# Designing a Hospital Waste Collection Network Under the Uncertainty of Waste Generation Peak and Collection Time Window

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### **Abstract**

In the contemporary world, optimal hospital waste management has emerged as a complex challenge. With the increasing production of hospital waste, including hazardous materials and various pollutants, the need for smart and effective management is more pressing than ever. This study aims to present a mathematical model for optimizing hospital waste management, focusing on collection timing and uncertainty management in peak waste generation. The proposed model seeks to significantly improve efficiency and reduce costs by addressing fluctuations in waste production and collection scheduling. The model assists managers and decision-makers in hospital management in facing the challenges and complexities arising from uncertainty in waste generation and collection. The results of this study demonstrate that utilizing a mathematical model to optimize hospital waste management can markedly enhance efficiency and reduce costs. This innovation not only improves the quality of healthcare services but also plays a crucial role in environmental preservation. This applied research proposes a scientific solution for optimizing hospital waste management and introduces a new approach to improving management processes in this field. The analysis and modeling results indicate that accounting for uncertainty in peak waste generation and collection timing can fundamentally transform the optimal management of hospital waste.

**Keywords**: Hospital Waste Management, Waste Collection Network, Uncertainty in Waste Generation, Collection Time Window

### 1. Introduction

As a vital matter, the optimal management of hospital waste has become a complex challenge in the present era (Cook, 2023). As society grows day by day and many medical advances are made, a large amount of hospital waste is generated (Obubu, 2023). These wastes, which contain hazardous materials and high pollution potential, require a smart and effective management approach (Nassour, 2023). The proper management of hospital waste not only improves the quality of healthcare services but also socially preserves the environment (Pasha, 2023). There are different types of waste in medical centers and hospitals due to their specific characteristics and the prevalence of diseases there (Jain, 2023). As the most important hospital wastes, hazardous wastes include materials such as medical needles, chemical solutions, expired drugs, and explosives. Due to the presence of polluting and dangerous substances, these wastes require special and safe management and disposal (Osman, 2023). In addition to hazardous wastes, non-hazardous wastes are generated in hospitals. These wastes include general waste such as paper, plastic, and disposable packaging and require proper management to prevent environmental pollution and create hygiene in the hospital. Wastes with special characteristics such as sharp objects are another group of hospital wastes. They include medical tools such as scalpels, needles, and other sharp objects, whose management requires the use of safe packaging and special collection devices (Cook, 2023). Finally, ordinary medical waste is among the types of hospital waste as well. This group includes disposable clothes for the personnel, hygiene supplies, and tools used in the process of treating patients. The management of these wastes requires their correct separation and safe disposal to ensure the health of the environment and the treatment staff. So, hospital waste collection highlights the weaknesses and challenges in the management of this group of waste from different aspects (Ghali, 2023). These challenges can significantly contribute to the performance of the hospital waste management system. One of the most important challenges is the high amount of waste generated by hospitals. The generation of hazardous and non-hazardous waste in hospitals has increased due to the increase in the number of patients and medical advances. Uncertainty in peak waste generation is a major challenge too. Since hospital activities are associated with fluctuations, the density of waste at a given time cannot be accurately predicted. This uncertainty can lead to incorrect setting of collection plans and, subsequently, waste of resources and increased costs (Bamakan, 2022). Another important challenge in hospital waste collection concerns hazardous materials and limited time shifts of some collection operations. The safe and healthy implementation of hazardous waste collection operations requires accurate timing and the use of appropriate equipment and tools. This challenge can complicate the collection process and increase health hazards. Another important challenge in this regard is environmental pollution and its negative effects on public health. Incorrect use of landfills or other waste disposal methods may harm water, soil, and air resources. Consequently, appropriate measures should be taken to reduce these challenges and improve the management of hospital waste collection (Khan, 2020). One of the imperatives of hospital waste collection scheduling is that it acts as

preventive agent. Environmental pollution, accidents caused by waste disposal, and adverse health conditions can be prevented by developing accurate and effective plans. Hospital waste collection scheduling not only improves waste management but is also cost-effective. Costs can be reduced and resources can be optimally managed by optimizing the workflow in waste collection and disposal (Ghali, 2023). The time window of hospital waste collection plays a key role in this process. The most optimal time must be determined for waste collection in a way that is effective both economically and in terms of maintaining public health. So, it seems inevitable to design a waste collection network considering the uncertainty in the waste generation peak and the collection time window as an innovative and effective solution.

Accordingly, this study is done mainly to provide a mathematical model to optimize hospital waste management with an emphasis on collection timing and uncertainty management. This innovation is focused on improving efficiency and reducing costs in hospital waste management and emphasizes developing a robust model taking into account the uncertainty in waste generation peak and collection time window. What separates this model from others is paying attention to the uncertainty in the waste generation peak and the collection time window. In this way, managers and decision-makers in the field of hospital management can be helped to provide optimization solutions to prevent environmental pollution and improve economic performance using this model. Due to its direct impact on reducing costs and improving the quality of healthcare services, this innovation is of particular importance.

### 2. Literature Review

Taşkın and Demir (2020) conducted a case study in Kayseri, Bolivia, titled Life Cycle Environmental and Energy Impact Assessment of Sustainable Urban Municipal Solid Waste (MSW) Collection and Transportation Strategies. Three separate scenarios were designed for eleven district municipalities to reveal a workable MSW system integration model. The results showed that the establishment of transfer stations instead of landfill sites reduces the environmental effects for all affected groups and also reduces the rate of cumulative energy demand. Obubu (2023) emphasized the importance of waste generation and disposal as a fundamental issue in maintaining environmental sustainability and the future of the planet. This study mentions the importance of waste management in all sectors, especially the health sector. The management of waste generated by medical centers should be given more attention to avoid the risks of improper biomedical waste management. In this study, the performance of waste management in medical centers in Lagos state was investigated using a descriptive survey method, and a total of 1256 medical centers in this state were examined. According to the results, 98.4% of the studied facilities are registered with the Lagos State Waste Management Authority (LAWMA) and 1.6% are not registered. The results generally showed that a policy should be established to regulate hospital waste and staff should be regularly trained in this area. Healthcare personnel were recommended to carefully follow the instructions and use color-coded waste bags to separate materials. The findings also showed that proper waste management in medical centers should be considered to prevent the occurrence of a public disaster. Afsi (2023) investigated different segregation methods, storage configurations, collection, and waste disposal systems in tertiary hospitals in Karachi.

A cross-sectional survey was conducted in ten tertiary hospitals in Karachi using convenient sampling. In this study, a researcher-made questionnaire was used to collect data about hospital waste management. According to the results, many components of the hospital waste management system interact with each other and should be comprehensively investigated to ensure the performance of each component. Many factors, including the number of beds in hospitals, are influential. According to Afsi (2023), solid waste management has recently gained significant importance due to rapid urbanization, improvement of living standards, and increasing attention to environmental quality. Many argue that collection accounts for a significant portion of total solid waste management costs because any improvements in collection system design can result in significant cost savings. Yadav and Karmakar (2019) conducted a study titled Sustainable Collection and Transportation of Municipal Solid Waste in Urban Centers. They used different T&C techniques and provided recommendations on their suitability for specific urban areas in developing or developed countries and acceptable approaches for mathematical or computational modeling for their greater efficiency. Holeczek et al. (2021) examined how different factors affect the risk values and outcomes of the hazardous materials vehicle routing problem. In this study, the establishment of the route between the customer and warehouse nodes in realistic urban road networks was considered the basis of HMVRP, and the effect of fleet size was investigated. Mojtahedi et al. (2021) conducted a study titled Sustainable Vehicle Routing Problem for Coordinated Solid Waste Management to introduce a new coordinated framework for a practical and efficient vehicle routing problem concerning the triple bottom line of sustainability to represent the solid waste management problem. The CSWM multiple objective functions applied in this study incorporate financial, environmental, and social considerations to develop a sustainable vehicle routing problem considering heterogeneous vehicle fleets operating across a multiechelon logistics network with optimization goals. Theurich et al. (2021) formulated the scheduling problem to reduce travel costs and time-dependent costs caused by bad road conditions as a vehicle routing problem with additional customer costs. Using a partition and permutation model, they developed a complex branch and bound (B&B) method and proposed two B&B strategies where the first task at the end of the path added one job at each B&B stage and the second task at each B&B stage included one job. The performance of the B&B method was analyzed and compared with a commercial solvent. In their study titled A Novel Reinforcement Learning-Based Hyper-Heuristic for Heterogeneous Vehicle Routing Problem, Zhuang et al. (2021) studied a heterogeneous vehicle routing problem that involves routing a predefined fleet with different vehicle capacities to serve a series of customers to minimize the maximum vehicle routing time. In this study, a mixed-integer linear programming (MILP) model was formulated to obtain optimal solutions for small-scale problems. Recent studies on hospital waste management were discussed above. More studies are separately evaluated in Table 1.

Table 1. A review of the literature

				ective ction			ion	a	aste	e.	uc	Un	certa	inty	ilon- f a	ıtion
Row	Year	Author(s)	Single-objective	Multi-objective	Routing	Location	Routing and location	Decision criteria	Municipal solid waste (MSW)	Hazardous waste	Defensive position	Probabilistic	Fuzzy	Robust	Exact solution (Epsilon- Delta definition of a	Meta-heuristic solution
1	2017	Taşkın and Demir		<b>✓</b>		<b>✓</b>			<		✓	✓			<b>✓</b>	
2	2020	Hina et al.	✓		✓						✓	✓			✓	
3	2020	Pérez et al.	✓		✓						✓	✓			✓	
4	2019	Yadav and Karmakar	✓		✓						✓	<b>✓</b>			✓	
5	2017	Lella et al.	✓				✓				✓	✓			✓	
6	2021	Theurich et al.		✓	✓						✓	✓			✓	
7	2021	Chabane et al.	✓	✓	✓											
8	2018	Li and Zhang		✓	✓						✓					✓
9	2020	Vega et al.		✓	✓									✓	✓	
10	2020	Shi et al.	✓		✓											
11	2018	Subramaniam et al .	✓		✓									<b>✓</b>	✓	
12	2020	Yazdani et al.	✓			✓					✓	✓				✓
13	2021	Cao et al.	✓				✓	✓			✓	✓			✓	✓
14	2020	Arnold and Sörensen	✓				✓	✓			✓	✓				✓
15	2021	Eren and Tuzkaya	✓		✓						<b>→</b>	✓			✓	
16	2020	Liu and Liao		✓	✓						✓	✓				✓
17	2021	Pourhejazi et al.		✓	✓					✓	✓	✓			✓	
18	2020	Hanan et al.		✓	✓				✓		✓	✓			✓	
19	2020	Babaei et al.	_	✓	_		✓	✓			✓	✓			✓	
20	2021	Eidi and Ghaseminejad		✓	✓						✓	✓				<b>√</b>
21	2020	Wu and Hifi	✓		_									✓	✓	
22	2020	Alao et al.	✓					✓	✓			✓			$\checkmark$	
23	2021	Tarakish et al.	✓					✓	✓			✓			$\checkmark$	

Row	Year	Author(s)		Multi-objective ction	Routing	Location	Routing and location	Decision criteria	Municipal solid waste (MSW)	Hazardous waste	Defensive position	Probabilistic u	certai Łnzzy	Robust	Exact solution (Epsilon- Delta definition of a	Meta-heuristic solution
24	2023	Obubu	02	<b>√</b>			<b>√</b>			<b>√</b>			<b>√</b>			✓
25	2023	Afsi	✓			✓			✓				✓			✓
26	2023	Sabih		✓		✓			✓					✓		✓
27	2024	This study		✓			✓	<b>√</b>		✓				✓	✓	✓

According to the surveys, the development of urban waste management is a vital factor in urban services. The models provided in this regard are vehicle routing. Most of the studies have addressed the vehicle routing problem for waste collection, and few studies have discussed routing and location in the time window of hazardous waste disposal. Additionally, the behavior of the mathematical model in hazardous waste collection in time windows has not been analyzed, and waste collection outside the time window has been shown to increase system costs. This is a research gap that is addressed in this study. Thus, the innovation of the study lies in this routing and location problem considering uncertainty.

### 3. Problem Statement

Urban waste has become a problem not only in developing countries but also in most industrialized countries. In Mexico, with a population of 107 million, only three institutions are responsible for the safe transportation and disposal of municipal and hospital infectious waste, which, according to the Minister of Health, is 700,000 tons and, in other words, 1.5 kg for each hospital bed. This is becoming a problem in Germany as well (Moslemi, 2017). The Germans have recently presented many plans for this purpose, such as the "Waste Economic Plan", one of the important goals of which is the proper management of urban garbage (Delido, 2020). Since residential houses are the main generators of urban waste, they are responsible for waste disposal or recycling, which means keeping waste separately. Waste separation in Germany is not only limited to houses. According to statistics, hospitals are the fifth largest generators of waste in the Federal Republic of Germany with the generation of 1.1 million tons of waste. However, 65% of these wastes have the characteristics of household waste (Letelier, 2021). A set of laws, technical training, and guidelines have been established in most countries in the past years to curb the growing pressure on nature. One of the biggest obstacles to environmental protection is the economy. This factor becomes doubly important through the budget. However, there are many common points between economy and ecology. A lot of costs can be saved in the separation of waste by taking reasonable and beneficial measures to protect the environment, for example, the separation of recyclable

materials. Management of solid waste collection chains is one of the most important concerns of human societies today. Waste collection and disposal have become more complicated with the increase in the volume of waste on the one hand and their diversity on the other hand. Nowadays, traditional waste collection and disposal systems are no longer responsive and cannot prevent environmental pollution caused by chemical, microbial, and radioactive waste, etc. (Mehdipour, 2016). Despite its shortcomings, the approval of the waste management law can be one of the important steps in improving the status of solid waste management in the country. In this section, a comprehensive plan is provided for hospital waste management.

# **Model Assumptions**

- Vehicles have a limited capacity to receive waste and have a time limit for waste collection.
- Since hospital wastes are highly hazardous, their normalization process and incineration cycle should be evaluated. This is not in the scope of this study and will be addressed in the upcoming recommendations.
- Each urban area and hospital visited by a vehicle that collects its hazardous waste.
- The capacity of collecting waste generated by urban areas and hospitals in areas does not exceed the time capacity of vehicles.
- In each tour, all the declared wastes are transported.
- Each vehicle can collect the waste before the time window announced by the urban areas and hospitals.
- Waste can be burned after collection.

### **Indices and Sets**

i, j: Nodes of urban areas and hospitals generating waste and recycling centers

V: Vehicles for transporting industrial waste

# **Parameters**

 $TL_i$ : the lower limit of the window of receiving the waste of area i

 $TU_i$ : the upper limit of the window of receiving the waste of area i

 $TIR_i$ : The duration of the discharge waste of area i

 $C_{ij}$ : The emission cost of traveling between areas i and j

 $TTR_i$ : The pollution level of the location of area i for the construction of a recycling center

 $TVR_v$ : The emission level of the selection of vehicle v for waste collection

 $TD_i$ : The time it takes for the vehicle to arrive at the hospital to receive waste i

Dtd<sub>i</sub>: Highly hazardous waste of COVID-19 in area i

Din<sub>i</sub>: General hazardous waste from hospitals involved in COVID-19

 $D_i$ : The maximum level of waste that must be collected from area i

$$D_i = Dtd_i + Din_i$$
(1)

The level of waste generated in each area consists of two parts, one of which contains the general waste of hospitals for disposal.

 $V_v$ : The capacity of vehicle v for waste collection

 $T_{Vi}$ : The time required for the discharge of waste of area i by vehicle v

 $TT_{vij}$ : The time required to move vehicles v from area i and j

*M*: A very large number

 $w_i$ : Prioritizing care for hospital i based on the level of personnel involvement with COVID-19 patients

 $M_i$ : The cost of constructing area i for waste disposal

 $Te_{\nu}$ : The maximum available time of vehicle v for waste collection

## **Decision Variables**

 $x_{Vij}$ : A paired variable that is 1 if the vehicle v moves from area i to area j and 0 otherwise.

 $O_i$ : A paired variable that is 1 if the waste collection vehicle meets area i in the specified time interval  $[0, TL_i]$  and 0 otherwise.

 $Q_i$ : A paired variable that is 1 if the waste collection vehicle meets area i in the specified time interval  $[TL_i, TU_i]$  and 0 otherwise.

 $Y_i$ : A paired variable that is 1 if the waste collection vehicle meets area i in the specified time interval  $[0, TU_i]$  and 0 otherwise.

 $Z_i$ : A paired variable that is 1 if the waste collection vehicle meets area i after  $TU_i$  and 0 otherwise.

 $TD_i$ : The time of vehicle entering area i for waste disposal

 $XTD_i$ : 1 if area i is located for waste disposal and 0 otherwise

 $SH_i$ : The amount of uncollected waste from area i

### **Mathematical Model**

$$MIN z_1 = \sum_{I} \sum_{I} \sum_{V} C_{IJ} * X_{VIJ} + \sum_{i} XTD_i * M_i$$
 (2)

$$MIN z_2 = MAX(0, TU_i - TD_i) + MAX(0, TD_i - Tl_i)$$
 (3)

$$MAX z_{3} = \sum_{I} w_{i} ((O_{I} * DTD_{I} + Q_{I} * \left(\frac{TU_{I} - TD_{I}}{TU_{I} - TL_{I}}\right) * DTD_{I}) - SH_{i})$$
(4)

$$MIN z_4 = \sum_{I} \sum_{I} \sum_{V} (TTR_i + TVR_v) * X_{VIJ}$$
(5)

The first objective function seeks to minimize transportation pollution costs and waste recycling center location costs, the second objective function seeks to minimize early and late delivery of services (the MAKESPAN objective function), the third objective function seeks to maximize the level of hospital waste collection from the areas (reducing the level of uncollected waste), and the fourth objective function seeks to reduce the level of pollution in locating and routing.

s.t

$$\sum_{I=1}^{N} \sum_{V} X_{VIJ} = 1 \qquad \forall J = 2, 3, \dots, N$$
 (6)

$$\sum_{I=1}^{N} \sum_{V} X_{VIJ} = 1 \qquad \forall I = 2, 3, \dots, N$$
 (7)

It ensures that every vehicle leaves the recycling center to collect hospital waste.

$$\sum_{I=1}^{N} X_{VIJ} - \sum_{I=1}^{N} X_{VJI} = 0 \quad \forall J = 1, 2, \dots, N \quad V = 1, 2, 3 \dots.$$
(8)

It ensures that there is no loop between urban areas and hospitals for waste collection.

$$\sum_{I=1}^{N} T_{VI} * \sum_{J} X_{VIJ} + \sum_{I} \sum_{J} TT_{vij} * X_{VIJ} \le Te_{v} \quad V = 1,2,3 \dots$$
(9)

It ensures that the total time required to collect waste from area i and the time required to travel between areas i and j do not exceed the maximum available time of vehicle v.

$$TD_J = \sum_I TID_I * \sum_V X_{VIJ} + \sum_{I=1}^{N} \sum_V (T_{VI} + X_{VIJ}) * X_{VIJ} \quad \forall J = 2,3, \dots, N$$
 (10)

Constraints on the timing of the arrival of vehicles for hospital and urban waste collection

$$\sum_{I=2} (D_I - (DTD_I * Z_I) + SH_i) * \sum_{I=1} X_{VIJ} \le K_V \qquad V = 1,2,3 \dots.$$
(11)

It guarantees that the duration of waste collection by vehicle v is less than the scheduled capacity of that vehicle.

$$(TU_i - TD_I) - M * (Y_I) \le 0 \quad \forall I = 1, 2, \dots, N$$
 (12)

It ensures that if the waste collection vehicle arrives at hospital i earlier than the time window, it will dispose all the waste of hospital i.

$$(TU_i - TD_I) + M * (Z_I) \ge 0 \quad \forall I = 1, 2, \dots, N$$
 (13)

It ensures that the waste vehicle will not be able to move all the waste after the designated time window of the hospital if it visits that hospital.

$$Z_I + Y_I = 1 \qquad \forall I = 1, 2, \dots, N \tag{14}$$

One of the following situations will occur for waste collection.

$$(TU_i - TD_I) + M * (1 - Q_I) \ge 0 \quad \forall I = 1, 2, \dots, N$$
 (15)

$$(TL_i - TD_I) + M * (1 - O_I) \ge 0 \quad \forall I = 1, 2, \dots, N$$
 (16)

$$(TL_i - TD_I) - M * (O_I) \le 0 \quad \forall I = 1, 2, \dots, N$$
 (17)

$$Q_I + O_I = 1 \qquad \forall I = 1, 2, \dots, N \tag{18}$$

It ensures that all hazardous and general hospital wastes are transported within the specified time window.

$$\sum_{I=2} X_{VI2} = 1 \qquad \forall V = 1, 2, 3, \dots, N$$
 (19)

It ensures that the vehicles start moving from the recycling center and that they transport all the waste there.

$$\sum_{I=1}^{N} \sum_{i=1}^{N} X_{VIJ} \le |S| - R(S) \qquad \forall S \in A - \{1\}$$
(20)

It ensures that a site is selected for hospital waste recycling.

$$TD_i \le XTD_i \qquad \forall i \tag{21}$$

# **Mathematical Model Robusting**

As mentioned earlier, the model presented in the previous section is linear. In this section, the uncertainty in the maximum time available to vehicles is added to the model using robust scheduling and Bertsimas and Sim's approach. In this way, Constraint 4 is modified as Bretsimas' model. So, the proposed model is linear. According to the surveys, the maximum available time parameter is one of the important parameters whose values may exceed the nominal values. Therefore, considering this parameter under uncertainty can bring the proposed model closer to the reality of the problem. Robust scheduling and Bertsimas and

Sim's approach are used to consider uncertainty in the maximum available time. The robust optimization method looks for optimal or near-optimal solutions that are feasible with a high probability. Bertsimas and Sim's approach is one of the four main approaches to considering uncertainty in robust scheduling. In this section, this approach is briefly mentioned. The following linear programming model is considered for this purpose:

$$Min \sum_{j} c_{j} x_{j}$$

$$s t$$

$$Ax \le b$$
(22)

In this model, it is assumed that only the coefficients on the right side in the constraints, i.e. matrix A, have non-deterministic values and that the entries of this matrix, i.e.  $a_{ij}$ , fluctuate , where  $\tilde{a}_{ij}$  and  $\hat{a}_{ij}$  are the nominal values and the maximum in the interval  $[\tilde{a}_{ij} - \hat{a}_{ij}, \tilde{a}_{ij} + \hat{a}_{ij}]$  deviation of parameter  $a_{ij}$ , respectively. The robust model proposed by Bertsimas and Sim is as follows:

$$Min \sum_{j} c_{j} x_{j}$$

$$st. \sum_{j} \tilde{a}_{ij} x_{j} + z_{i} \Gamma_{i} + \sum_{j \in j_{i}} \mu_{ij} \leq b_{i} \quad \forall i$$

$$z_{i} + \mu_{ij} \geq \hat{a}_{ij} x_{ij} \quad \forall i, j$$

$$z_{i}, \mu_{ij} \geq 0 \quad \forall i, j$$

$$(23)$$

Where  $z_i$  and  $\mu_{ij}$  are dual covariates and the parameter  $\Gamma_i$ , which is called the uncertainty budget, shows the level of conservatism that is chosen according to the importance of the constraint and the risk-taking of the decision-maker. Finally, the proposed model will be as follows:

### **Parameters**

 $\widehat{Te}_v$ : The tolerance in the maximum available time of vehicle v for waste discharge

 $\Gamma_v$ : Uncertainty budget

### **Variables**

 $p_v$  and  $q_v$ : The robust model variables

$$\sum_{I=1}^{I} T_{VI} * \sum_{J} X_{VIJ} + \sum_{I} \sum_{J} TT_{vij} * X_{VIJ} \leq Te_v + \Gamma_v p_v + q_v \quad V = 1,2,3 \dots$$

$$p_v + q_v \geq \widehat{Te}_v \quad \forall v \in V$$

$$p_v, q_v \geq 0$$

$$(24)$$

The above model is a linear mixed integer programming model and can be solved accurately at high speed.

# Non-Dominated Ranking Genetic Algorithm (NRGA)

Omar Al Jadaan et al. (2008) successfully developed a multi-objective evolutionary algorithm called genetic algorithm based on non-inferior ranking for optimization of non-convex, non-linear, and discrete functions. They developed a new approach by combining the ranking-based roulette wheel selection algorithm and the Pareto-based population ranking algorithm based on the problems in the previous approaches. In most cases, this algorithm can achieve a better range of solutions in the Pareto frontier and also converge earlier to the optimal Pareto frontier compared to other multi-objective evolutionary algorithms (MOEAs). However, NRGA and NSGAII algorithms differ in their selection strategy. In the NRGA algorithm, the ranking-based roulette wheel operator is used instead of the swarm competition operator. This operator is designed in such a way that better-fit members are more likely to be selected for reproduction and formation of the next generation. Figure 1 shows the pseudocode for this algorithm.

```
Initialize population P
 Generate random population with size N
 Evaluate objective values
 Assign rank (level) based on Pareto dominance
Generate child population O
    Rank based on roulette wheel selection recombination and mutation
for i=1 to N_F do
    for each member of the combined population
   (PUQ) do
       Assign rank (level) based on Pareto - sort
       Generate sets of non – dominated fronts
       Calculate the crowding distance between members on each front
      (elitist) Select the member of the combined population based on the least dominated N solution
      to make the population of the next generation. Ties are resolved by taking the less crowding
   Create next generation
     Rank based on roulette wheel selection recombination and mutation
End for
```

Figure 1. The NRGA pseudocode

# Multi-Objective Simulated Annealing algorithm (MOSA)

The multi-objective simulated annealing algorithm (MOSA) is one of the most successful single-solution algorithms. This algorithm uses simple mathematical logic to search. Many people have praised the efficiency of this algorithm in various research problems in operations and engineering sciences. First introduced by Kirkpatrick et al. (1983), the algorithm uses the logic of annealing the crystallized metal at high temperatures. The algorithm starts searching with a random high-temperature solution. A neighborhood is created for the solution of the previous step in each step of the algorithm. A new solution is accepted if the solution is improved. Otherwise, it is accepted with probability controlled by the current temperature using the Boltzmann distribution function. This logic enables the algorithm to accept the bad solution first with a high probability and then with a lower probability and escape from the local optimum. Figure 2 shows the pseudocode of the introduced algorithm.

- 1. Parameter setting
- 2. Initialize and evaluation fitness functions  $(x, f_i(x))$
- 3. Best solution =  $(x, f_i(x))$
- 4. For 1 to max-iteration
  - 4.1. Do mutation operator (x')
  - 4.2. Calculate the fitness function and  $(\Delta f_i)$

4.3.1. If 
$$\Delta f_1 \leq 0 \&\& \Delta f_2 \geq 0$$

*Update the Best solution* = $(x', f_i(x'))$ 

*Update the solution* x=x'

4.3.2. Else if 
$$\Delta f_1 \ge 0 \&\& \Delta f_2 \ge 0 \mid |\Delta f_1 \le 0 \&\& \Delta f_2 \le 0$$

Put this solution in Pareto set

4.3.3. Else 
$$\Delta f_1 \ge 0 \&\& \Delta f_2 \le 0$$

$$P_1 = \exp\left(\frac{-\Delta f_1}{T}\right)$$
,  $P_2 = \exp\left(\frac{-\Delta f_2}{T}\right)$ ,  $h=rand$ 

If 
$$h < P_1 & & h < P_2$$

*Update the solution* x=x'

- 5. Update temperature  $(T=\alpha*T)$
- 6. Do non-dominate sorting in this Pareto set.
- 7. If stopping criteria are satisfied, stop, if not, go to step 4.1.

Figure 2. The MOSA pseudocode

# Validation of the Mathematical Model

The problem under study discusses and examines the process of dispatching medical staff to care for patients at home. Three sizes are evaluated and analyzed according to the

mathematical model presented in the previous chapter to evaluate the problem in terms of comprehensiveness.

Table 2. Sizes of the problem

Sizes	Total medical staff	Total patients needing care
Small	3	10

The indices and parameters of the mathematical model are first introduced and the decision variables are then analyzed according to the mathematical model.

The number of available vehicles is 3 and the number of medical centers that need to collect waste is 10 in the small sizes of the problem.

Table 3. The lower limit of the time window for the provision of vehicle services for transporting waste to medical centers i

Schedulin	Cente									
	r 1									
The lower limit	2	5	7	5	8	8	5	8	10	10

Table 4. The upper limit of the time window for the provision of vehicle services for transporting waste to medical centers i

Schedulin	Cente									
g	r 1	r 2	r 3	r 4	r 5	r 6	r 7	r 8	r 9	r 10
The upper limit	4	7	9	7	9	10	8	10	12	12

Routing of waste vehicles to provide services to medical centers

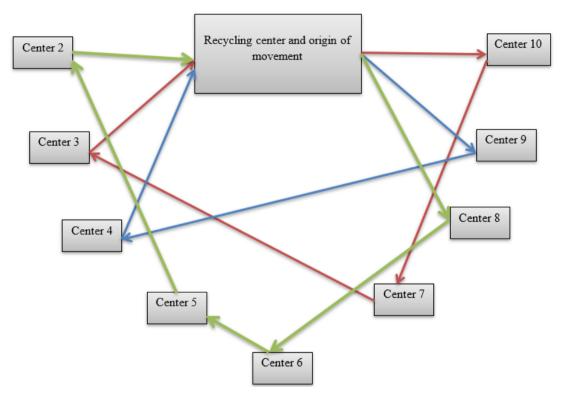


Figure 3. routiong

As can be seen, in the small size of the mathematical model, all three waste vehicles serve the medical centers for waste collection. Vehicle 1 first goes to Medical Center 10 and collects waste. It then goes to Medical Centers 7 and 3. Finally, it sends the waste to the waste recycling site. Moreover, vehicle 2 first goes to Medical Center 9 then to Medical Center 4 and finally returns to the waste recycling site. Vehicle 3 first goes to Medical Center 8 then to Medical Centers 6 and 5 and finally to Medical Center 2 and recycling site and transfers waste.

# Taguchi Methods

Taguchi proposed new statistical concepts in the late 1940s, and these concepts were then proved to be valuable tools in quality control and improvement. Since then, many Japanese craftsmen have been using this method to improve the quality of products and processes. The increase in the quality of cars made in this country can be strongly attributed to the widespread use of this method. The Taguchi methods concern quality control in manufacturing industries and are based on three simple main concepts:

Quality should be designed during production rather than checked during the product manufacturing process.

The product must be designed to be safe against uncontrollable environmental factors.

The cost of quality should be measured as a function of deviation from the standard state, and losses should be measured across the system.

### **Terms**

**Factor**: A factor is a controllable experimental variable that affects the output. It can be considered an independent variable.

Factor level: It is a specific value of a factor.

**Solution**: The solution (output) is a measurable phenomenon that examines the effect of factors on it during a series of experiments.

Experimental design: An experimental design is the result of arranging factors and their levels.

According to Taguchi, the best way to improve design quality is to create it in the product. Taguchi combined and created special groups of orthogonal arrays (OA) to present his experiments. OAs make it easier to design experiments. Designing an experiment includes choosing the most suitable OA, determining the factors with suitable columns, and the location of experiments (experimental conditions). In this study, the smaller-the-better equation of the Taguchi method is used.

$$SN_S = -10\log\left(\frac{1}{n}\sum_{i=1}^n y_i^2\right) \tag{25}$$

Since the mathematical model of this study is multi-objective, an equation using standard multi-objective measures is used as a response in the Taguchi method. This equation is as follows.

$$MCOV = \frac{MID}{MS} \tag{26}$$

Standard multi-objective measures should now be identified.

Standard Multi-Objective Measures

One of the ways to deal with these problems is to use performance measures. Some of these measures are explained below:

### 1. The Number of Pareto Solutions

This measure is equal to the number of output solutions each time the algorithm is executed. This measure is defined as the number of output solutions of the execution of each algorithm in the comparison between several algorithms. The more Pareto solutions of a method, the more desirable that method is.

## 2. Mean Ideal Distance (MID)

This measure is used to calculate the mean distance of the Pareto solutions from the origin of the coordinates. According to the following equation, the lower this measure is, the higher the efficiency of the algorithm.

$$MID = \frac{\sum_{i=1}^{n} \sqrt{\left(\frac{f \ 1_{i} - f \ 1_{best}}{f \ 1_{total}^{max} - f \ 1_{total}^{min}}\right)^{2} + \left(\frac{f \ 2_{i} - f \ 2_{best}}{f \ 2_{total}^{max} - f \ 2_{total}^{min}}\right)^{2}}{n}}$$
(27)

# 3. Algorithm Execution Time (CPU Time)

One of the important measures in big problems is their execution time. So, algorithm execution time is a quality measure.

# 4. Maximum Scatter (MS)

This measure is defined as follows:

$$MS = \sqrt{\sum_{i=1}^{I} \left( \min f_i - \max f_i \right)^2}$$
 (28)

## 5. Spread of Non-Dominated Solutions (SNS)

This measure is introduced to identify the spread and variety of Pareto solutions obtained.

$$SNS = \sqrt{\frac{\sum_{i=1}^{n} (MID - C_i)^2}{n-1}}$$

$$C_i = \sqrt{f \, 1_i^2 + f \, 2_i^2}$$
(29)

## The Results of the Design of the Experiments

After designing the problem, the experiment is designed using the Taguchi method. As mentioned before, the Taguchi method reduces the parameter setting time by reducing the number of experiments. First, we specify the parameters we want to adjust in each algorithm. The parameters to be set in each algorithm are determined first. Parameter levels and OAs for experiments are obtained using Minitab software. After determining the number of experiments for each algorithm, the algorithms were tested with the specified levels and executed ten times. The results of these experiments were averaged and unweighted. Thus, better S/N plots and parameters were obtained.

First, the levels of each algorithm should be obtained and listed. For this purpose, relevant studies were reviewed and candidate levels were identified among them according to Table 5.

Table 5. Different levels of each algorithm's parameters

Algorithms	Algorithm		Parameter level	
Aigoriums	parameters	Level 1	Level 2	Level 3
	Pc	0.7	0.8	0.9
NSGA-II	Pm	0.05	0.1	0.15
NSGA-II	N-pop	50	100	150
	Max-iteration	2*(i+j+k+l+o)	3*( <i>i</i> + <i>j</i> + <i>k</i> + <i>l</i> + <i>o</i> )	4*(i+j+k+l+o)
	$T_0$	30	40	50
MOSA	$\alpha$	0.99	0.9	0.88
	Max-iteration	4*( <i>i</i> + <i>j</i> + <i>k</i> + <i>l</i> + <i>o</i> )	8*( <i>i</i> + <i>j</i> + <i>k</i> + <i>l</i> + <i>o</i> )	12*(i+j+k+l+o)

Finally, the experiments were designed and L9 orthogonal arrays were selected for the NSGA-II algorithm using Minitab 16. The solution values for the Taguchi method were obtained after executing the algorithm for each experiment. Table 6 shows these values and OAs.

Table 6. L9 orthogonal array and computational results for the NSGA-II algorithm

Pc	Pm	N-pop	Max-iteration	NSGA-II Response
1	1	1	1	4.497e-006
1	2	2	2	9.3753e-006
1	3	3	3	3.857e-006
2	1	2	3	7.0235e-006
2	2	3	1	9.5885e-006
2	3	1	2	1.594e-005
3	1	3	2	3.8218e-006
3	2	1	3	4.3919e-006
3	3	2	1	1.417e-005
	1 1 2 2 2 3 3	1 1 1 1 1 1 3 2 1 1 2 2 2 2 3 3 3 1 3 2	1     1       1     2       1     3       2     1       2     2       3     1       3     1       3     2       1     1	1     1     1     1       1     2     2     2       1     3     3     3       2     1     2     3       2     2     3     1       2     3     1     2       3     1     3     2       3     2     1     3

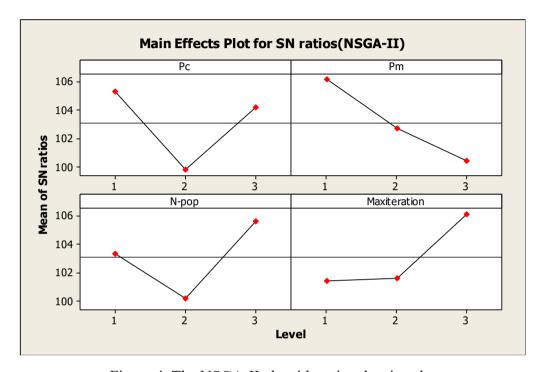


Figure 4. The NSGA-II algorithm signal-noise plot

Table 7. L9 orthogonal array and computational results for the MOSA algorithm

Experiment	$T_0$	α	Max-iteration	MOSA Response
1	1	1	1	1.3846e-005
2	1	2	2	9.9297e-006
3	1	3	3	1.3376e-005
4	2	1	2	9.7577e-006
5	2	2	3	1.5536e-005
6	2	3	1	1.6989e-005
7	3	1	3	5.4162e-006
8	3	2	1	1.1872e-005
9	3	3	2	8.9511e-006

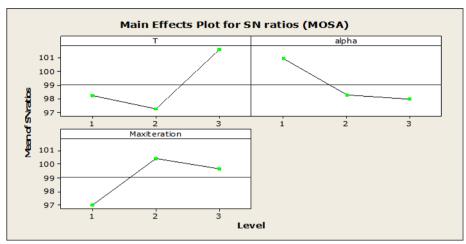


Figure 5. The NSGA-II algorithm signal-noise plot

### The Results of Numerical Problems

The suitable parameters in the algorithm are determined after designing the experiment and setting the parameters. The algorithms are executed for the problems and compared. In this way, 12 problems are executed with the algorithm. A view of the problem chromosome and allocation algorithms can be seen in Figure 6.

Cycle	·	Part 1							P	art	2						Par	rt 3				
Ç,	i				j				j		1	,				i+j	<i>i</i> + <i>v</i>				1	,
1	0.072	:   [-	0.958	0.799	0.641	0.385	0.650	0.897	.674	0.115	0.243	0.122	0.490	0.986	).218	0.516	0.026	0.346	1961	352	0.310	).288
2	0.526 0.072	2090	0.562	-	0.544 (	0.738	.726	0.783 (	098	0.168	.469	0.989 (			.2390		0.723 (		.144	8300	.023 0	926
3	09900	0340	0.3830	0.339 0.123	4310	0.997	3500	7920	3130	0.8520	5120	3910	3820	4720	3040	8940	0.4710	6350	0.0430.	5340	0.494   0.023	1200
4	0.317 0.0	$\sim$			0.230 0.321 0.431	.982 0.	.195 0.	0.336 0.090 0.240 0.760 0.792	.302 0.	0.678 0.	.0006.	.010 0.	0.602 0.991 0.787 0.382 0.716	0.513 0.448 0.472 0.289	0.929 0.467 0.001 0.700 0.304 0	0.611 0.246 0.894 0.457	.913 0.	0.458 0.203 0.635 0.199	.211	0.955  0.231  0.545  0.919  0.534  0.830	0.449 0.	.949 0.
5	0.077 0	0.2150	0.814 0.492	0.055 0.892	0.230	0.934 0.982	0.830	0.240	0.6060	0.5430	0.721	0.773 0	0.991	0.5130	0.001	0.611 0	0.156 0.913	0.458	0.4650	0.545	0.097	0.316
6	0.286	).252	0.185	0.587		0.454	946(	060.0	.951	0.958	.598	).067	0.602	0.593	.467	0.307	0.568	0.523	0.845	.231	0.070	0.057
7	941		0.965	0.178	.777	0.637	.004	.336	.366	0.251	.206	.234	.201	0.193	.929	.647	0.755	0.553	0.638	.955 (	0.746 0.070 0.097	936
8	0.222 0.227 0.286 0.077 0.317 0.099 0	0.8940	0.777	0.3390	0.757 0.777 0.861	0.6170	0.296   0.004   0.946   0.830   0.195   0.350   0.726	0.205	0.771 0.366 0.951 0.606 0.302 0.313 0.860 0.674	0.2580	0.236 0.206 0.598 0.721 0.900 0.512 0.469	0.347   0.234   0.067   0.773   0.010   0.391	0.862  0.201	0.402 0	0.901	0.777 0.647	0.607	0.733 0	0.687	0.3310	0.238 0	0.988 0.936 0.057 0.316 0.949 0.120 0.976 0.2

Figure 6. A view of the problem chromosome

According to Figure 5, the problem chromosome has three parts, all of which are filled with random numbers between 0 and 1. They are then sorted, and the resulting sequences are used as allocation sequences. It is worth mentioning that this study used the priority-based coding method.

for t = 1 to T

Inputs: I: A set of sources

J: A set of applicants

 $D_j$ : demand of applicant j

Ca<sub>i</sub>: capacity of source i

V(I+J): Encode solution of period t

Outputs: Xaloc<sub>ij</sub>: The amount of shipment between nodes

 $Y_{j:}$  binary variable shows the opened applicant

while 
$$\sum_{i} Ca_{i} \geq 0$$

*Step 1:*  $Xaloc_{ij} = 0 \ \forall i \in I, \forall j \in J$ 

Step2: select the value of the first column of sub-segment I for index i

select the value of the first column of sub-segment J for index j

Step3:  $Xaloc_{ij}=min(Ca_i, D_j)$ 

Update demands and capacities

 $Ca_i = Ca_i - Xaloc_{ij} D_j = D_j - Xaloc_{ij}$ 

Step4: if  $Ca_i=0$  then V(1,I)=0

if  $D_i = 0$  then V(1,J) = 0

End while

Step 5: for j = 1 to J

 $if \sum_{I} Xaloc_{ij} > 0 then Y_{j}=1$ 

End for

End for

Figure 7. Algorithm for allocation of the first part of the chromosome

for t = 1 to T

Inputs: I: A set of sources

J: A set of applicants

*D<sub>i</sub>*: demand of applicant j

Ca<sub>i</sub>: capacity of source i

*V(I+J): Encode solution of period t* 

 $INV_{it-1}$ = The amount of saved goods in storage i at period t-1

Outputs: Xalocij: The amount of the shipment between nodes

INV<sub>it</sub>: amount of remaining goods in storage i at period t

 $Ca_i = Ca_i + INV_{it-1}$ 

while 
$$\sum_{i} D_{i} \geq 0$$
 or  $\sum_{i} Ca_{i} \geq 0$ 

Step 1:  $Xaloc_{ij} = 0 \ \forall i \in I, \forall j \in J$ 

Step2: select the value of the first column of sub-segment I for index i

 $select\ the\ value\ of\ the\ first\ column\ of\ sub\ segment\ J\ for\ index\ j$ 

Step3:  $Xaloc_{ij}=min(Ca_i, D_j)$ 

Update demands and capacities

 $Ca_i = Ca_i - Xaloc_{ij} D_i = D_i - Xaloc_{ij}$ 

Step4: if  $Ca_i=0$  then V(1,I)=0

if  $D_j = 0$  then V(1,J) = 0

End while

Step 5:  $INV_{it} = Ca_i$ 

End for

Figure 8. Algorithm for allocation of the second part of the chromosome

for t = 1 to T

Inputs: I: set of source

J: set of applicant

*D<sub>i</sub>*: demand of applicant j

Ca<sub>i</sub>: capacity of source i

*V(I+J): Encode solution of period t* 

Outputs: Xalocij: amount of shipment between nodes

 $Y_{i:}$  binary variable shows the opened applicant

while 
$$\sum_{j} D_{j} \geq 0$$

Step 1:  $Xaloc_{ij} = 0 \ \forall i \in I, \forall j \in J$ 

Step2: select value of first column of sub-segment I for i index

select value of first column of sub-segment J for j index

Step3:  $Xaloc_{ij}=min(Ca_i, D_j)$ 

Update demands and capacities

 $Ca_i = Ca_i - Xaloc_{ij} D_i = D_i - Xaloc_{ij}$ 

Step4: if  $Ca_i=0$  then V(1,I)=0

if  $D_i = 0$  then V(1,J) = 0

End while

*Step5:* for j = 1 to J

 $if \sum_{I} Xaloc_{ij} > 0 then Y_j = 1$ 

End for

End for

Figure 9. Algorithm for allocation of the third part of the chromosome

The first part of the chromosome concerns the allocation between the levels of "medical center index" and "medical center index + waste vehicle" in 8 cycles. The second part of the chromosome concerns the allocation between the levels of "waste vehicle" and "medical center" in 8 cycles. The third part of the chromosome concerns the allocation between the levels of "medical center + waste vehicle + recycling centers" and "time window" in 8 cycles.

The results of solving the proposed mathematical model using the mentioned methods are given in Table 8.

Table 8. The computational results of the algorithms for 12 sub-problems

	NPS	CPU Time	MID	Problem	MS	SNS
Problem	NSGA-II	NSGA-II	NSGA-	Tionem	NSGA-II	NSGA-II
			II	1	364337.8753	252546.1615
1	12	52.8815	1.4990	2	673114.5623	656675.3457
2	12	115.6659	1.119	3	724566.1986	896440.1103
3	15	199.8113	2.1143	4	1017213.025	1434697.2836
4	8	302.7046	3.6118	5	574956,4112	1984306.2674
5	11	746.2813	3.6959	6	968246.0069	2481696.6479
6	11	989.7569	3.1876	7	1057282.628	2868420.3377
7	6	1154.2441	5.0146	8	919442.925	3506257.3114
8	10	1939.262	5.8759	9	1865527.2988	5375823.3732
9	9	4644.4178	4.8438	10		
10	17	5114.7147	3.9634	11	1839931.0865	5473421.1723
11	10	7779.6802	5.8276	12	1399581.8508	6090874.8803
12	20	12039.6386	4.8702	12	1761960.461	6249123.7952

Some Pareto plots of the problem in different sizes are provided in Figures 10 to 12 for better understanding.

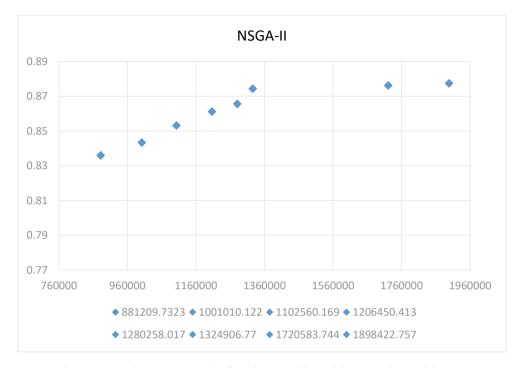


Figure 10. The Pareto plot for the small problem (sub-problem 4)

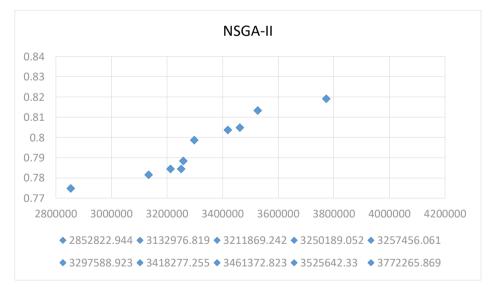


Figure 11. The Pareto plot for the medium problem (sub-problem 8)

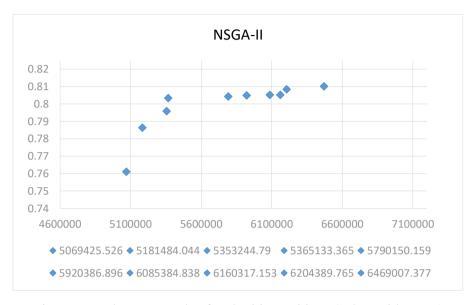


Figure 12. The Pareto plot for the big problem (sub-problem 11)

After calculating the standard indices for these methods, it can be seen that the results of the proposed algorithm are close to the  $\varepsilon$ -constraint method or even better than it in some cases. This can be seen in Table 9.

Table 9. The model validation results

Methods	NP	S↑	CPU t	ime↓	MID↓		MS↑		SNS↑		
Wiethous	value	gap	value	gap	value	gap	value	gap	value	gap	
ε-constraint	10	0.17	261.94	15.86	1.38	0	369174.06	0	296471.86	0.18	
NSGAII	12	0	90.14	4.80	1.41	0.02	360138.19	0.024	280685.49	0.23	

Where the gap is calculated using the following formula.

$$gap = \left| \frac{a \lg - best}{best} \right|$$
(30)

# **Discussion and Conclusions**

This study examined the challenges and requirements of hospital waste management, taking into account the uncertainty in the waste generation peak and the collection time window. An attempt was made to manage the uncertainty in waste generation and collection peak and to optimize the collection schedule by providing a mathematical model to optimize waste

management. Having robust and innovative features, this model can make a significant improvement in the optimal management of hospital waste due to the challenges of uncertainty.

As an innovation in hospital waste management, the model can help managers and decision-makers face the challenges and complexities of uncertainty in waste generation and collection in the best possible way. This applied study suggests a scientific and proposed solution for optimizing hospital waste management and brings a new way to improve management processes in this field. Analyzsis and modeling in the study show that providing a mathematical model to optimize hospital waste management by considering uncertainty can significantly improve efficiency and reduce costs.

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