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Modelling the social Internet of Things network using random graphs

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Abstract

The Social Internet of Things (SIOT) is a paradigm in which things communicate with one another based on social relationships. The goals are for things to search for services and retrieve and provide users with them independently from their owners. The existing approaches lack models for analyzing the efficiency of SIOTs. The objective of the present research is to model SIOTs with regard to various relationships and their different topological properties. In this approach, the graphs related to the SIOT are modeled based on the similarity of their topological properties and random graphs in such a way that these properties are preserved by increasing the size of the network, the intended topological properties are preserved. For this purpose, first, the topological properties of real SIOT graphs have been extracted. Then, using numerical and intuitive comparisons, the degree of resemblance between the SIOT topological properties and random graphs has been examined. In order to prove this resemblance and network scalability, the connection between the average route length and the descending gradient algorithm has been implemented. The obtained results have shown the resemblance of ownership object relationship (OOR) real graph to the ErdoS Renyi (ER) random graph at p=0.9, the POR graph to the ER random graph at p=0.009, the CLOR graph to the ER random graph at p=0.00009 and the SOR graph to the Barbasi Albert (BA) random graph at m=50. In order to evaluate the proposed framework, the real SIOT dataset has been used, and the scalability and maintenance of the topological properties have been proven.

Key words: Modeling، SIOT، Random graph، Topological properties, Scalability

1. Introduction

The Internet of Things (IOT) is an emerging network which is formed by billions of things connected to the Internet. In this network, smart things might be heterogeneous in terms of their nature, operating system and manufacturer's special protocols; they might also be able to generate a large amount of data in a physical space [1]. In the SIOT network, the goal is for

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things to search for services independent of their owners, retrieve services and provide users with them. In order to search for services, things in this network can conduct network crawling using friends and things' friends' friends; they can retrieve the required services and provide users with them through social relationships independent of their owners [2].

The relationships between things in this network can be of different forms. They are categorized as CLOR, OOR, POR, Co work object relationship (CWOR), and SOR in [3] and divided into 6 categories in [4], namely, CLOR, OOR, POR, CWOR, friendship object relationship (FOR), and community of interest (COI). In the CLOR relationship, things cooperate with each other to undertake a task in a shared location. In an OOR relationship, things owned by the same person connect with each other. In the POR relationship, things having the same brand connect with one another. In the CWOR relationship, things cooperate with each other to perform a shared task to achieve a shared goal. Finally, in the SOR relationship, things whose owners had social relationships with each other once, connect with one another. Things with one kind of relationship can connect to each other virtually and transfer data.

The dynamic nature of smart things, the relationships existing between them, and having heterogeneous things with different kinds of relationships in SIOT cause a complicated structure. The complexity of the SIOT network increases with the increase in the number of nodes [5]. As an instance, in article [6], the number of connected devices in the SIOT was mentioned as 30 billion.

In a SIOT network, each relationship is demonstrated using a graph; the nodes, network devices and edges determine these relationships. Each of these graphs has special topological properties of their own based on their varied relationships. Some of the most prevalent properties of the network are the clustering coefficient, grade distribution and average route length [7]. A higher grade distribution is indicative of more relationships among nodes for exchanging data with each other; a higher clustering coefficient for each node suggests a stronger tendency of nodes to join the cluster; and a shorter average route length between two nodes results in transferring the data from a shorter route and, consequently, an increase in the efficiency of the network. As an instance, considering the fact that things with the same owner connect with each other in the OOR relationship, a full graph can be generated, and the clustering coefficient in this graph is nearly 1. Given the fact that the number of things owned by a person is not very high and that all the nodes are connected to each other, the grade distribution is not very high, and it is reasonable for nodes to use a route with a shorter average route length to transmit data. In the CLOR relationship, things in the same location connect with each other, and this location might have large dimensions; therefore, in this type of relationship, the clustering coefficient and grade distribution are low, while the route length is high. In addition, in the SOR relationship, the clustering coefficient and grade distribution are low, and the average route length is short due to the corresponding properties. Finally, the graph related to the POR relationship is not complete due to an increase in the number of things with the same brand and

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the probability of a lack of relationships between all of them. Furthermore, the clustering coefficient (cc) and grade distribution (d) are low, and the average route length (l) is rather short in order to facilitate data transmission.

Considering the significance and necessity of internet networks for social things, extensive research has been conducted in a variety of areas, including service offering systems in networks, identifying appropriate services in networks, and trust checks and management etc. Researchers have implemented modeling and simulation approaches to evaluate their network methods. For instance, the author of [8] generated a network using a BA scalable random graph model in order to examine his work.

Given the fact that, with an increase in the size of the network, there is a dramatic change in the simulation and modeling results of most researchers, generating a scalable framework which produces logical results with larger networks is a major challenge. For instance, the author of [5] utilized a framework which is not scalable for evaluating and comparing his results.

Random graph models play an important role in analyzing complicated networks. These models can answer questions such as 1. How are social relations organized in social networking? 2. How are the information flows distributed, and how are they to be managed? Random graphs help in understanding, controlling and predicting phenomena occurring in social media, etc. They also have an influential role in identifying the formation mechanisms of network topology [9].

There are several random graphs available, including those of Erdos Renyi, Watts Strogatz and Barbasi Albert. In an ER random graph, the new node is connected to the existing nodes using an edge and equal probability. This network initiates with nodes without edges. An edge is added between a pair of nodes at random times. The addition of random edges to a node is based on 2 parameters: 1. the number of nodes and (n) 2. Probability (p). This graph is defined as G (n,p), in which n refers to the number of nodes and p represents the probability of a relationship between the edges. In this graph, the number of triangles is high, which leads to a higher clustering coefficient; the problem with this graph is a low grade distribution in comparison with networks in real life [7].

Watts Strogatz (WS) introduced a model called the small world. In the small world related to social media, people are connected to each other by a short chain of acquaintances. In this model, a normal network is created using a pair of nodes; then, one pair of nodes is chosen randomly using the shortcuts, and their relationship is made. In other words, in this model, the nodes are formed in a ring first, and then they are wired with random long-range edges in the following steps. The produced graph has a high clustering coefficient, but its grade distribution does not follow the power law [15].

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A BA random graph is an algorithm for producing scale-free random networks using a preferred attachment mechanism. In the preferred attachment mechanism, the new node tends to connect to nodes with higher grades. In this graph, the new node is connected to the network based on the degree m. The m parameter determines how many edges are required for the new node to connect to the existing nodes. This graph has the features of a no-scale network but does not have a high clustering coefficient [7].

Since considering topological properties in producing a random graph leads to an understanding and analysis of the efficiency of the network, the random graph should be able to maintain the topological structure of the system even when the number of nodes and relationships increase.

In order to propose a random graph with the ability to maintain the properties of various SIOT relationships even with an increase or decrease in size, the following steps were taken in this research:

First, a criterion is needed to detect SIOT topological properties. Therefore, these properties are extracted using a graph resulting from a real implementation of the SIOT.

Next, the degrees of resemblance between different kinds of SIOT graphs and common random graphs are represented using numerical and intuitive comparisons.

After due evaluation, the ER and BA graphs represented the closest results to those for the real SIOT graph, on condition that the p parameter in the ER and the m parameter in the BA were chosen correctly. Therefore, in the last step and by implementing the descending gradient algorithm, optimal parameters to prove the maximum level of resemblance to a real graph are extracted for ErdoS Renyi and Barbasi Albert's graphs and the relationship between the average route lengths of these graphs.

The objective of the descending gradient algorithm is to maximize the resemblance between random graphs and real SIOT graphs or minimize the distance using the parameters of random graphs and real available values. In this process, the number of nodes in the random graph is fixed, and the purpose is to optimize other parameters. After detecting the optimal parameters, we can fix them and change the nodes for future implementation.

The innovative part of the research is that, first, the SIOT and its various relationships are examined, and then, it is determined which random graph is suitable for each type of relationship. In the following step, the random graph is customized in such a way that it can have the most resemblance with the real SIOT graph by adjusting the parameters.

The other sections of the paper are organized as follows: In section 2, a review of the literature is conducted. In section 3, first, the extraction of topological properties related to different kinds of real SIOT graphs is described. Based on numerical and intuitive comparisons, the resemblance between real SIOT graphs and random graphs was examined. Then, a scalable

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framework is proposed using random graphs constructed by Watts Strogatz, Barbasi Albert and Erdos Renyi by implementing a descending gradient algorithm and extracting optimal parameters for different kinds of social relationships related to the SIOT network. The experimental results on the real SIOT network dataset are presented in section 4, and the conclusions of this paper are presented in section 5.

2. Review of the Literature

Given the fact that SIOT is a cutting-edge paradigm and that most related studies have focused on detecting possible policies as well as methods and techniques for establishing independent relationships between smart things without human involvement, insufficient attention has been given to modeling and analyzing the results of SIOT networks [5]. Therefore, in order to conduct an analysis of the new programs and protocols in SIOTs, a structure is needed to facilitate experimentation. The research conducted in this area can be categorized into two sections.

The first category includes those that have examined the process of modeling and producing SIOTs, and the second category belongs to those studies that have conducted graph simulation. In this section, we aim to evaluate the papers which have investigated SIOT modeling. In the modeling process of the examined studies, the following modeling tools were introduced: 1. a bipartite graph, 2. Random graphs, 3. Link prediction.

Therefore, in this section, some parts of the conducted studies that have used these tools have been examined.

In [10], the authors first modeled the SIOT network using a flexible bipartite graph to conduct their research. This model comprises 2 sets of nodes, including 1. Service providers and 2. Service requesters. In this research, the social relationships between service requester nodes are created using the Hellinger distance.

In [11], social media and the relationships between users were created and modeled using the ErdoS Renyi random graph. In the ErdoS Renyi random graph, the relationship probability of each edge corresponds to the p probability, and in every step, one edge is added to the network to connect to the new node.

In order to form social relationships among thing owners in article [12], the Watts Strogatz random graph [15] was implemented. In this approach, several parameters are set to determine the number of thing owners, except for state-owned things. In addition, the probability of adding a new edge is considered to be 0.5, which is indicative of a small-world network between thing owners' friends.

In article ,[13] mobility, physical contact and relationships between users were implemented using a random graph. This model can effectively demonstrate a history of users' contact and proximity in an abstract way. In this model, users' contact and proximity with special rates

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result in the appearance or disappearance of edges. With this model, the call tracking graph of users can be analyzed by changing the grade distribution and density.

The authors in article [14] modeled the SIOT network by merging the concepts of the small world. The objective of a small-world network is to identify existing relationships in the real world, particularly within a group of people. In fact, in small-world phenomena, people are considered to be connected to each other via a small chain.

In addition, in article [5] , the authors modeled the network to predict the relationships among SIOT network things. The authors have taken the following 3 major steps to model this network:

i) collecting raw movement data of IoT things
ii) generating temporal sequence networks
iii) predicting future relationships among things

In phase 1, information such as longitude and latitude and the timestamp related to moving things in the network were gathered. In the next phase, the raw data extracted in the previous phase, the extracted locations, as well as the tagging of each stay were performed by identifying the stays, and the temporal sequence related to things was calculated. In the second phase, the encounter between things is calculated using the overlap sweep line time algorithm; then, the time sequence of the SIOT network is created. Finally, in the third step, by implementing the time sequence created in the previous step and using the Bayesian nonparametric prediction model, the links between things are extracted using the Dirichlet distribution, and eventually, the SIOT network is produced. In this process, the prediction probability of a link between two things is relevant to the grade of those two things.

Based on the review of the literature, it was evident that network scalability was not considered when modeling the SIOT network or the relationships between users. Network scalability in this study means that the network can maintain its topological properties in the event of an increase in the number of nodes. Based on these findings, the present study is among the first studies in which the SIOT network and its relationships were modeled with random graphs in such a way that with an increase in the number of nodes, the topological properties of the network were optimally maintained.

3. Modeling the SIOT structure

Considering the fact that, the size of the real SIOT is large, random networks can be appropriate tools for modeling this network. Random graphs are varied, and the way in which they are produced results in the development of different topological properties. SIOT modeling with random graphs should be performed in such a way that the resulting graph has the highest level of resemblance to the network. In order to extract the resemblance of the topological properties of the SIOTs and random graphs, two methods were employed: an intuitive method and a numerical method.

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In an intuitive comparison, the degree of resemblance between the topological properties of the existing relationships in the SIOT and different random graphs was experimentally examined; in addition, via a numerical method, the degree of their resemblance was measured using the existing mathematical relationships. Each of these methods will be explained in the following section.

3.1.Intuitive method

Examining the structure of the SIOT revealed that this network is not formed by a specific graph; in fact, considering the various relationships in this network, several graphs should be created. In other words, a particular type of random graph should be considered for each type of relationship. Given the fact that one SIOT node is simultaneously present in several different graphs depending on the kind of links, each of these links might have different topological properties.

The relationships between nodes in the SIOT can be of different forms, including OOR, CLOR, CWOR, SOR and POR[3].

Given the fact that in the OOR relationship, things with the same owner connect with one, a full graph can be generated, and the clustering coefficient in this graph is nearly 1. Given that the number of things owned by a person is not many and that all the nodes are connected to each other, the grade distribution is not very high, and it is reasonable for nodes to use a route with a shorter average route length in order to transmit data.

On the one hand, in the CLOR relationship, things in the same location connect with each other, and this location might have large dimensions; therefore, in this type of relationship, the clustering coefficient and grade distribution are low, while the route length is high. Furthermore, due to the corresponding properties of the SOR relationship, the clustering coefficient and grade distribution are low, and the average route length is short. Finally, the graph related to the POR relationship is not complete due to an increase in the number of things with the same brand and the probability of a lack of relationships between all of them. In addition, the clustering coefficient and grade distribution are low, and the average route length is rather short to facilitate data transmission.

On the other hand, ER, WS and BA can be mentioned among the random graphs. In an ER random graph, the new node is connected to the existing nodes by an edge and equal probability. This network starts with nodes which are without edges. Each edge is added to a pair of nodes at random. The addition of random edges to a node is based on 2 parameters: 1. the number of nodes 2. Probability. This graph is defined as G (n,p), in which n refers to the number of nodes and p represents the probability of a relationship between the edges. In this graph, the number of triangles is high, and therefore, there is a higher clustering coefficient; the problem with this graph is a low grade distribution in comparison with real-life networks.

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The grade distribution, average route length and clustering coefficient in this graph were obtained using equations (3.1.1), (3.1.2), and (3.1.3) [7].

$$\bar{d}=p\,(n-1) \tag{3.1.1}$$

$$\bar{l}=\frac{\ln(n)-\gamma}{\ln(\bar{d})} \tag{3.1.2}$$

$$CC=p \tag{3.1.3}$$

Watts Strogatz proposed a model called the small world model. In the small world related to social media, people are connected to each other by a short chain of acquaintances. In this model, a regular network is generated using a pair of nodes; then, one pair of nodes is randomly selected using the shortcuts, and their relationship is made. In other words, in this model, the nodes are first formed in a ring, and in the following steps, they are wired with random long-range edges. The produced graph has a high clustering coefficient; however, its grade distribution does not follow the power law. In this research, the average route length of this graph was calculated experimentally and by implementation in the network x library in the python environment [15].

The BA random scale is an algorithm for generating no scale random networks using a preferred attachment mechanism. In this mechanism, the new node tends to connect to nodes with higher grades. In this graph, the new node connects to the network based on the degree m. The m parameter determines the number of edges needed for the new node to connect to the existing nodes. This graph has the properties of a no-scale network despite the fact that it does not have a high clustering coefficient. Parameters related to this graph are obtained using equations (3.1.4), (3.1.5), and (3.1.6) [7].

$$\bar{d}=2m \tag{3.1.4}$$

$$\bar{l}=\frac{\ln(n)-\ln\left(\frac{m}{2}\right)-1-\gamma}{\ln(\ln(n))+\ln\left(\frac{m}{2}\right)}+1.5 \tag{3.1.5}$$

$$CC=\frac{(m-1)}{8}\,\frac{(\ln(n))^2}{n} \tag{3.1.6}$$

Now, based on the comparisons made between real SIOT graphs and random graphs, it can be concluded that the graph related to the OOR relationship can be implemented by an ER random graph, and the graphs related to CLOR, SOR and POR can be modeled based on the comparison of the relevant clustering coefficient with the BA random graph.

3.2.Numerical methods

Real implementation of the SIOT is conducted with 16216 nodes, and 5 graphs related to different kinds of relationships are extracted. In this study, 4 kinds of relationships are introduced, namely, OOR, POR, CLOR, and SOR; 16216 nodes are utilized, and their topological properties are calculated based on Table 1.

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Table 1. Topological properties extracted from real SIOT graphs

Clustering coefficient mean(cc)	Average route length ($\bar{l}$)	Grade distribution (d)	The kind of graph
0.9	1	8.17	OOR
0.009	0	2.62	POR
0	27.3	3.3	CLOR
0.9	1	7.17	SOR

The topological properties of the ER graph for different p values are calculated based on equations (3.1.1)-(3.1.3); the topological properties of the BA graph for 16216 nodes are measured based on equations (3.1.4)-(3.1.6); and the topological properties of the WS graph are demonstrated by simulation and are presented in Tables 4 and 5. In this simulation, n refers to the number of nodes (16216), p denotes the probability of an edge relationship, k is the grade mean, cc refers to the clustering coefficient, $\bar{l}$ denotes the average route length and d is the denotative of the grade distribution. In this algorithm, first of all, a regular ring network with a grade distribution is formed in which each node is connected to its neighboring k; in other words, there is a k/2 node on each side of the node. Then the process of rewiring with a probability of p is gradually conducted to generate a random graph. During the process of wiring, each (i,j) edge is consistently replaced with an (r,j) edge in such a way that r is randomly selected from among existing nodes; therefore, r≠j.

Table 2. BA graph for different m

m	2	5	10	15	20	25	30	40	50	60	70	100	200	500	1000	5000
d	4	10	20	30	40	50	60	80	100	118	138	196	386	931	1767	3000
$\bar{l}$	5.1	3.8	3.2	2.9	2.7	2.6	2.6	2.5	2.4	2.3	2.2	2.1	2.01	1.8	1.7	1.6
C	0.0006	0.002	0.005	0.008	0.01	0.01	0.018	0.02	0.03	0.04	0.04	0.07	0.1	0.3	0.7	0.9

Table 3. ER graph for different p

P	0.00009	0.0001	0.0003	0.0009	0.002	0.007	0.009	0.03	0.05	0.08	0.1	0.5	0.7	0.9
D	1.4	1.6	4.8	14.5	32.43	113.5	145.9	486.4	810.75	1297.2	1621.5	8107.5	11350.5	14593.5
$\bar{l}$	30.33	22.75	6.06	3.5	2.6	1.9	1.8	1.4	1.3	1.2	1.2	1.01	0.9	0.9

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cc	0.00009	0.0001	0.0003	0.0009	0.002	0.007	0.009	0.03	0.05	0.08	0.1	0.5	0.7	0.9

Table 4. WS graph for different p and k values

p	0.00001	0.0001	0.0001	0.001	0.01	0.01	0.01	0.01	0.01	0.1	0.1	0.1	0.1	0.1	0.1
k	20	80	500	120	20	80	120	500	1000	20	40	80	120	500	1000
d	20	80	500	120	20	80	120	500	1000	20	40	80	120	500	1000
$\bar{l}$	269.3	13.9	3.3	4.6	7.7	3.8	3.2	2.6	2.17	4.6	3.6	2.9	2.8	2.01	1.9
cc	0.7	0.7	0.0001	0.7	0.6	0.7	0.7	0.7	0.7	0.5	0.5	0.5	0.5	0.5	0.5

Table 5. WS graph for different p and k values

p	0.1	0.5	0.5	0.5	0.5	0.5	0.5	0.9	0.9	0.9	0.9	0.9	0.9	0.99
k	2000	20	120	80	500	1000	2000	20	80	120	500	1000	2000	80
d	2000	20	120	80	500	1000	2000	20	80	120	500	1000	2000	80
$\bar{l}$	1.8	3.6	2.4	2.7	1.9	1.9	1.8	3.5	2.6	2.4	1.9	1.9	1.8	2.6
cc	0.5	0.08	0.09	0.09	0.1	0.1	0.18	0.001	0.005	0.008	0.03	0.06	0.12	0.004

The results obtained from mathematical equations demonstrated that the OOR graph is close to the ER graph at p=0.9 and it is similar to the WS graph at p=0.01 and k=500, 1000; the POR graph is close to the ER graph at p=0.009 and to that of WS per k=120 and p=0.9; however, the OOR and POR graphs are highly similar to the ER graph. The graph related to the CLOR relationship resembles an ER random graph at p=0.00009; in addition, the SOR graph is similar to the BA graph at m=50.

In the following section, in order to prove this resemblance, the graphs related to social relationships are modeled with similar random graphs per the values obtained from mathematical equations.

3.3.Finding maximum resemblance of random graphs to real SIOT graph using optimal parameters provided that $0<p<1$ and $m>0$

Considering the dataset related to the graphs for social relations in the real SIOT network, the optimal p and m parameters pertaining to each of the real graphs are determined using formulas

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(3.1.2) and (3.1.5), which are related to the route length of the ER and BA graphs and the descending gradient algorithm; then, with the implementation of the algorithm pertinent to forming the ER and BA random graph, the graph related to each relationship was generated.

The aim of the descending gradient algorithm is to maximize the resemblance between random graphs and real SIOT graphs or minimize the distance using the parameters of random graphs and real available values. In doing so, the number of nodes in the random graph is fixed, and the goal is to optimize the other parameters. After detecting the optimal parameters, we can fix them and change the nodes for future implementation. In the descending gradient algorithm, this can be performed by finding the local minimum of the average route length function using appropriate steps with a negative gradient. To put it more simply, the average route length derivative should be calculated based on point m in the BA random graph, and the average route length derivative in the ER random graph should be computed based on p. The objective here is to find an optimal m and p point which minimizes the $\bar{l}$ function to the lowest possible degree. For points p and m, the direction to which the function value decreases with the highest slope is against the direction of the function gradient at that point, which means vectors $-\nabla(\mathrm{m})$ and $-\nabla(\mathrm{p})$.

The descending gradient algorithm moves against the direction of the gradient to some degree each time alternatively, meaning mb+1 ← mb-η$\frac{\delta \bar{dl}}{\delta m}$ in the BA random graph and pb+1= pb-η$\frac{\delta \bar{dl}}{\delta P}$ in the ER random graph until it reaches the optimal level.

In the formula, η (stride length) is the value by which the algorithm moves against the direction of the gradient each time; this value is usually low and is determined experimentally. In convex functions, the function value remains the same or is reduced each time; in other words, $\bar{l}$ (m0)>= $\bar{l}$ (m1)>= $\bar{l}$ (m2)>=…. and $\bar{l}$ (p0)>= $\bar{l}$ (p1)>= $\bar{l}$ (p2)>=…. approaching the optimal point, since the gradient approaches zero, mb and Pb reach stability, and the changes are negligible. Therefore, the average route length derivative is first calculated based on m in the BA random graph and based on p in the ER random graph.

The derivative of formula 3.1.5 is:

$$\frac{\partial \bar{l}}{\partial m} = \frac{-\frac{1}{m}[\ln\left(\ln(n)+\ln\left(\frac{m}{2}\right)\right]-\frac{1}{m}[\ln(m)-\ln\left(\frac{m}{2}\right)]}{(\ln\left(\ln(n)+\ln\left(\frac{m}{2}\right)\right))^2} \tag{3.3.1}$$

$$\frac{-\frac{1}{m}[\ln\left(\ln(n)\right)+ln\frac{m}{2}+\ln(n)\big]-\ln\left(\frac{m}{2}\right)]}{(\ln\left(\ln(n)+ln\frac{m}{2}\right))^2}= \tag{3.3.2}$$

$$\frac{-\frac{1}{m}[\ln(\ln(n))+\ln(n)]}{(\ln\left(\ln(n)+ln\frac{m}{2}\right))^2} = \tag{3.3.3}$$

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$$\frac{-[\ln(\ln(n))-\ln(n)]}{m(\ln\left(\ln(n)+ln\frac{m}{2}\right))^2} = \quad (3.3.4)$$

$$mb+1=mb- \eta\frac{-[\ln(\ln(n))-\ln(n)]}{m(\ln\left(\ln(n)+ln\frac{m}{2}\right))^2} \quad (3.3.5)$$

Here, mb values relevant to each of the relationships were first calculated per n=16216 using formula 3.1.5 and the obtained values in Table 2; then, the optimal m value pertaining to each of the real graphs related to social relations was computed by implementing the descending gradient algorithm mb+1.

Algorithm 1: Calculate the gradient of the function $\bar{l}$(m) related to the length of the path in BA graph at the point mb and determination the optimal value of m

η ← 0.01

precision ← 0.001

max_iters ← 100

iters_count ← 0

previous_step_size ← 1

Program while _step_size > precision and iters_count < max_iters

$mb+1 \leftarrow mb-\eta\frac{\delta \bar{dl}}{\delta m}$

End While

In algorithm 1, stride length was considered to be 0.001 based on the conducted research. Precision refers to the accuracy level of the algorithm; max_iters denotes the number of steps in the algorithm; and previous_step_size is implemented for the purpose of comparison. Previous_step_size and iters_count are considered to prevent the ring from entering the eternal loop.

The optimal m is calculated in Table 6 for different mb values and descending gradient algorithm 1.

Table 6. Optimal m values from algorithm 1

mb	1	15	50	100	500	1000	1300
optimal m	7	17	50	100	500	1000	1300

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Then, the process started using the algorithm related to the generation of BA random graphs and the obtained value pertaining to the optimal m.

Algorithm 2: BA Random graph generation algorithm for each social relationship for values of m obtained in algorithm 1

Input: A list of Relationships and their corresponding m values

Output: The graph related to each of the social relations by maintaining the topological properties

1 Begin
2 for each Ri in R do
3 Consider a small graph G0 with n0 vertices V0 and no edges. At each step s > 0:
4 Min-cc ← The minimum value of the clustering coefficient of nodes
5 max-cc ← The maximum value of the clustering coefficient of nodes
6 cc ← max-cc
7 Calculate the gradient of the function $\bar{l}$(m) related to the length of the path in BA graph at the point m and determination the value of m
8 Add a new vertex vs to Vs-1
9 Select a vertex u from Vs-1 that is not adjacent to vs and with a probability proportional to its degree *d* (u). Add edge <vs, u>.
10 Add the remaining m– 1 edge as follows:
11 a) If m - 1 edges have been added, continue with Step 13. Otherwise, proceed with the next step.
b) With probability q: select a vertex w that is adjacent to u, but not to vs. If no such vertex exists, continue with Step 10c. Otherwise ,add edge <vs ,w> and continue with Step 10 a.
12 c) Select a vertex u' from Vs-1 that is not adjacent to vs and with a probability proportional to its degree *d* (u') .Add edge <vs, u'> and set u ← u' Continue with Step 10 a.
13 Stop when n vertices have been added, otherwise repeat from Step 8
14 end for
15 calculated value of clustering coefficient, degree distribution and average of minimum of path length

The BA random graph starts the process with a small G0 graph with n0 node v0 and without any edges. In this algorithm, one new node is selected to be added to the previous nodes in each step. Then, a u node, which does not neighbor the new node vs, is selected from among the existing nodes of (Vs-1) based on its grade, and the edge is set between them. Next, in order to add m-1 to the remaining edge, the following steps are implemented.

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Considering the probability of q (which is considered to be 0.5 in this study), another node, which neighbors u but not neighbors vs, is selected from the existing graph based on its grade, and the edge is set between them. If such a node does not exist, another node which does not neighbor the new node is selected from the existing graph due to consideration of its grade, and the edge is subsequently added. After adding n nodes, the clustering coefficient mean, grade distribution and average route length related to each graph are calculated based on Table 7.

Then, the process has started using Algorithm 2 related to the generation of the BA random graph, and the values are obtained from parameter m; parameters related to the clustering coefficient, average route length and grade distribution pertaining to the graph SOR are calculated.

The results obtained from Algorithm 2 per number of optimal m are presented in Table 7.

Table 7. Topological properties extracted from algorithm 1 per n=16216

M	7	17	50	100	500	1000	1300
$\bar{l}$	3	2.7	2.3	2.1	1.8	1.7	1.7
CC	0.009	0.01	0.03	0.07	0.3	0.7	0.9
$\bar{d}$	14	39.5	98.7	195.99	930.9	1767.47	2001.21

The results obtained in Table 7 are similar to those in Table 2 for m=50 and for the value of m.

In order to model the CLOR, POR and OOR graphs, the derivative of equation 3.1.2 was implemented based on p to be utilized in the descending gradient algorithm. After determining p, the ER algorithm is modeled based on the obtained p.

Formula 2 derivative:

$$\frac{\ln(n)-\gamma}{\ln(p(d)} + 0.5$$

$$+ 0.5\frac{\ln(n)-\gamma}{\ln(p(n-1)}$$

$$\times\frac{1}{p(n-1)}\frac{\partial\bar{l}}{\partial p} = -\frac{\ln(n)-0.5}{(\ln(p(n-1))^2}$$

Algorithm 3: Calculate the gradient of the function $\bar{l}$(P) related to the length of the path in ER graph at the point Pb and determination the optimal value of p

```
η ← 0.01
precision ← 0.02
max _iters ← 100
```

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```
iters_count ← 0
previous_step_size ← 1
Program while _step_size > precision and iters_count < max_iters
    pb+1 ← pb-η δdl/δp
    End While
```

$$p_{b+1} \leftarrow p_b - \eta \frac{\delta \overline{dl}}{\delta p}$$

In algorithm 3, stride length is considered to be 0.001 based on the conducted studies. As mentioned earlier, modeling with an ER graph for an OOR graph should be performed per p0=0.9, while a POR graph should be modeled per p0=0.009; this process for generating a CLOR graph should be conducted with a value of p0=0.00009. Therefore, the optimal p values are first computed based on Algorithm 3.

Table 8. Optimal p value per n=16216

pb	0.00009	0.0003	0.001	0.009	0.1	0.5
Optimal p	0.8	0.001	0.002	0.009	0.1	0.5

After determining the optimal p values, they were added to algorithm 4 ,[15] and the related parameters were calculated.

Algorithm 4: ER Random graph generation algorithm for each social relationship

Input: A list of Relationships and their corresponding p values
Output: The graph G (n, p) related to each of the social relations by maintaining the topological properties

```
Begin
for each Ri in R do
   Consider a small graph G0 with 0 vertices
      for i in range(n):
         for j in range(i+1,n):
            if random number €[0,1) <= p:
                Add edge(i,j)
            end if
         end for
      end for
end for
calculated value of clustering coefficient, degree distribution and average of minimum of
path length
```

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An ER random graph starts the process with an empty graph. In this algorithm, a new node is added to the existing nodes in each step using an edge and with the probability of a new vertex p. After adding n nodes, the clustering coefficient mean, grade distribution and average route length related to each graph are calculated based on Table 9.

The extracted results from Algorithm 4 per p are presented in Table 9.

Table 9. Topological properties extracted from algorithm 4 per n=16216

P	0.00009	0.002	0.009	0.05	0.08	0.1	0.9
$\bar{l}$	30.45	2.8	2.25	2.02	1.99	1.8	0.95
CC	0.00009	0.002	0.009	0.05	0.08	0.1	0.9
$\bar{d}$	1.6	45.46	146.7	820.85	1301.4	1621.83	15635.8

3.4. Evaluating the proposed random graph

Considering the fact that the objective of this study is to prove the scalability of the modeled SIOT networks with random graphs, parameters have been evaluated and compared in order to examine algorithms 1, 3 and 5 for different values of n, which refers to the number of nodes in a network.

The results extracted from algorithm 1 are presented in Table 10 for p=0.01 and k=1000 in the OOR graph and for different nodes.

Table 10. Extracted results from the WS random graph algorithm [7] for different numbers of nodes

N	1000	5000	10000	16216
$\bar{l}$	1	1.8	1.9	2.17
cc	1	0.7	0.7	0.7
$\bar{d}$	999	1000	1000	1000

The results obtained from the WS random graph algorithm for different n are presented in Table 11 for p=0.9 and k=120 in the POR graph.

Table 11. Results extracted from the WS random graph algorithm for different numbers of nodes. The results obtained from Algorithm 2 for different n are shown in Table 12 for m=50 in the SOR graph.

N	1000	5000	10000	16216
$\bar{l}$	1.8	2.02	2.2	2.4
cc	0.1	2.02	0.01	0.008
$\bar{d}$	120	120	120	120

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Table 12. Results extracted from Algorithm 2 for different numbers of nodes

N	1000	5000	10000	16216
$\bar{l}$	1.9	2.1	2.3	2.3
CC	0.2	0.08	0.05	0.03
$\bar{d}$	90.02	96.9	98.19	98.7

It is concluded from the obtained results that the changes in topological properties of the network that occur in the event of an increase in the number of nodes are favorable.

In this section, the results are calculated using different numbers of nodes to evaluate the scalability of the graph related to social relationships using random graphs.

The extracted results from Algorithm 4 for different values of n are shown in Table 13 for Pb=0.00009 in the OOR graph.

Table 13. Extracted results from algorithm 4 for pb= 0.00009

n	1000	5000	10000	16216
Optimal p	0.2	0.5	0.005	0.9
$\bar{l}$	1.7	1.4	2.3	0.9
CC	0.2	0.5	0.005	0.9
$\bar{d}$	2621.2	8110.8	121.52	14699.9

The results extracted from Algorithm 4 for different n values are shown in Table 14 for p=0.09 in the OOR graph.

Table 14. Extracted results from Algorithm 4 for p=0.09

n	1000	5000	10000	16216
$\bar{l}$	1.9	1.9	1.9	1.9
CC	0.09	0.09	0.09	0.09
$\bar{d}$	90.486	450.59	900.86	1260.52

The results extracted from Algorithm 4 for different n values are presented in Table 15 for p=0.9 in the OOR graph.

Table 15. Extracted results from Algorithm 4 for p=0.9

n	1000	5000	10000	16216
$\bar{l}$	1.09	1.09	1.09	1.09
CC	0.9	0.9	0.9	0.9
$\bar{d}$	900.12	4500.36	9000.65	13580.35

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It is inferred from the obtained results that the parameters did not change dramatically when the number of nodes increased and that network scalability was optimally maintained.

4. Dataset

The SIOT dataset was utilized to evaluate the proposed framework.1 These datasets are based on real IoT data existing in Standar city and a description of IOTs. The total number of IOTs is 16216.14600. The dataset includes a proximity matrix for different kinds of SIOT social relations using defined parameters.

5. Conclusion and suggestions for future studies

The problem that most researchers have in studying the SIOT network is the lack of a scalable framework for conducting experiments and evaluating the method which they use. The objective of this study is to model different graphs of this network using random graphs with due consideration of the variety of relationships and their resemblance to topological properties. In order to do so, first of all, the topological properties related to different relationships and graphs of the real SIOT network were extracted; then, by implementing intuitive and mathematical comparisons, the degree of resemblance between each graph and random graphs was determined. The results obtained from intuitive and mathematical comparisons proved that OOR, POR, and CLOR graphs are highly similar to random ER graphs and that the SOR graph resembles the random BA graph. The optimal parameters for p and m were then calculated using the descending gradient algorithm to extract the maximum level of resemblance between the random graphs and real SIOT graphs. Afterwards, optimal values were added to the random graph algorithm to demonstrate the accuracy of the method and results. Then, the related topological properties were computed for comparison with the values obtained from real graphs. Finally, to prove the scalability of the network, the number of nodes was increased, and network scalability was examined. The results showed that the changes in the values related to topological properties remained optimal when the number of nodes increased. Future studies can focus on writing algorithms based on these random graphs and calculating the results for protocols proposed by researchers.

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