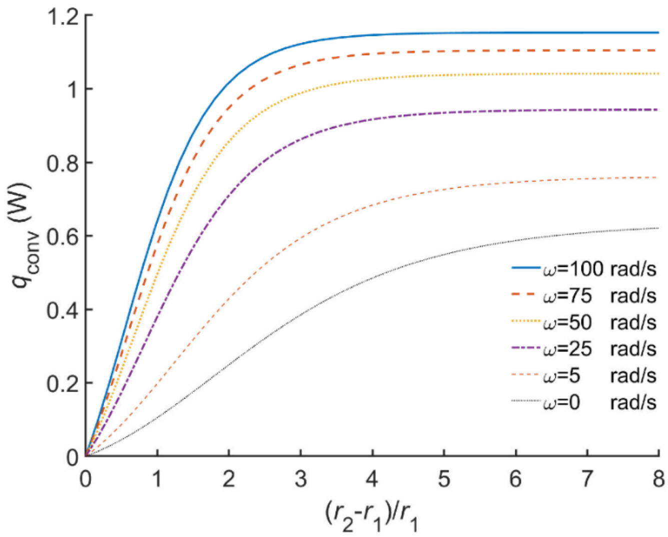

Figure 11. Rate of convection heat transfer of fin in terms of the dimensionless wing length of fin at different values of the angular velocity

It should be mentioned that the tangential velocity component in the fin is obtained from the product of the radial location and the angular velocity. Therefore, an increase in radial location and angular velocity increases the tangential velocity, and an increase in the tangential velocity leads to an increase in convection heat transfer. Especially, the increase of convection heat transfer is higher at high values of radial location and angular velocity.

In Figure 12, the contour of fin entropy generation in terms of the dimensionless wing length of fin and the angular velocity is shown.

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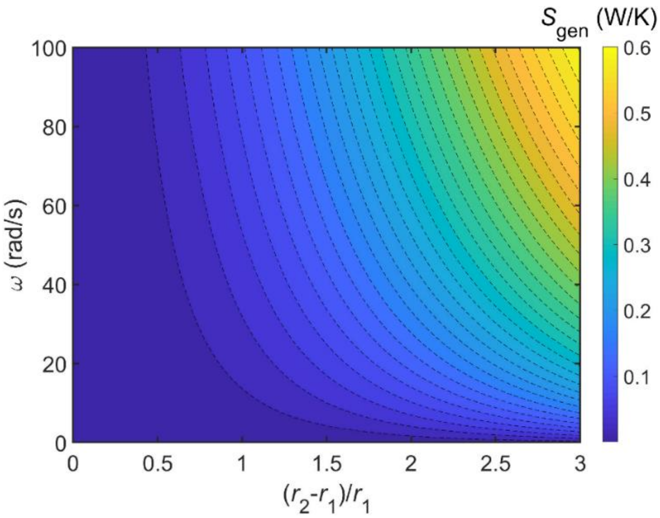

Figure 12. Contour of fin entropy generation in terms of the dimensionless wing length of fin and the angular velocity

According to Figure 12, the entropy generation increases with the dimensionless wing length of fin and the angular velocity. By increasing the wing length of fin and the angular velocity, the temperature difference between the fin and the ambient temperature increases and the entropy generation increases. Increasing temperature difference is one of the main factors of increasing irreversibility rate.

In order to study further, in Figure 13, the changes of entropy generation rate in terms of the dimensionless wing length of fin and the angular velocity are drawn.

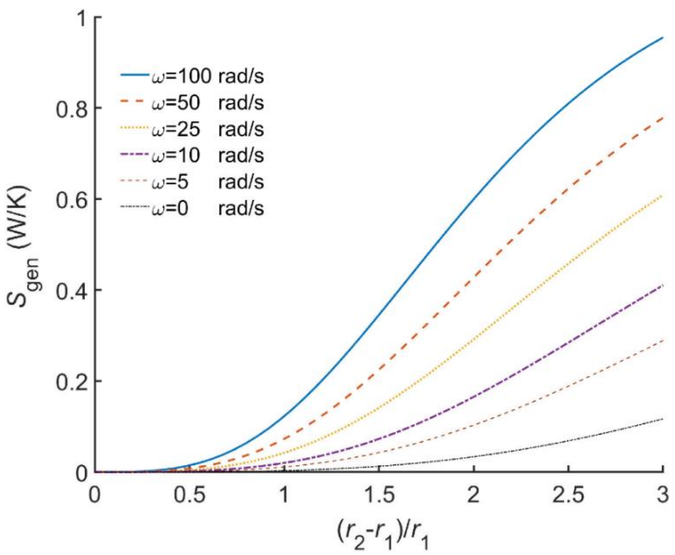

Figure 13. Changes of entropy generation rate in terms of the dimensionless wing length of fin and the angular velocity

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According to Figure 13, the increase in temperature difference caused by the increase in angular velocity and wing length of fin has increased entropy generation rate. As a general rule, thermal systems with lower temperature differences are closer to thermodynamic equilibrium conditions and have lower entropy generation rate.

Conclusion

The main conclusions of the present study are as follows

- The comprehensive analytical solution of the present research is able to predict the temperature of the fin in different states such as stationary fin.
- Due to the symmetry in the fin geometry and physical conditions, the dependence of the fin temperature distribution on the tangential location is not significant.
- An increase in angular velocity leads to a decrease in fin efficiency and an increase in entropy generation rate. Therefore, to establish a balance between increasing efficiency and decreasing entropy, a suitable angular velocity should be selected.
- Increasing the angular velocity increases the convection heat transfer and decreases the fin temperature. This is an important advantage. By connecting the fin to the rotating heating systems, you can use the advantage of self-cooling and increase the heat removal from the system.

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