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Low Power Approximate Computation of Logarithm

Hafiz Muhammad Imran[1,*], Khalid Mahmood Aamir[2], Muhammad Ilyas[1], Mohamed Deriche[3]

[1]*Department of Computer Science, University of Sargodha, Sargodha, Pakistan.*
[2]*Department of IT, University of Sargodha, Sargodha, Pakistan.*
[3]*Artificial Intelligence Research Center, Ajman University, Ajman, UAE*
imran.wahab@uos.edu.pk, khalid.aamir@uos.edu.pk, m.ilyas@uos.edu.pk,
m.deriche@ajman.ac.ae

Abstract

J. Napier first introduced logarithms in 1614. Many algorithms have evolved to compute logarithms. Several algorithms compute the approximate values of logarithms. The higher the complexity, the more power is required for computations. State-of-the-art algorithms compute logarithms in $O\ (\ log_2^2(n))$. We have introduced a method to calculate the approximate logarithm value (base 2) with the computational complexity of *O*(1). This indicates that the proposed algorithm requires very low power compared to the existing algorithms.

Keywords*: Lowe power computation, logarithm, computational complexity, approximate algorithm*

1. Introduction

John Napier introduced logarithms in 1614 as a way to make calculations easier [1]. To make high-accuracy computations easier to complete, navigators, scientists, engineers, surveyors, and others quickly embraced them. An instance of a transcendental function is the logarithm. According to the Gelfond-Schneider theorem, logarithms typically take transcendental values [2].

Logarithmic scales compress expansive quantities into narrower ranges. For instance, the decibel (dB) is a logarithmic measure utilized primarily to quantify ratios, particularly for signal power and amplitude (with sound pressure being a typical illustration). The pH is a logarithmic scale used in chemistry to quantify the level of acidity in a water-based solution. Logarithms are commonly used in scientific equations, as well as in quantifying the intricacy of algorithms and fractal geometries. They are utilized to quantify frequency ratios of musical intervals, are present in formulas for calculating prime numbers or estimating factorials, contribute to certain models in psychophysics, and can assist in forensic accounting. Approximating factorials is a useful technique that may be applied in several fields such as psychophysics and forensic accounting.

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The concept of logarithm, which functions as the reciprocal operation of exponentiation, also applies to other mathematical systems. In everyday circumstances, the logarithm function typically has many values. The complex logarithm is the inverse of the complex exponential function, although it has several values. The discrete logarithm is the mathematical operation that undoes the exponential function in finite groups. It is commonly used in public-key cryptography.

The logarithm with a base of 10 is commonly known as the decimal or common logarithm, and it is extensively used in science and engineering. The natural logarithm, with the base e = 2.718, is extensively utilized in mathematics and physics because of its easily calculable derivative. The binary logarithm, using a base of 2, is frequently employed in computer science.

The logarithm table was a highly helpful instrument that facilitated the practical application of logarithms in real-life scenarios [3]. Henry Briggs pioneered the creation of the inaugural version of these tables in 1617.

A logarithm function expressed as a series is called a logarithm power series. Typically, it manifests as an unending series of variable powers that approaches the logarithm function within a specified range of values. The Taylor series expansion of the natural logarithm function ln(x) is a typical example of a power series for logarithms [4].

Feynman stated that each given number can be represented as the multiplication of integers in $1+2^{-k}$, where k is an integer. We can employ a binary numeral system to represent integers to resolve this issue. Once we have identified the elements, we can calculate the logarithm by summing the precalculated logarithms of the individual components. This strategy was highly efficient on the connection machine, as each CPU could access the compact database that stored the logarithms $1+2^{-k}$. Notably, the division process has higher running time than the entire computation.

Rest of the paper is organized as follows. In the next section, we present the properties of logarithms followed by our proposed method. Comparative results are presented in Section 4 and discussed in Section 5. Conclusions are drawn in Section 6.

2. Properties of Logarithm

There are several important formulas, also known as logarithmic identities or laws that are related to logarithms. The logarithm of a ratio is the difference between the logarithms of two numbers. Similarly, the sum of the logarithms of the numbers being multiplied is the logarithm of their product. The logarithm of a number divided by p is the logarithm of the p-th root of that number, and the logarithm of a number raised to the power of p equals p times its logarithm. These identities are displayed in Table 1 below [5].

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Table 1 Identities of Logarithm

Formula	
Product	$\log_b(xy) = \log_b x + \log_b y$
Quotient	$\log_b \frac{x}{y} = \log_b x - \log_b y$
Power	$\log_b(x^p) = p \log_b x$
Root	$\log_b \sqrt[p]{x} = \frac{\log_b x}{p}$

3. Proposed Method

This study presents a matrix multiplication algorithm that avoids the use of the multiplication operation. The algorithm uses a fixed binary pattern in table-2 to take $log_2$and add the result. The anti-log of log_2 is2^x, which is equivalent to the bit-shift operations is used to calculate anti-log. The proposed algorithm is given below. In the following equations, base of logarithm is 2.

Define: $x = 2^k \frac{x}{2^k}$ (1)

Taking log on both the sides

$$\log x = k + \log\left(\frac{x}{2^k}\right) \quad (2)$$

Redefine *x* as: $x = 2^k + 2^k\left(\frac{x}{2^k} - 1\right)$ (3)

We define f as: $f\left(2^k(1+\alpha)\right) = k + \sqrt{\alpha}$ (4)

Therefore, $f(x) = f\left(2^k + 2^k\left(\frac{x}{2^k} - 1\right)\right)$ (5)

$$f(x) = k + \sqrt{\frac{x}{2^k} - 1} \quad (6)$$

It is obvious that $f(x) = \log x$ (7)

If $\sqrt{\frac{x}{2^k} - 1} = \log\left(\frac{x}{2^k}\right)$ (8)

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Squaring both sides

$$\frac{x}{2^k} - 1 = \left(\log \frac{x}{2^k}\right)^2 \quad (9)$$

$$\frac{x}{2^k} - 1 = (\log x - k)^2 \quad (10)$$

$$\frac{x}{2^k} - 1 = (\log x)^2 - 2k(\log x) + k^2 \quad (11)$$

$$(\log x)^2 - 2k(\log x) + k^2 - \frac{x}{2^k} + 1 = 0 \quad (12)$$

solutions are:

$$\log x = \frac{2k \pm \sqrt{4k^2 - 4\left(k^2 - \frac{x}{2^k} + 1\right)}}{2} \quad (13)$$

$$logx = k \pm \sqrt{k^2 - \left(k^2 - \frac{x}{2^k} + 1\right)} \quad (14)$$

$$logx = k \pm \sqrt{\frac{x}{2^k} - 1} \quad (15)$$

We take positive root only to coincide with Eq. (6).

$$\log x = k + \sqrt{\frac{x}{2^k} - 1} \quad (16)$$

We compute approximate value of log(x) using Eq. 16. This requires computation of a square root. For fast computation, we compute square root of x using a lookup table. An example of a lookup table with six decimal bits is show in Table 2.

Table2: Example of a lookup table for finding a square root.

Number	3-bit Binary Value	6-bit Binary pattern
0	000	000000
1	001	000001
2	010	000100
3	011	001001
4	100	010000
5	101	011001
6	110	100100
7	111	110001

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4. Results

Figure 1 shows a comparison between actual values of $\log_2(n)$ and approximate values of $\log_2(n)$ computed using Eq. 16. Existing state-of-the-art algorithm computes $\log_2(n)$ in bit computations with a complexity of $O\ (\ log_2^2(n))$. $\log_2(n)$ is calculated using this tabular method in $O(1)$ bit computations.

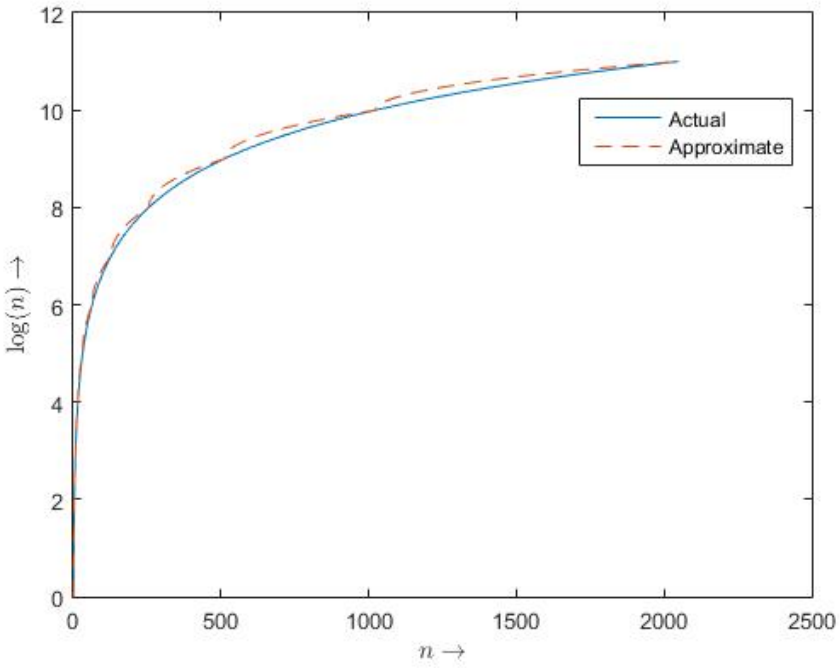

Figure 1: Comparison between actual and approximate values of log(n).

5. Discussion

The computational complexity for existing state-of-the-art algorithms is computed as follows.

Number of computations=$O(\log_2 n)$

Length of n = $O(\log_2 n)$ bits

Computational complexity: $O\ (\ log_2^2(n))$

The proposed algorithm's computational complexity is based on the length of n (in bits) and computing the square root. Both operations have computational complexity of $O(1)$.Therefore, overall computational complexity of the proposed method is $O(1)$.

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6. Conclusion

The computational complexity of state-of-the-art algorithms is $O\ (\ log_2^2(n))$ whereas the proposed method computes an approximate value of $\log_2 n$ with computational complexity $O(1)$. As the complexity of the proposed algorithm is much smaller, the computational cost (and hence power requirement for computation) is reduced.

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