



A Conservative Graph-Theoretical-Based Framework for Solving Supply-to-Demand Matching and Transmission Line Usage Allocation Problems in Electrical Networks

Akintunde Samson Alayande¹, Olawale Popoola², Claudius Ojo Aremu Awosope³

1. Department of Electrical and Electronics Engineering, University of Lagos, Akoka, Lagos, 100213, Nigeria
aalayande@unilag.edu.ng
2. Centre for Energy & Electric Power, Department of Electrical Engineering, Tshwane University of Technology, Pretoria, 0002, South Africa
popoolao@tut.ac.za
3. Department of Electrical and Information Engineering, Covenant University, Ota, Nigeria
awosope1946@gmail.com

Correspondence:

Akintunde Samson Alayande

Department of Electrical and Electronics Engineering, University of Lagos, Akoka, Lagos, 100213, Nigeria

Email: aalayande@unilag.edu.ng

Abstract

Power system operation and planning are greatly influenced by efficient matching of network demands with the available supply as well as allocation of network usage to the participants in a fairer manner. This has contributed a lot of challenge to an effective operation of modern power systems worldwide. This paper, therefore, presents an alternative approach of the graph theory-based method, which provides a simple but efficient solution to supply-to-demand matching as well as the network usage allocation problems for a secure and reliable operation of power systems most especially, during critical outages. The problems are formulated from the approach of the basic circuit theory and network topology perspectives. An Allocation Factor (AF) matrix, which captures the structural interconnections of various system components such as network nodes and transmission lines, is developed. The Generator Allocation Factor (GAF) matrix, which has the capability of matching the generation to load as well as solving transmission usage allocation problem within the power network, is then formulated using the developed AF matrix. The study uses a simple IEEE 5-bus system and the application is also extended to the practical



system of the Nigerian 28-bus. The results show that the approach has the capability of matching the supply to demand plus losses in an efficient manner. The results obtained show the merit of the approach in solving the supply-to-demand matching and network usage allocation problems, most especially during critical outages, for efficient transmission pricing by the market regulators.

Keywords: Network topology matrix, network usage allocation, generator factor allocation, graph theory

1. INTRODUCTION

Electricity Supply Industry (ESI) is currently undergoing a revolutionary transformation throughout the world [1]. This restructuring is aimed to introduce competition within the industry so as to improve the operation of the new power system [2], [3]. This new evolution allows non-discriminatory access of network participants to transmission line usage. Consequently, various families of new challenges such as fair network usage charge allocation [4], transmission loss allocation [5], [6], congestion management [7], supply-demand matching [8], transient stability enhancement [9], [10], and economic with reliability performances [11], [12] issues have been introduced into the operation of the new power systems. These problems are becoming more important as the level of competition increases. The challenge is further burdened and exacerbated by the increase in deep penetration of renewable energy sources in the network [13]. This is due to the fact that there is a change in the power flows and more additional buses are introduced into the network. The network supply is, therefore, expected to be dynamically managed to meet network demands. In resolving such issue, it is important that the contributions of each generator be determined with respect to network loads [8], [14]–[16].

Traditionally, efficient power system operation requires that the generated power should be matched with the demand power and the losses [8], [14], [17]. Intuitively, for the power system to be operated reliably, it is expected that the losses incurred during power transmission need to be effectively and accurately compensated for by increasing the power generation above that which is required to serve the network loads. This increase in power generated, however, increases the generation cost. Meanwhile, proper and efficient allocation of these charges in a deregulated economy requires determining network usage allocation by the participants within the system as well as supply-to-demand matching within the network [18], [19]. Moreover, to promote transparent competition in the operation of a restructured power system, each market participant needs to be charged in a way that corresponds to the use of transmission lines [4]. Fair apportionment of network usage charges to various market participants still remains an unresolved issue in a deregulated economy.



In order to promote a reasonable economic decision, it is pertinent to determine the actual cost of unbundled resources provided. This can be efficiently handled by determining the amount of contribution that each generator is delivering to serve the network demands. This is because it allows the operators to include the level of transmission line usage for determining the charge associated with the transmission network services in a more efficient manner [16], [20]. There is a need to determine how the electricity supply can be matched with the demand for effective network operation and how the network usage can be transparently allocated among the participants.

Furthermore, the challenge of generation mismatch to load as well as network usage charge allocation within power grids has increasingly become an area of great interest to power system researchers in recent times [21]. In the quest for finding better and alternative approaches to solving these network usage allocation and supply-to-demand matching problems, a considerable number of studies using different methodologies have been deployed in the literature [22]–[34]. However, there are certain bottlenecks associated with these existing approaches, which limits their wide and real-time practical applications. One of such challenges is the selection of a suitable reference bus in the system as the selection of an unsuitable slack bus has a greater influence on the results which could be misleading [13], [35]. The mathematical complexity of the problem formulation is another issue, which limits the applicability of such methods in large-sized power systems [36]–[38]. High computational complexity due to non-linear mathematical equations as well as factorization of the Jacobian matrix will require high computer memory and storage devices. This could lead to local optimal results instead of global results due to truncation of the optimal results obtained since the methods of solution are iterative-based. In [39], a power-flow analysis, which is dependent on the network topological approach, is explored. A technique for determining the contribution of each network participant to line flow, which is dependent on distribution factors, is developed. Although the approach provided promising results, it increases the network size by introducing some virtual nodes to the system. Consequently, there is increase in the mathematical complexity associated with the problem. Several other iterative-based methods have been documented in the literature most of which are reviewed in [40]. These are corroborated by the comprehensive review of the existing methodologies presented by the authors in [41]. The method developed in [42] is based on the postage-stamp concept in which the network users are charged considering the power transacted. This approach has been widely applied using various hypothetical networks due to its simple structure. However, in terms of fairness and accuracy of usage allocation, this approach is not suitable for practical applications. In order to overcome this challenge, a novel optimization-based min-max two-stage approach to transmission usage pricing is proposed in [43]. This approach provides solutions to the challenges such as numerical stability and scalability issues. The main challenge associated with this method is that the complexity of the approach, in terms of the computation and computer memory space, is relatively high as it involves searching for



the global optimal solution. In [44], a contract path-based approach is presented. A two-stage approach for determining the usage allocation cost is presented in [45]. Another important technique in use by many researchers, due to its simplicity, is the proportional sharing principle method [46]–[48]. Although this easy-to-understand method formulates the problem based on circuit theory law as it assumes nodes within the network to be a perfect mixer, this principle has some unfairness and arbitrariness. As such, the solution provided using this method may not be efficient and its application may not be suitable when considering large-sized modern practical power systems.

To overcome these challenges for effective transmission pricing, this paper investigates the role of a graph-theoretical approach, as an alternative method for providing an effective solution to power system operational problems such as network usage allocation and supply-to-demand matching problems in a practical grid network. To the best knowledge of the authors, no research work has been carried out in resolving the issue of the supply-to-demand matching problem from a graph theory perspective, which serves as the major contribution of this paper to the stream of power system research. Moreover, the effectiveness of this approach is tested on a real power system of the Nigerian 28-bus grid for the first time. Section 2 presents both the theoretical and mathematical formulations from a graph-theory-based perspective. Results and discussion of results are presented in section 4 while the study is concluded in section 4.

2. METHODOLOGY

This section presents the methodology adopted used in this study. The approach is basically dependent on the graph-theoretical scheme. In this section, the mathematical framework for the approach is presented.

2.1 Theoretical Framework and Mathematical Formulations Based on Graph-Theoretical Perspective

Any given complex network such as an electric power network can be modelled as a weighted directed graph $G := (V(G), E(G), f)$ where $V(G) = \{v_1, v_2, \dots, v_m\}$ is a set of vertices, $E(G) = \{e_1, e_2, \dots, e_m\}$ is a set of edges such that $E(G) \subseteq V \times V$ and f is a function, which maps $e(i, j)$ the elements of $E(G)$ ($i, j \in E(G)$ where $(i \neq j)$) to each pair of vertices in $V(G)$ [6]. The structural connections within the network are captured in the sparse adjacency matrix $A(G)$ with a dimension of $N \times N$ and whose elements are set to 1 if an edge exists between any two nodes i and j and are set to 0 if otherwise. In general, for an N -bus interconnected power network consisting of $E(G)$ edges and $V(G)$ vertices, the adjacency-based Network Topology (NT) matrix for the network graph is estimated from



$$NT(G) = D_{in} - A(G) \quad (1)$$

where D_{in} is the in-degree matrix given as

$$D_{in} = \text{diag} \left(d^-(v_1), d^-(v_2), \dots, d^-(v_N) \right) \quad (2)$$

Alternatively,

$$NT(G) = \begin{cases} d^-(v_i) & \text{for } i = j \\ -a_{ij} & \text{for } i \neq j \end{cases} \quad (3)$$

where $d^-(v_i)$ is the inward demi-degree of vertex i defined by

$$d^-(v_i) = \sum_j^N |a_{ij}| \quad (4)$$

a_{ij} denotes the (i, j) th element in the adjacency matrix $A(G)$ of the network graph defined by

$$a_{ij} = \begin{cases} 1, & \text{if there exists a directed edge} \\ & \text{from vertex } i \text{ to vertex } j \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

The information provided by power flow solution using Newton-Raphson iterative procedure combined with the (NT) matrix equation are used to formulate the Allocation Factor (AF) matrix for the network graph, which can be expressed by

$$AF(G) = \begin{cases} -P_{ij} & \text{for } i \neq j \text{ and } P_{ij} > 0 \\ P_{Ti} & \text{for } i = j \\ 0 & \text{otherwise} \end{cases} \quad (6)$$



where P_{ij} represents the off-diagonal element of $AF(G)$ and signifies the real flow from any bus i to bus j in the network while P_{Ti} is the main diagonal element and represents the net inflow at any bus i . The actual and counter flows within the network are used to determine the off-diagonal elements of $AF(G)$ while the main diagonal elements are determined from the net flows at all nodes as

$$P_{Ti} = P_{gi} + \sum_{k=1}^N P_{ki} \text{ for } P_{ij} > 0 (i, j \in V(G)) \quad (7)$$

One main inherent characteristic of the Network Topology (NT) matrix is that its N rows are linearly independent and hence, $\det[NT] = 0$. The Network Topology (NT) matrix, as shown in equations (1) and (3), captures the interconnectivity of the network elements such as buses and transmission lines. It, therefore, serves as the basis for the formulation of the Allocation Factor (AF) matrix, which can be used to provide an effective solution to various allocation problems between power system participants. Although supply-to-demand allocation problem and transmission network usage allocation problem are addressed in this paper, other allocation problems such as loss allocation problem, and transmission capacity margin allocation problem can also be solved using the Allocation Factor (AF) matrix.

The AF matrix formulated in equation (6) is a square, symmetric, invertible, and, semi-definite matrix. Consequently, the sum of all the elements in the k th column of AF matrix equals the total real power generated at any bus k . Also, the sum of all the elements in the k th row equals the total sum of all real power demanded at the bus k . Based on the foregoing, we can write

$$[AF]^T [I] = [P_g] \quad (8)$$

$$[AF][I] = [P_d] \quad (9)$$

where $[I]$, $[P_g]$ and $[P_d]$ are the unit vector, column vector of the network generators and column vector of network demands respectively.



Let

$$P_{gg} = \text{diag}(P_{g1}, P_{g2}, \dots, P_{gN}) \quad (10)$$

$$P_{dd} = \text{diag}(P_{d1}, P_{d2}, \dots, P_{dN}) \quad (11)$$

It should be noted that at any load bus i , $P_{gi} = 0$ and at any generator bus i where no load is located, $P_{di} = 0$.

From equations (8) and (9),

$$[I] = [AF]^{-1} [P_d] = [AF^{-1}]^T [P_g] \quad (12)$$

Consequently,

$$[P_g] = [P_{gg}] [I] \quad (13)$$

By using equations (12) and (13), it is straight forward to show that

$$P_{gi} = \sum_{k=1}^N GAF_{ik} \times P_{dk} \quad (14)$$

where $[GAF]$ is the Generator Allocation Factor matrix defined as

$$[GAF] = [AF]^{-1} [P_{gg}] \quad (15)$$

It can be seen that equation (14) gives the real power contribution from the generator placed at bus i to the demand located at bus k while equation (15) presents the proportion of real power from the network generators that should be allocated to network demands.

The information obtained from GAF are very important for the following two reason. First, the information serves as a signal for scheduling the generators in the network economically so as to meet network loads plus losses. Second, it presents an alternative way of allocating the usage cost associated with the network usage to the network participant in an economic manner.



Therefore, the contribution of a generator, placed at any bus i towards the usage of a transmission line that connects buses j and k , can easily be estimated using equation (14) as

$$\left[P_{gi \rightarrow j-k} \right] = \left[GAF_{ij} \right] \times \left[LF_{jk} \right] \quad (16)$$

where LF_{jk} is the line flow on a branch that connects buses j and k within the network and GAF_{ij} is estimated from equation (16).

3. Results and Discussions

This section presents illustrative examples using a simple IEEE 5-bus and a practical Nigerian 28-bus system case studies to demonstrate the effectiveness of the approach presented in this paper. The single-line diagram for the IEEE 5-bus system and the associated data are extracted from [49]. This system consists of seven transmission lines, two generators located at buses 1 and 2 respectively while the remaining three buses 3, 4 and 5 are load buses. The single-line data diagram and the data for the practical Nigerian 28-bus network are adapted from [50]. This network consists of thirty-one transmission lines, ten generators located at buses 1 to 10 and the remaining 21 buses being load buses. In this study, MATLAB-based programme is developed and implemented for the power-flow simulations, which are executed on an Intel Core i5 Quad Core processor, 3.60 GHz, 8 GB RAM personal computer.

The methodology presented in this paper involves three major analyses, which follows a step-by-step procedural algorithm. The first step in the analysis involves power flow analysis. The Network Topology (NT) matrix, which captures the strength of network interconnectivity between the buses and transmission lines is then formulated. The results of the Network Topology (NT) matrix are then used in conjunction with the power flow results to estimate the Allocation Factor (AF) matrix for the network. Thereafter, the AF matrix is used with the amount of real power generated by each generator to provide the GAF matrix, which presents the proportion of power that each generator needs to contribute in order to meet up with the network demands. The GAF matrix is then applied to the network loads in order to determine the required supply-to-load matching within the network. In order to estimate the transmission line usage allocation from the network generators, the line flows obtained from the power flow results are also used in conjunction with the GAF matrix. The numerical results obtained using two case studies are presented and discussed in the subsections that follow:

3.1 The IEEE 5-bus Network

In this section, a simple 5-bus network is considered to illustrate the approach presented in this paper in a step-by-step manner. The results obtained from the converged power-flow analysis are presented in



Table 1. The convergence to the solution for the results presented in Table 1 was obtained after three iterations using Newton-Raphson iterative technique.

Table 1: Power-flow-based results using a simple IEEE 5-bus system

Bus	Voltage Magnitude (pu)	Voltage angle (Radians)	Generator		Load	
			MW	MVAR	MW	MVAR
1	1.060	0.000	0.578	0.500	0.000	0.000
2	1.030	-0.007	0.908	7.666	0.000	0.000
3	1.011	-0.058	0.000	0.000	0.450	0.149
4	1.009	-0.061	0.000	0.000	0.400	0.050
5	1.001	-0.070	0.000	0.000	0.600	0.100

With reference to the power-flow results (Table 1) obtained using a simple IEEE 5-bus system and the Network Topology results presented in Table 2, the Allocation Factor (AF) matrix results are obtained as presented in Table 3. From the results obtained for the AF (Table 3), the total real power generation required to meet up with the demand as well as the network losses can be estimated. For example, from the AF matrix presented in Table 3, it can be seen that the addition of the entries column-wise for the first and second columns gives 57.8MW and 90.8MW respectively while the summation of the elements column-wise for each of the other three columns gives zero respectively. This is because, the network generators are located at buses 1 and 2 while the remaining three buses are load buses. In other words, since no generator is attached to buses 3, 4 and 5, power generation at these buses is zero. However, if the entries of the matrix AF are added row-wise, it can be seen that bus 3 gives 46.3MW, bus 4 gives 40.6MW and bus 5 gives 61.1MW. The results obtained for the GAF matrix for the 5-bus system under consideration are presented in Table 4. These results show the proportion of real power that should be contributed to each network demand by each of the two generators if the allocation is to be fair. Obviously, the algebraic sum of the entries in each of the column equals unity. Furthermore, as presented in Table 4, based on the network topological structure, it can be seen that any load placed at bus L1 will definitely be fed by only generator G1 (100% supply from G1). The implication of this is that the load placed at the bus L1 serves as a local load to generator G1. Consequently, such load will be totally fed by the generator G1. This, however, is not the case for other loads L2, L3, L4 and L5 located at buses 2, 3, 4, and 5.

Table 2: Network Topology (NT) matrix for the IEEE 5-bus network

	G1	G2	L1	L2	L3
G1	0.00	-1.00	-1.00	0.00	0.00
G2	0.00	1.00	-1.00	-1.00	-1.00
L1	0.00	0.00	2.00	-1.00	0.00

L2	0.00	0.00	0.00	2.00	-1.00
L3	0.00	0.00	0.00	0.00	2.00

Table 3: Allocation Factor (AF) matrix for IEEE 5-bus network

	G1	G2	L1	L2	L3
G1	57.80	-27.90	-29.80	0.00	0.00
G2	0.00	118.70	-29.60	-31.90	-56.70
L1	0.00	0.00	59.40	-13.10	0.00
L2	0.00	0.00	0.00	45.00	-4.40
L3	0.00	0.00	0.00	0.00	61.10

Table 4: Generator Allocation Factor (GAF) matrix for the IEEE 5-bus network

Generators Bus	Load Bus				
	L1	L2	L3	L4	L5
G1	1.000	0.235	0.619	0.347	0.243
G2	0.000	0.765	0.381	0.653	0.757
*	0.000	0.000	0.000	0.000	0.000
*	0.000	0.000	0.000	0.000	0.000
*	0.000	0.000	0.000	0.000	0.000

Based on Table 4, 23.5% of the load L2 will be fed through the generator G1 while the remaining 76.5% will be supplied by the generator G2. The load L3 located at bus 3 will be supplied by 61.9% of G1 and 38.1% of G2. The load at L4 will be supplied by 34.7% of G1 and 65.3% of G2 while the load placed at bus 5 will be supplied by 24.3% of G1 and 75.7% of G2. Based on this proportion, the supply-to-demand matching problem is solved and the results are presented in Table 5. In other words, the network demands are supplied by generators G1 and G2 in the proportion dictated by the GAF matrix as presented in Table 5.

Table 5: Supply-demand matching based on GAF for the IEEE 5-bus network

		Loads Bus			Total
Generator Bus		L3	L4	L5	MW
	G1 (MW)	27.8465	13.8706	14.5855	56.3026
	G2 (MW)	17.1535	26.1294	45.4145	88.6974
Total (MW)		45	40	60	145



The asterisks (*) shown in Table 5 show that no generator is placed at buses 3, 4 and 5 within the simple 5-bus network under consideration. It can be seen that the total real power contributed by generator G1 adds up to 56.3MW while that of G2 sums up to 88.7MW as predicted by the GAF matrix. Also, the total real power demands at buses 3, 4, and 5 are 45MW, 40MW and 60MW respectively. This therefore shows that the real power loss allocation within the system can be easily predicted by comparing the results presented in Table 3 and that of Table 5. It can be seen that the total real power generated from Table 3 amounts to 148.6MW while from Table 5 is 145MW. This, therefore accounts for the real power loss within the system, which amounts to 3.6MW.

Table 6: Estimation of usage allocation based on GAF matrix for the IEEE 5-bus network

Line	Usage Allocation (MW)		Total (MW)
	G1	G2	G1 + G2
1-2	26.800	0.000	26.800
1-3	27.750	0.000	27.750
2-3	7.875	26.634	34.509
2-4	9.489	28.876	38.365
2-5	14.737	28.687	43.424
3-4	7.572	0.000	7.572
4-5	2.376	3.386	5.762
Total	96.599	87.583	184.182

Using the IEEE 5-bus system, under consideration, the results of the analysis obtained for the usage allocation, through the use of GAF-based results, are as presented in Table 6. From the results, it is obvious that the total contribution from the two generators G1 and G2 is 184.182 MW. Out of this total contribution to the load demands on the network, G1 contributes 96.599 MW while G2 contributes 87.583 MW. More so, as can be seen from the results, 100% transmission line usage is associated with the lines 1-2 and 1-3. This means that lines 1-2 and 1-3 are employed to transport 100% of the power by G1. AS such, no contribution is made to the lines 1-2 and 1-3 by the G2. Consequently, the main charges associated with the transmission line usage is expected to be completely handled by generator G1. It can also be seen that the transmission line 2-5 contributes the larger part of the power to network usage with 43.424 MW while transmission line 4-5 has the least contribution of 5.762 MW to the network usage. The results also show that generator G1 made its largest contribution of 27.750 MW and its least contribution of 2.376 MW to the transmission lines 1-3 and 4-5 respectively. In the same vein, generator G2 made its largest contribution of 28.876 MW and least contribution of 0.0 MW to the transmission lines 2-4 and (1-2, 1-3 and 3-4) respectively. Figure 1. Depicts



the total real power contributed by the two generators and allocated to the transmission lines in the network.

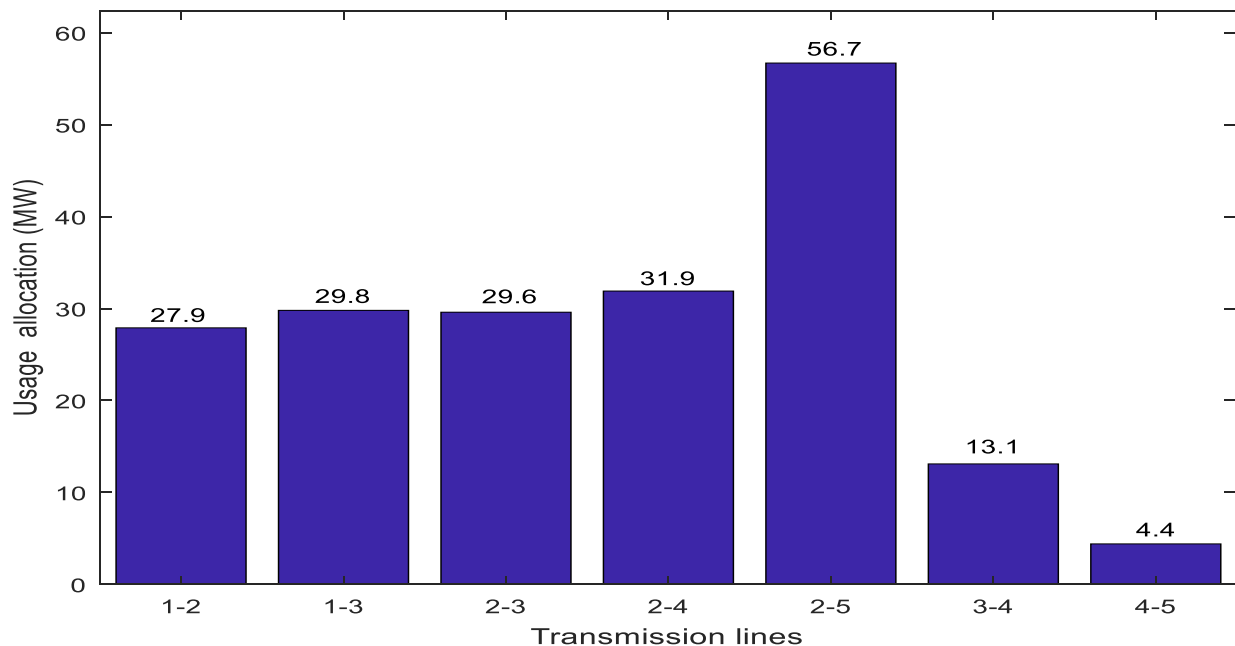


Figure1: Transmission line usage allocation in the IEEE 5-bus network

It can be seen that the most used transmission line by the two generators is the transmission line connecting buses 2 and 5 with the total allocation of 56.7MW while the least allocation is apportioned to the line connecting buses 4 and 5 with a total real power of 3.7617MW. This information helps in the allocation of the charges to the network participants in a fairer and transparent manner. It can be seen that the most used transmission line by the two generators is the transmission line connecting buses 2 and 5 with the total allocation of 56.7MW while the least allocation is apportioned to the line connecting buses 4 and 5 with a total real power of 3.7617MW. This information helps in the allocation of the charges to the network participants in a fairer and transparent manner.

3.2 Practical Nigerian 28-bus Network

As discussed in the previous illustrative example, similar procedures are also followed using the Nigerian 28-bus network. First, the converged power-flow solution is obtained using Newton-Raphson technique. The results of power-flow solution. Based on GAF matrix for this network, the contribution of each generator to the network demands are estimated.



The results obtained when the suggested approach is applied to solve the supply-to-demand matching problem based on GAF matrix are presented in Figure 2. It can be seen that for a balanced operation of the system, larger percentage of the real power needs to be supplied by the AES GS which amounts to 735.6MW followed by Okpai GS which contributed 690.87MW. The least contribution to the network demands is 46.07MW which is supplied by Egbin GS. The distribution of the supply allocations to network demands based on GAF matrix are presented in Table 7. It can be seen that some generators do not contribute to some of the loads within the system. For example, the local load connected to the generator bus 1 (Egbin GS) is not supplied by generators G3 (Okpai GS), G6 (Afam GS), G7 (Calabar GS) and G8 (Shiroro GS). This is based on the nature of the structural interconnections of network participants as captured by the GAF matrix.

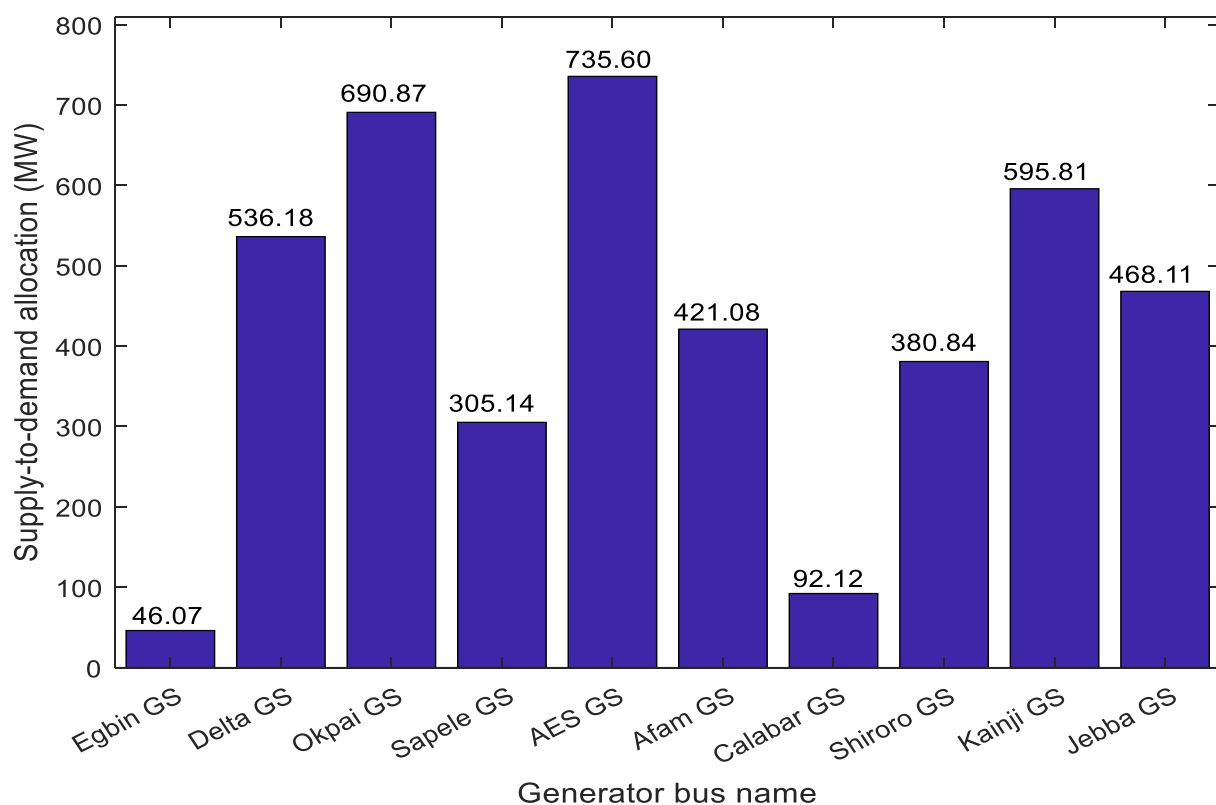


Figure 2: Estimation of supply-to-demand matching in the nigerian 28-bus network

Furthermore, it can be seen from the results of Table 7 that a load connected to a generator bus is not only supplied by local generator at that bus but also supplied by one or more other generators within the network. Examples of these are the loads LG1, LG4, LG6, LG7, LG8, and LG9 connected to generators



G1 (Egbin GS), G4 (Sapele GS), G6 (Afam GS), G7 (Calabar GS), G8 (Shiroro GS), and G9 (Kainji GS). This shows that the GAF matrix provides an economic way of generator scheduling for meeting the set of network loads. This method is, therefore, a good signal or measure which could be useful for allocating cost within deregulated power system. Also, it will provide the system operators and utilities an insight as to how the network participants (electric consumers) could be efficiently charged (tariff). This will help the system operation to be more stable and reliable.

Table 7: Supply-demand matching based on GAF matrix for the nigerian 28-bus network

Bus Code	Generator Contribution (MW)									
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
LG1	9.25	7.37	0.00	5.07	34.41	0.00	0.00	0.00	6.44	6.36
LG2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LG3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LG4	0.00	10.53	0.00	10.08	0.00	0.00	0.00	0.00	0.00	0.00
LG5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LG6	0.00	0.00	0.00	0.00	0.00	52.50	0.00	0.00	0.00	0.00
LG7	0.00	0.00	8.82	0.00	0.00	0.00	1.18	0.000	0.00	0.00
LG8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	44.88	12.80	12.63
LG9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00
LG10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L11	36.83	29.36	0.00	20.19	137.05	0.00	0.00	0.00	25.66	25.32
L12	0.00	42.60	0.00	29.29	198.85	0.00	0.00	0.00	37.22	36.74
L13	0.00	78.26	0.00	53.80	365.28	0.00	0.00	0.00	68.38	67.48
L14	0.00	8.18	0.00	5.62	0.00	0.00	0.00	0.00	0.00	0.00
L15	0.00	96.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L16	0.00	227.14	0.00	156.16	0.00	0.00	0.00	0.00	0.00	0.00
L17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.54	5.46
L18	0.00	11.97	0.00	8.23	0.00	0.00	0.00	0.00	128.65	126.96
L19	0.00	0.00	62.03	0.00	0.00	0.00	8.27	0.00	0.00	0.00
L20	0.00	0.00	73.05	0.00	0.00	0.00	9.74	70.36	20.06	19.80
L21	0.00	8.73	0.00	6.00	0.00	0.00	0.00	0.00	93.85	92.62
L22	0.00	0.00	83.50	0.00	0.00	0.00	11.13	80.42	22.93	22.63
L23	0.00	0.00	194.82	0.00	0.00	206.20	25.98	0.00	0.00	0.00
L24	0.00	7.63	75.29	5.25	0.00	79.68	10.04	0.00	0.00	0.00
L25	0.00	7.92	78.13	5.45	0.00	82.69	10.42	0.00	0.00	0.00
L26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	185.19	52.80	52.11
L27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	114.50	0.00
L28	0.00	0.00	115.24	0.00	0.00	0.00	15.37	0.00	0.00	0.00

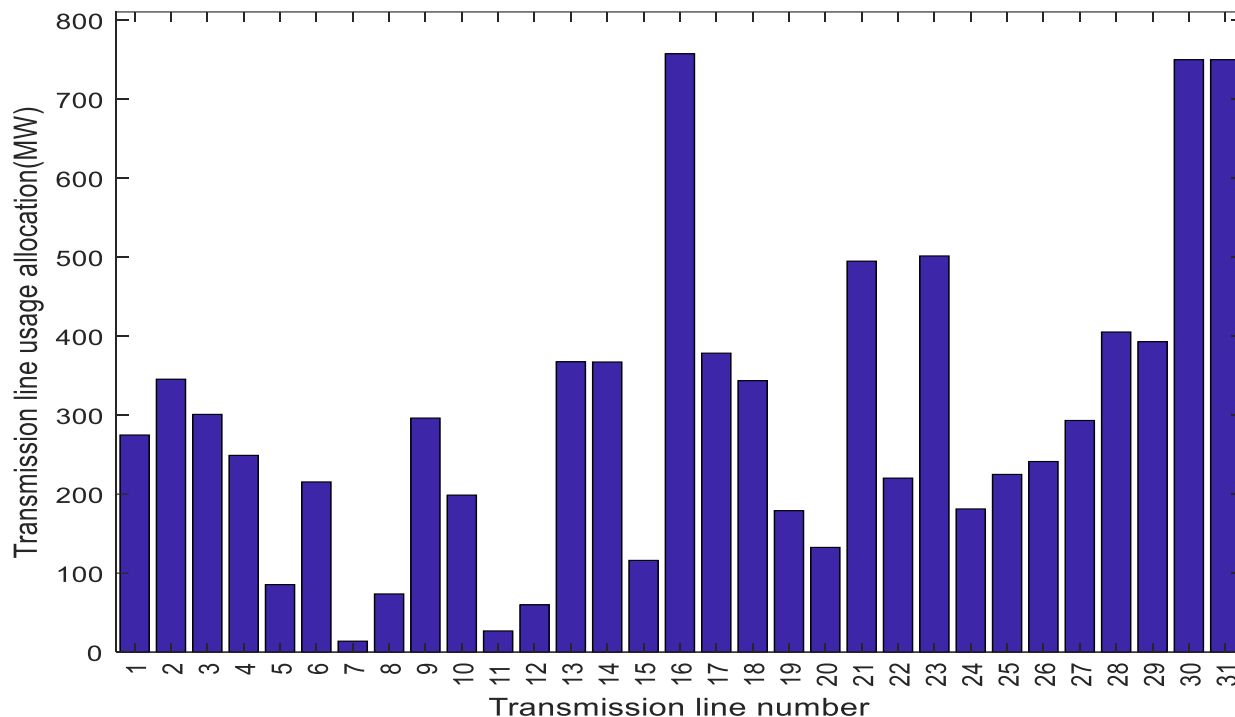


Figure 3: Transmission line usage allocation to network participants in the Nigerian 28-bus network

The results obtained for contributions of the network generator to each transmission line usage based on GAF matrix are presented in Table 8 and depicted in Figure 3 while the distribution of the generator contribution to network transmission lines are presented in Table 9. Table 8 shows the ranking of the usage allocation associated with the transmission line in an ascending order. The transmission line associated with the least usage allocation (13.8810MW) is line number 7 which is the transmission line that connects Benin TS bus with the Ajaokuta TS bus within load-to-load region in the network and it is therefore ranked number one. Also, the transmission line associated with the highest usage allocation (757.4860MW) is line number 16 (Jebba TS – Osogbo TS) within the load-to-load region and therefore ranked 31. As seen from Table 8, out of the 31 transmission lines existing in the Nigerian 28 bus system, 16 are either within the generator-to-load region or load-to-generator region which constitutes 52% of the total of the total network region. Out of these 16 transmission lines, the capacities of 13 of them (lines 1, 3, 9, 13, 17, 21, 22, 23, 26, 27, 28, 29, and 31) are heavily used, while the capacities associated with the remaining 2 (lines 8 and 15) are weakly or lightly used with reference to the allocation of real power from the generators.



Table 8: Estimation of Transmission line allocation from the network generators

Line no.	Bus name – Bus name	From bus code – To bus code	Region of influence	Transmission Usage (TU)	TU ranking
1	Egbin GS – Ajah TS	G1-L11	Gen -Load	274.8720	16
2	Ikeja West TS – Akangba TS	L13-L12	Load - Load	345.5291	21
3	Ikeja West TS – Egbin GS	L13-G1	Load – Gen	300.9810	19
4	Benin TS – Ikeja West TS	L16-L13	Load – Load	249.1000	15
5	Ayede TS – Ikeja West TS	L18-L13	Load – Load	85.5091	5
6	Osogbo TS – Ikeja West TS	L21-L13	Load – Load	215.4809	11
7	Benin TS – Ajaokuta TS	L16-L14	Load – Load	13.8810	1
8	Delta GS – Benin TS	G2-L16	Gen -Load	73.6830	4
9	Delta GS – Aladja TS	G2-L15	Gen -Load	296.3170	18
10	Aladja TS – Sapele TS	L15-L4	Load – Load	198.7940	10
11	Benin TS – Onitsha TS	L16-L25	Load – Load	26.8310	2
12	Benin TS – Osogbo TS	L16-L21	Load – Load	59.8410	3
13	Sapele GS – Benin TS	G4-L16	Gen -Load	367.5470	23
14	Osogbo TS – Ayede TS	L21-L18	Load – Load	367.3031	22
15	Kainji GS – Birnin-Kebbi TS	G9-L27	Gen -Load	116.1370	6
16	Jebba TS – Osogbo TS	L17-L21	Load – Load	757.4860	31
17	Afam GS – Alaoji TS	G6-L23	Gen -Load	378.5000	24
18	Alaoji TS – Onitsha TS	L23-L25	Load – Load	343.6700	20
19	Onitsha TS – New Haven TS	L25-L24	Load – Load	179.0319	8
20	Jos TS – Gombe TS	L19-L28	Load – Load	132.5750	7
21	Jebba GS – Jebba TS	G10-L17	Gen – Load	495.0000	27
22	Jebba TS – Shiroro GS	L17-G8	Load - Gen	220.3140	12
23	Kainji GS - Jebba TS	G9-L17	Gen -Load	501.5630	28
24	Jos TS – Kaduna TS	L19-L20	Load – Load	181.2750	9
25	Kaduna TS – Kano TS	L20-L22	Load – Load	224.9850	13
26	Shiroro GS – Kaduna TS	G8-L20	Gen -Load	241.3221	14



27	Shiroro GS – Abuja TS	G8-L26	Gen -Load	293.2680	17
28	Calabar GS – Alaoji TS	G7-L23	Gen -Load	405.2970	26
29	Calabar GS – Jos TS	G7-L19	Gen -Load	392.9800	25
30	Okpai GS – Calabar GS	G3-G7	Gen -Load	750.0000	29
31	AES GS – Ikeja West TS	G5-L13	Gen -Load	750.0000	30

It can also be observed from the results displayed in Table 8 that only one transmission lines within the generator-to-generator region in the network. This is the transmission line that connects Okpai GS located at bus 3 and Calabar GS on bus 7. Based on the ranking of the transmission line usage allocations as presented in Table 8, it can be seen that line 30 is ranked 29, which shows that its capability is fully or heavily utilized in the network. Also, by considering the load-to-load region within the network, it can be seen that out of the 15 transmission lines within the load-to-load region, the capacities of 7 transmission lines (lines 2, 4, 6, 14, 16, 18 and 25) are heavily used, the capacities of 3 transmission lines (lines 10, 19 and 24) are moderately loaded or used while the capacities of 5 transmission lines (lines 5, 7, 11, 12 and 20) are weakly loaded.

Based on the foregoing analyses, it can be seen that the total transmission lines that are heavily used within the Nigerian 28-bus network are 21 out of the total 31 existing transmission lines (1 within generator-to-generator region, 13 within the generator-to-load region and 7 within load-to-load region) in the system. The explains why the Nigerian 28-bus network may not be able to cope with further additional load in the future, which means that any further increase could cause the system to experience voltage collapse. Consequently, the topology of such network will be inherently weak. Such system is liable to frequent system collapse. For the system operation to be effective, the system needs to be reinforced.

The contribution of each generator to transmission line usage in the network of Nigerian 28-bus system is presented in Figure 4. It can be seen that the generator associated with the highest share in the transmission line usage is Okpai GS with a share of 2049MW followed by Calabar GS with a contribution of 173.2MW and so on. Also, it can be seen that the generator G1 (Egbin GS) has the least contribution of 36.9MW to the transmission network usage in the network followed by Kainji GS with 1637.7MW and then Jebba GS with 1501.7MW. Therefore, based on each generator share to transmission lines usage allocation, we can arrange the network generators in order of decreasing criticality as Okpai GS < Kainji GS < Jebba GS < AES < Delta GS < Afam GS < Shiroro GS < Sapele GS < Calabar GS < Egbin GS. However, during a critical outage condition, the most critical generator based on usage allocation is Calabar GS followed



by Shiroro GS and then Sapele GS. This is because, Calabar GS has three different transmission lines connected to it. These include Calabar GS - Alaoji TS (line 28), Calabar GS - Jos TS (line 29) and Okpai - Calabar GS (line 30) with the associated usage rankings of 26, 25 and 29 respectively as presented in Table 8.

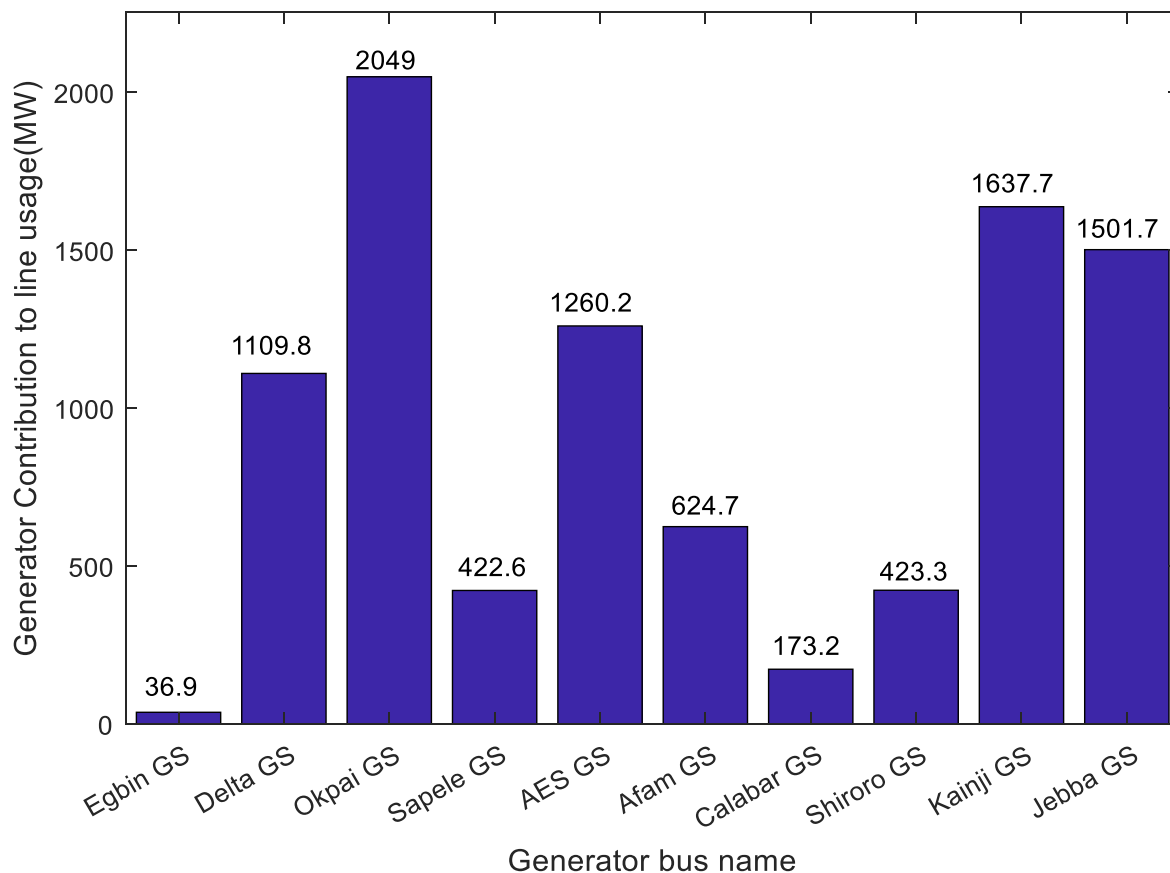


Figure 4: Total contribution of each generator to transmission line usage in the Nigerian 28-bus network

Table 9: Generator allocation to transmission line usage based on GAF matrix for the Nigerian 28-bus network

Line No.	Line Usage (MW)									
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
1	36.89	29.41	0.00	20.22	137.29	0.00	0.00	0.00	25.70	25.36
2	0.00	42.70	0.00	29.36	199.33	0.00	0.00	0.00	37.31	36.83
3	0.00	37.20	0.00	25.57	173.63	0.00	0.00	0.00	32.50	32.08
4	0.00	147.61	0.00	101.49	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	3.71	0.00	2.55	0.00	0.00	0.00	0.00	39.89	39.36



6	0.00	9.35	0.00	6.43	0.00	0.00	0.00	0.00	100.51	99.19
7	0.00	8.23	0.00	5.66	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	73.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	296.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	198.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	15.90	0.00	10.93	0.00	0.00	0.00	0.00	0.00	0.00
12	0.00	35.46	0.00	24.38	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	187.79	0.00	179.76	0.00	0.00	0.00	0.00	0.00	0.00
14	0.00	15.94	0.00	10.96	0.00	0.00	0.00	0.00	171.32	169.08
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	116.13	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	381.23	376.25
17	0.00	0.00	0.00	0.00	0.00	378.50	0.00	0.00	0.00	0.00
18	0.00	0.00	156.80	0.00	0.00	165.96	20.91	0.00	0.00	0.00
19	0.00	7.68	75.77	5.28	0.00	80.19	10.10	0.00	0.00	0.00
20	0.00	0.00	116.98	0.00	0.00	0.00	15.60	0.00	0.00	0.00
21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	495.00
22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	110.88	109.43
23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	501.56	0.00
24	0.00	0.00	159.95	0.00	0.00	0.00	21.33	0.00	0.00	0.00
25	0.00	0.00	85.15	0.00	0.00	0.00	11.35	82.01	23.38	23.08
26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	154.05	43.92	43.35
27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	187.21	53.38	52.68
28	0.00	0.00	357.62	0.00	0.00	0.00	47.68	0.00	0.00	0.00
29	0.00	0.00	346.75	0.00	0.00	0.00	46.23	0.00	0.00	0.00
30	0.00	0.00	750.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
31	0.00	0.00	0.00	0.00	750.00	0.00	0.00	0.00	0.00	0.00

Based on these usage allocation rankings, it can be seen that all these three transmission lines are heavily utilized. The same is applicable to Shiroro GS; it also has three transmission lines (Jebba TS -Shiroro GS (line 22), Shiroro GS – Kaduna TS (line 26) and Shiroro GS -Abuja TS (line 27)) connected to it with the associated rankings of 12, 14 and 17 respectively. Based on these rankings, it can be seen that the transmission lines connected to Shiroro GS are not as utilized as those connected to Calabar GS. This implies that Calabar GS is the most critical generator bus considering transmission usage allocation during critical outage of a multi-node generator. This information provided a useful insight as to which of the network generators and



transmission lines requires more or additional attention in terms of for efficient and reliable operation of the system.

4.0 Conclusion

The application of a graph theoretical-based approach, for solving supply-to-demand matching and transmission line usage allocation problems, has been presented in this paper. The theoretical and mathematical formulations for the approach are presented. The simplicity of the approach and its capability in providing a quick and efficient solution to supply-to-demand matching and usage allocation problems is demonstrated through a step-by-step procedure using the IEEE 5-bus system. The suggested approach is then extended to a relatively large-sized practical network of Nigerian 28-bus system. The results obtained shows the suitability and applicability of the method for fair and equitable (1) allocations of generation to load for efficient power system operation and also (2) estimating the network usage by the participants, which is a useful signal for estimation of relevant usage costs associated with the network usage for fairer allocation to the consumers of electricity. Consequently, the information obtained could be of help to power utilities from two perspectives: firstly, in determining the capacity of the generators required to serve network demands plus the associated losses. It could also be useful in the efficient allocation of charges associated with the transmission line usage to the network participants. More conclusively, this approach could be useful in efficient transmission line usage allocation for a fairer allocation of usage charge to network participants since the market participants are charged in a way that correlates to their use of the system. It also helps in identifying the generators and transmission lines that require additional protection for a reliable operation of the network.

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Appendices

Appendix A

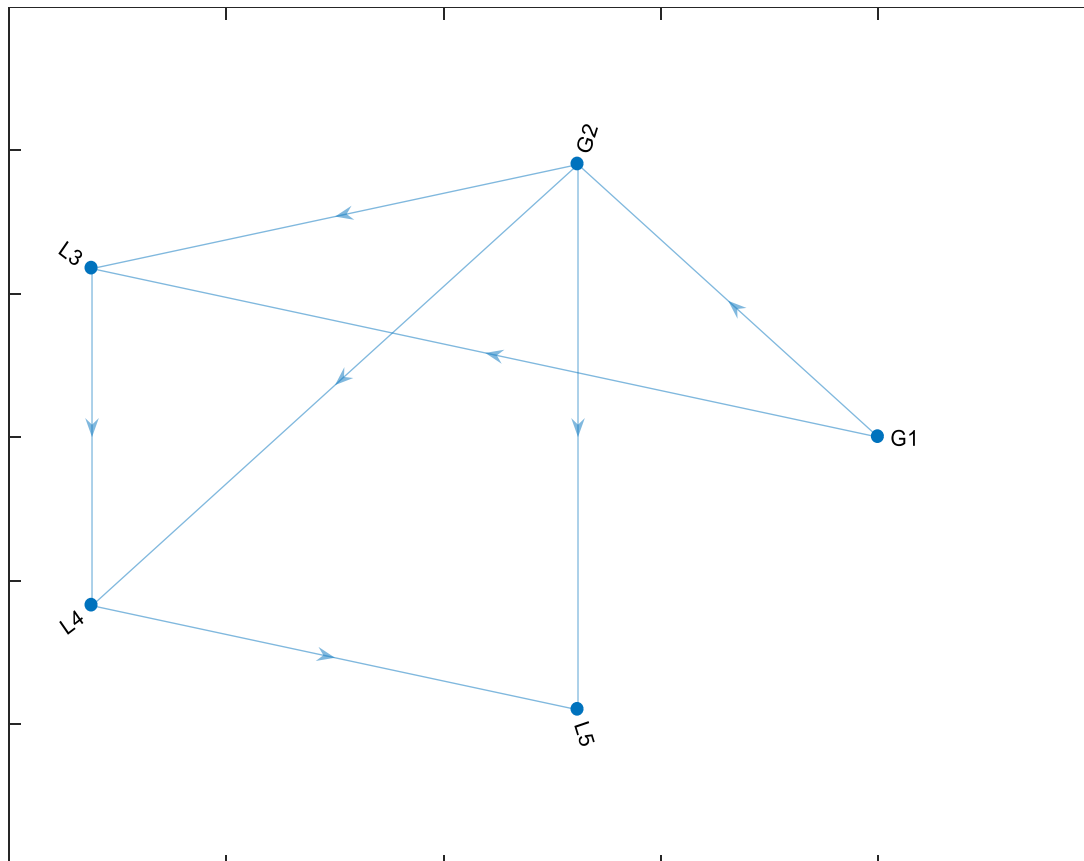


Figure appendix A: The directed graphical model of the IEEE 5-bus network

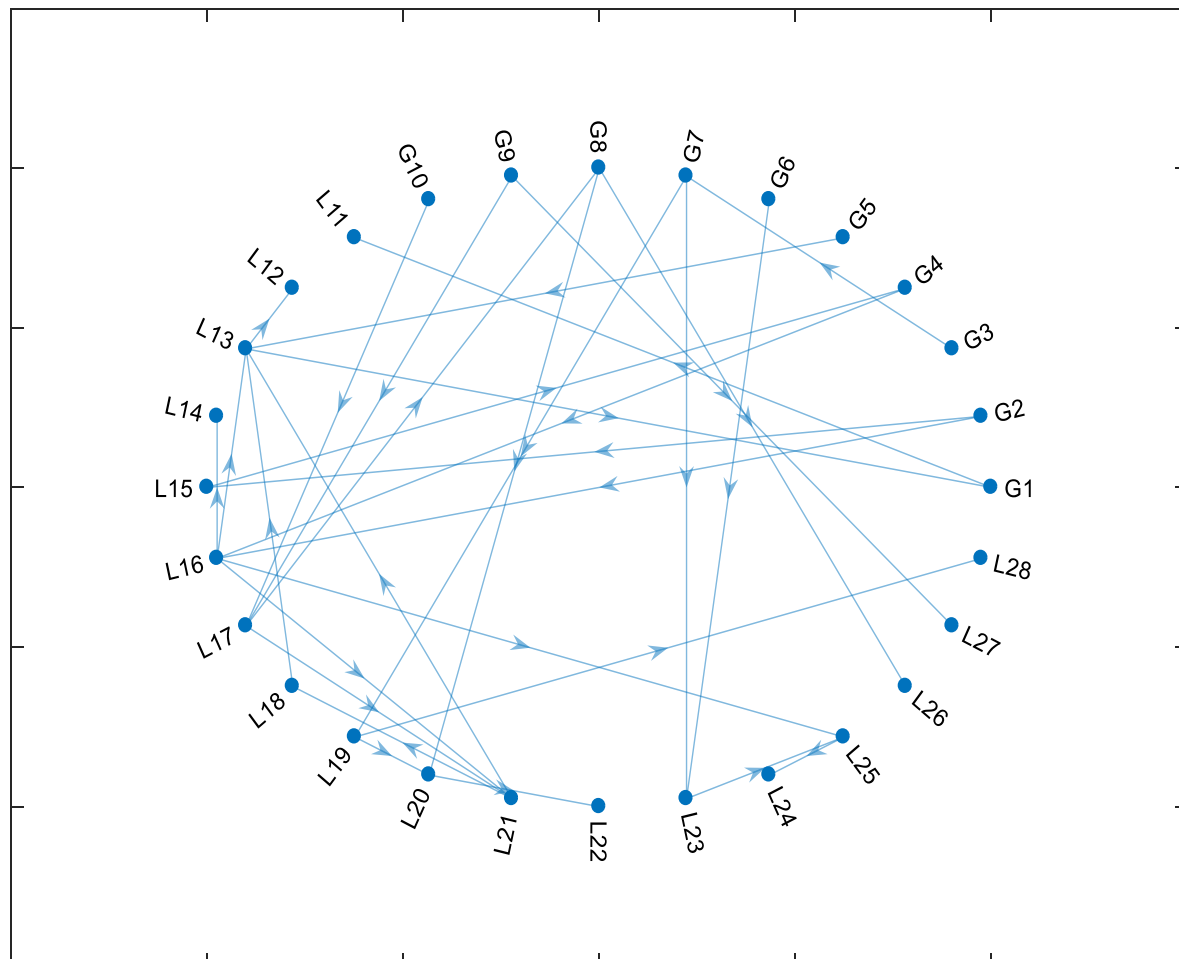


Figure appendix B: The directed graphical model of the Nigerian 28-bus network