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Calculation of the Energy Release Rate for Dynamic Crack Propagation (Kinking)

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Abstract: In fact, if the load or geometry of the structure is asymmetric with respect to the crack axis, the fracture will occur in a mixed mode and the crack will not propagate in a straight line. Besides, the use of bifurcation criterion has shown to be necessary to determine the new propagation direction. However, for fast loading or when crack growth rate is important, the moment of inertia must be taken into account in the formula. Then, this is called dynamic crack growth. In virtue of which, a new model was developed in this paper to evaluate the energy release rate G in the case of dynamic loading. This model is based on the association of the special mixed finite element RMQ-7 and the virtual crack propagation technique. As for the performance of the model, it was validated by studying a cracked plate under dynamic load with different levels of velocity.

Keywords: KINKING; FRACTURE; CRACK; DYNAMIC; RMQ-7.

1. INTRODUCTION

Fracture mechanics propose to describe the steps of crack initiation and propagation according to the behavior of the material, which can present two types of fractures, the brittle fracture (Linear fracture mechanics) and ductile fracture (Nonlinear fracture mechanics) [1].

Indeed, the first work in fracture mechanics aimed to accurately determine the mechanical field in the vicinity of a static crack. On the other hand, the difficulty of this kind of research lies in the accurate calculation of mechanical parameters, in respect such as stress intensity

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factor or energy release rate considering the singularity introduced by the crack tip. Consequently, with these mechanical parameters, it is possible to predict whether the crack will grow or not [2, 3].

Nonetheless, when the loading or geometry of a structure is not symmetrical with respect to the crack axis, the fracture occurs in mixed mode and the crack does not propagate in a rectilinear way. In that case, it is necessary to take bifurcation criteria into consideration. For determination purpose of the new direction of propagation, several authors have developed methods to predict the angle of kinking [4].

In fact of matter, the kinking occurs either due to the geometric configuration or because of any loading, it develops in mixed mode I (crack opening) and in mode II (slide in the plane) [5,6].

For fast loading problems, when the crack growth rate is important, the inertia term must be considered in the formulation, when solving the problem, we are talking about dynamic crack growth [7].

Additionally, the study of dynamic fracture stands for a branch of mechanics in which the influence of inertia exceeds stress, and the strain rate has a significant influence on the properties of the material and the evolution of fracture.

Most importantly, the onset of dynamic fracture or the dynamic propagation of an existing crack can occur under the influence of the relevant structure or the sudden application of a potentially damaging load and create, as a consequence, a discontinuity in the material. Further, dynamic failure is studied for the first time and has essentially shown to be empirical. It refers to development of artillery for the purpose of perforating targets in the 19thcentury. Even so, as an engineering science, dynamic fracture did not become part of fracture mechanics until the 1940s.

Nevertheless, as it was proposed by author in [8], it is indeed possible to analyse an interface crack between dissimilar anisotropic materials by a numerical method being carried out. In this respect, the target problem to be solved is superimposed on the asymptotic solution of the displacement in the vicinity of an interface crack tip, which is described by use of the formalism of Stroh. Moreover, the proposed method accurately provides mode-separated stress intensity factors using relatively coarse meshes for the finite element method.

More to the point, H.G. Beomanalyzed an interface crack and a sub-interface crack in an orthotropic bi-material structure consisting of a thin film and a semi-planar substrate. Besides, the orthotropic bi-material structure is subjected to a compressive load and a bending moment per unit thickness. Likewise, they used the orthotropic rescaling technique to determine the explicit dependence of dimensionless parameters on an orthotropic parameter for the film. In virtue of which, they obtained through numerical calculations, the variations of the parameters without dimension with the other parameters of the material [9].

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In addition, V. Mantič and I.G. García presented a theoretical model for the prediction of the critical load generating crack initiation at the fiber-matrix interface under a biaxial transverse load at a distance. Above and beyond, they assumed a linear isotropic elastic behavior for the inclusion and the matrix. Hence, the analytical and semi-analytical expressions obtained make it possible to easily study the influence of all the dimensionless parameters governing the behavior of the fiber-matrix system. Accordingly, their study made it possible to predict the effect of the size of the inclusion radius on the critical load; as a result, the smaller inclusions being stronger and less dependent on the secondary load [10].

With regards to Jun Lei, he applied the hybrid time-domain boundary element method, together with the multi-regional technique, for simulation purpose of the dynamic process of propagation and/or twisting of an interface crack in a two-dimensional material. Nonetheless, a comparison was made between the numerical results of the crack growth trajectory for different materials' combinations. Therefore, corresponding experimental results have found good agreement between them, which shows that the numerical boundary element method can provide an excellent simulation of the dynamic propagation and deflection of an interface crack [11].

Author in [12] investigated the rate of energy relaxation associated with stress intensity factors at an arbitrary angle under mixed-mode loadings through the use of both a numerical method and a theoretical derivation. Thus, they confirmed that the deduced theoretical expression could provide results as accurate as the used numerical method and the experimental data.

As for Jenny Carlsson and Per Isaksson, 2019, they investigated the influence of pores in a material on crack dynamics in brittle fracture. Hence, a dynamic phase field finite element model was used in order to investigate the effect of pores with respect to the crack's pathing, the propagation velocity and the energy release rate in a strip sample geometry with circular pores. As a consequence, they noticed that as the porosity of an initially solid material increases, the crack tip is more and more likely to become shielded or stopped, which may be the key to the high relative strength often exhibited by porous materials. Additionally, they found that when a pore is the same size as the characteristic internal length then the pore does not localize the damage. Since the characteristic internal length only regulates the damage field and not the end-of-strain kinetic energy distributions, the crack dynamics are still affected by the small pores [13].

In the light of the facts set out in this paper, we propose a method for calculating the energy release rate G in the dynamic case, using a mixed finite element RMQ-7, which element was developed by Bouzerd H (1992), with a quadrilateral shape consisting of seven nodes with fourteen degrees of freedom. Likewise, Bouziane (2009) introduced the design of the finite element in natural (ξ,η) plane.

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Correspondingly in this work, this method is initially used in the study of bending in the dynamic case. Therefore, the obtained results are compared with those of the analytical solution.

2. METHOD USED FOR THE STUDY OF THE KINKING

Given their complexity, real structures cannot be analytically processed, thus leading researchers to turn to numerical methods to approach an exact solution. For this reason, two approaches can be distinguished for the study of singular bands:

- Local method: Presents the study of stress and deformation fields near the crack front [14]. This approach calculates the stress force factor from the displacement of the crack lip. Therefore, once they are determined, the energy release rate can be calculated by linking their relationship.

- Global or energetic method: is characterized by the study of the overall behaviour of ruptured structures at the energy level. Among these methods, the integral J method is based on Rice's formula, whereas the virtual crack propagation method directly provides the energy return rate G (Griffith's theory) [15].

3. CALCULATION METHOD

3.1. Stages of construction of the RMQ-7 element

The mixed variational method is the obligatory basis for constructing such an element. using the mixed Reissner principle (the Reissner elements generate a large number of degrees of freedom, which makes their use cumbersome, hence a high calculation time). To construct a reference element from which the final element was developed, two methods were used:

- The relocation technique, a method suggested by Verchery, which consists of moving certain static unknowns towards the interior or to one side of the element.
- The static condensation of the unknowns internal to the element.

This leads to the successive construction of an RMQ-5 element (reference element) then an element RMQ-11 (by the relocation technique), to obtain at the end the element RMQ-7 object of our work (by condensation), the reference element is a mixed element with five nodes, each of the four corner nodes presents the complete set of static and kinematic plane variables$(u_1 \,.u_2 \,.\sigma_{11}.\sigma_{12}.\sigma_{22})$ and the fifth node located in the middle of one of the horizontal sides only includes static variables [16].

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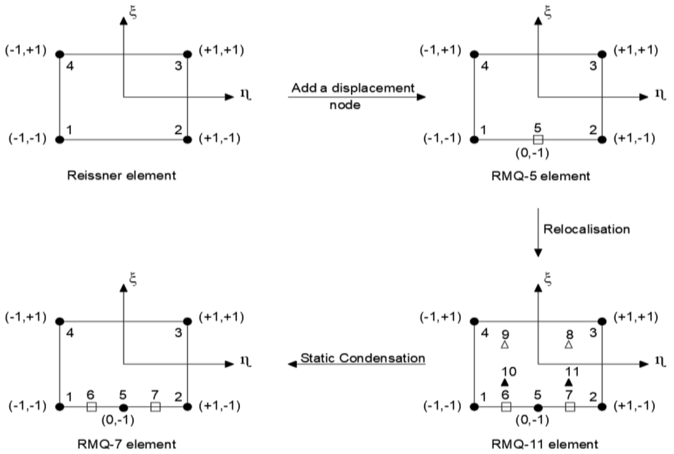

Figure 1: Different Stages construction of the RMQ-7 element

3.2. Element RMQ-7

Bouzerd H. developed the finite element RMQ-7 which has the ability to model coherent interfaces as well as fractured interfaces, and assumes linear elastic behaviour of the material at small displacements. Likewise, using the energy release rate can be estimated through the virtual crack propagation method. Consequently, the RMQ-7 element is a rectangular element with 07 nodes, 14 degrees of freedom; three of these sides are compatible with linear classical elements and therefore have a displacement node at each end. The fourth side in addition to its two kinematic endpoints offer three additional nodes, an intermediate displacement node and two static nodes in the middle of each half edge, that introduce components of the stress vector along the interface. As a result of which, the RMQ-7 element requires static variables σ_{12} and σ_{22} at constraint nodes 6 and 7, together with kinematic variables at the remaining nodes, as per illustrated in (Figure 2).

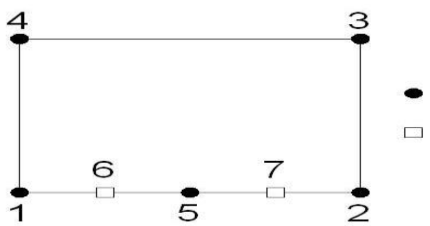

Figure 2: Configuration of the RAMQ-7 element

3.3. Energy Release Rate G

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The energy release rate G represents the energy required to propagate a crack per unit length [17]. For the evaluation of the energy recovery rate, there are several techniques, in respect such as the virtual crack propagation technique introduced by Hellen and Parks [18, 19], which makes it possible to calculate the variation in the total potential energy when a virtual crack is introduced. For a Crack propagation δa, the energy release rate is obtained from the following formula:

$$G_S = \frac{\delta\pi}{\delta a} \quad (1)$$

$$\delta\pi = \pi(a) - \pi(a + \delta a) \quad (2)$$

In the case of collinear crack propagation, (Bouzerd 1992), the virtual propagation method associated with the RMQ-7 element was used to calculate the energy release rate for the elbow bending case. More and more, linear elastic behaviour is assumed in a small range and the external load does not change during the crack increasing δa.

Calculation of the energy release rate:

$$G_S = -\frac{\pi(a+\delta a)-\pi(a)}{\delta a} \quad (3)$$

The strain energy is written as following:

$$\pi = \frac{1}{2}\sum_{i=1}^{ne}\{V\}_i^t\,[K]_i\{V\}_i \quad (4)$$

With:

$[K]_i$: Stiffness matrix element i, and exponent t indicates transposition.

$\{V\}_i$: Column vector containing the nodal values of element i.

ne :Total number of elements in the discretized structure.

t :Indicates transposition .

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In the case of dynamic propagation of the crack, the first analytical treatise of a dynamic fracture was made by Mott,who amended the Griffith's energy balance for a central crack in an infinite plate with the kinetic energy of a fracture event [20]. Therefore, the expression for the strain energy release rate in an elastic continuum reads:

$$G = \frac{dF}{da} - \frac{d\pi}{da} - \frac{dE_K}{da} \tag{5}$$

With:

π: Elastic energy, F: Energy of external forces, E_K: Kinetic energy.

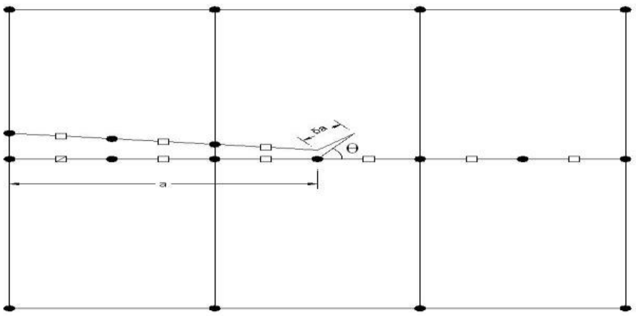

Figure 3: Dynamic propagation of an inclined crack (kinking)

The kinetic energy is calculated as follows:

$$E_K = \frac{1}{2} k^2 \rho a^2 \dot{a}^2 \left(\frac{\sigma}{E}\right)^2 \tag{6}$$

With:

ρ: Density,

a: The crack length,

$\dot{a}$: The crack speed.

k:The wave constant , E: Young's modulus.

σ:External load.

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Calculate the variation in the total kinetic energy when a virtual crack is introduced, for a crack propagation δa.

$$G_{Kin} = \frac{E_K(a+\delta a)-E_K(a)}{\delta a} \tag{7}$$

$E_K(a+\delta a)$ and $E_K(a)$ represent the kinetic energy of the cracked structure in the configurations $(a+\delta a)$ and (a) respectively.

Thus, the relation gives the expression of the total energy release rate G for calculation of the dynamic propagation of the crack (kinking):

$$G = G_S + G_{Kin} \tag{8}$$

$$G = \left(\frac{\pi(a+\delta a)-\pi(a)}{\delta a} + \frac{E_K(a+\delta a)-E_K(a)}{\delta a}\right) \tag{9}$$

3.4. Numerical Implantation

The developed model is valid on a real case which consists in studying the propagation of a crack in the dynamic case. In this respect, we propose a method for calculation of the energy release rate G in the dynamic case, using a mixed finite element RMQ-7, which element was developed by Bouzerd H (1992), with a quadrilateral shape consisting of seven nodes with fourteen degrees of freedom. Besides, Bouziane (2009) introduced the formulation of the finite element in natural (ξ,η) plane.

In the case of collinear crack propagation, (Bouzerd 1992), the virtual propagation method associated with the RMQ-7 element was used to calculate the energy release rate for the elbow bending case. In addition, linear elastic behaviour is assumed in a small range and the external load does not change during crack increasing δa. Thus, the energy release rates G is developed by Bouzerd, as it is only evaluated from one finite element analysis in the configuration (a+δa) with kinking. Above and beyond, the initial configuration is (a) implicitly considered in the same analyse by evaluating and storing the elementary matrices of the concerned elements. Indeed, the kinking extension leads to a much localised disturbance around the crack tip; the rest of the mesh remains unchanged. In consequence, only four elements are rearranged: the two elements (upper and lower) containing the crack tip and the two elements immediately related thereto in the direction of the evolution of the crack, as shown in (Figure 4).

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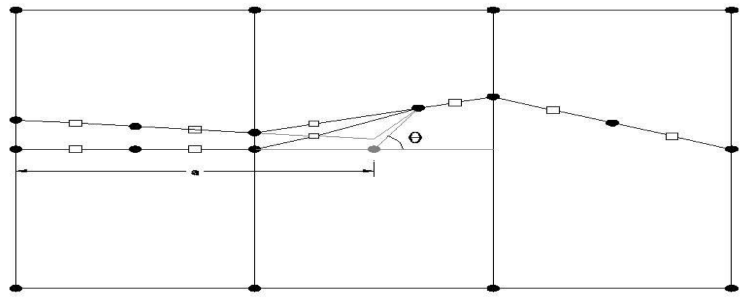

Figure 4: Reorganization of the mesh around the new tip crack

4. Validation Example

For validation purpose of the proposed model, a cracked plate under tensile stress was studied σ =1 Mpa. Besides, the plate is made of homogeneous isotropic material, its mechanical properties are: Young's modulus E=32 Gpa, Poisson's ratio υ=0.2, density ρ=2450 kg/m³, the geometric dimensions of the panel are: plate length L = 100 mm, plate width b = 40 mm, crack length a = 50 mm, crack angle 30°.

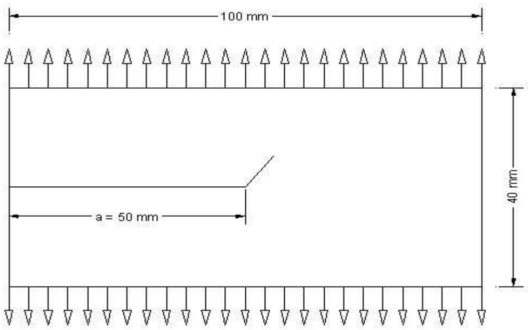

Figure 5: Studied plate

The dynamic energy release rate G in this case was analytically evaluated by theta method (Debruyne,J.Laverne) [21].

Energy balance:

$$G = -\left(\frac{\partial W_{el}}{\partial l} + \frac{\partial W_{ext}}{\partial l} + \frac{\partial W_{kin}}{\partial l}\right) \quad (10)$$

$$W_{el} = \frac{1}{2}\int Tr(\sigma \nabla u)d\Omega \quad (11)$$

$$W_{ext} = \int f.u\, d\Omega \quad (12)$$

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$$W_{kin} = \frac{1}{2}\int \rho.\dot{u}\dot{u}\, d\Omega \tag{13}$$

With:

W_{el}: Elastic energy.

W_{ext}: Energy of external forces.

W_{kin}: Kinetic energy.

The relation gives the expression of the energy release rate G analytical:

$$G = \int \sigma(u\,,\alpha).(\nabla u \nabla \theta) + \frac{1}{2}\rho\dot{u}\dot{u}div\theta - \frac{1}{2}\sigma(u,\alpha)\varepsilon(u)div\theta + \rho\ddot{u}\nabla u\theta + \rho\dot{u}\nabla\dot{u}\theta \tag{14}$$

With:

σ: Stress tensor, ε :Strain tensor, u : Displacement.

u̇: Crack velocity.

ü: Acceleration.

ρ: Density.

θ: Angle of bifurcation.

5. RESULTS AND DISCUSSION

The structure was discretized using the element RMQ-7. Upon conducting a study, the mesh consisting of 4200 elements and 10821 nodes was retained. Therefore, the obtained results were summarized in the following table:

Table01: Values of the energy release rate

V (m/s)	angle of bifurcation θ(°)	G calculated (m.J)	G analytic (m.J)	Error (٪)
18.18	30	0.119	0.115	+ 3.361
20	30	0.120	0.125	- 4.166

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25	30	0.119	0.110	+ 7.563
33.33	30	0.120	0.125	- 4.166
100	30	0.147	0.135	+ 8.163

After several tests with different speeds, the result obtained by the proposed approach has shown to be the closest to the analytical solution.

6. CONCLUSIONS

The proposed model for calculating the energy release rate G , utilizing the RMQ-7 element in conjunction with the virtual crack propagation technique, represents a significant advancement in the analysis of crack propagation, particularly under dynamic loading conditions. The RMQ-7 element is uniquely designed to evaluate the energy release rate G for scenarios involving crack kinking, a critical aspect of fracture mechanics where the crack path deviates from its original trajectory. This model is distinguished by its incorporation of both static and kinematic variables at specific nodes, ensuring a comprehensive assessment of the stress and strain fields around the crack tip.

The RMQ-7 element necessitates static variables σ_12and σ_22 at constraint nodes 6 and 7, respectively. These variables are essential for accurately capturing the stress state in the vicinity of the crack tip. The constraint nodes serve as critical points where the local stresses are evaluated, providing crucial information about the material's resistance to crack propagation. In addition to these static variables, the model also incorporates kinematic variables at the remaining nodes. This combination allows for a detailed description of the displacement field around the crack, which is vital for determining the deformation and subsequent energy release rate.

One of the significant strengths of the developed model is its applicability to real-world cases, particularly in dynamic scenarios. Crack propagation under dynamic loading conditions poses unique challenges, as the speed and nature of loading can significantly influence the fracture process. The proposed method addresses these challenges by enabling the calculation of the energy release rate G in dynamic cases. This is achieved through a mixed finite element approach using the RMQ-7 element, which has been validated and proven effective in handling cracking problems under dynamic loads.

The effectiveness of the RMQ-7 element in dynamic cases is underscored by its ability to provide accurate evaluations of the energy release rate across various test conditions. By varying the speed in each test, the model demonstrates its robustness and adaptability. The results indicate that the RMQ-7 element not only simplifies the calculation process but also

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enhances the accuracy of the energy release rate evaluation. This is particularly important in engineering applications where precise predictions of crack growth and material failure are crucial for safety and reliability.

Moreover, the integration of the virtual crack propagation technique with the RMQ-7 element facilitates a more streamlined and relevant calculation process. This technique allows for the simulation of crack growth without the need for complex remeshing, which is often required in traditional finite element methods. As a result, the proposed model significantly reduces computational effort and improves the efficiency of the analysis.

In summary, the proposed model leveraging the RMQ-7 element and the virtual crack propagation technique represents a notable advancement in fracture mechanics. By incorporating both static and kinematic variables, it provides a comprehensive assessment of the stress and displacement fields around the crack tip. Its application to dynamic cases demonstrates its versatility and effectiveness in handling real-world cracking problems. The model simplifies the calculation process while ensuring accurate evaluations of the energy release rate, making it a valuable tool for engineers and researchers in the field of fracture mechanics. This innovative approach not only enhances our understanding of crack propagation under dynamic loading but also contributes to the development of safer and more reliable materials and structures.

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