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Computational Insights into Entropy and Neighborhood Indices of Rectangular and Circumcoronene Bilayer Germanium Phosphide

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***Abstract*—** Molecular descriptors known as topological indices include important information about the chemical. By using these indices, researchers can compute numerical values that are correlated with different chemical and physical properties, enabling the investigation and forecasting of the behaviours and characteristics of the substances. We use entropy calculations and certain neighborhood topological indices to examine the rectangular and circumcoronene bilayers germanium phosphide in this work. Insightful trends and patterns are shown by our numerical analysis and graphical representations, offering important details on the reliability of these indices in forecasting these chemical structures potential for the manufacture of sustainable materials. This paper shows how entropy calculations and topological indices can speed up the search for new materials and lead to the development of cutting-edge, environmentally friendly products.

***Keywords*—**Bilayer Germanium Phosphide; Entropy; Topological Indices; Materials Science; Semiconductor Applications.

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1. INTRODUCTION

Within mathematical chemistry, the discipline of chemical graph theory applies graph theory to the modeling and analysis of chemical structures or networks. This method uses a graph to illustrate the structure of a chemical molecule, with atoms shown as points (nodes) and the bonds that connect them as lines (edges). Through the use of graph theory ideas, scientists can extract many numerical parameters from these graphical representations that are referred to as topological parameters. These parameters are graph invariants, which means that they do not change based on how the structure is shown visually Ali et al., [1].
While Baig et al., [2] computed various indices for DSL (dominating set of a line graph). In chemical graph theory, several topological characteristics are calculated to clarify physicochemical features, including configuration, enthalpy, and stability [3,4,5]. Various methods are used to derive these values; some rely on the number of edges that connect a vertex, while others measure the distance between a vertex and a particular point. In chemical graph theory, neighborhood indices are quantitative metrics that are used to describe the topological structure of molecules. They shed light on the symmetry and connectivity patterns of molecular graphs, which are essential for comprehending their characteristics and actions in a variety of applications. Here is a detailed introduction to some neighborhood indices: ABC_4 index, GA_5 index, and Sanskruti index [6,7]. Simon raj and George [10] investigated the physicochemical characteristics of dominating silicate networks. These factors have been the subject of several study articles, which have provided insightful information about various chemical structures, nanotubes, and networks, clarifying their physicochemical characteristics, including electron negativity and electron affinity. Liu et al., [11] calculated topological properties of concealed non-kekulean benzenoid hydrocarbon.

The ABC4 index is a neighborhood degree based index, this index has been utilized in the study of molecular descriptors and has shown applications in QSAR (Quantitative Structure-Activity Relationship) studies [8]. The GA_5 index, also known as the fifth Geometric-Arithmetic index, is another neighborhood degree based index used to quantify the molecular structure [9]. This index provides information about the symmetry and complexity of molecular graphs [12]. The Sanskruti index is a relatively newer neighborhood index in chemical graph theory. It incorporates topological features and has been applied in studies involving molecular descriptors and structure-property relationships [13].

$$N_u = \sum_{u \in \eta(v)} P_u \tag{1}$$

Where N_u represent open neighborhood degree of the vertex u.

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2. Preliminaries

Definition 2.1 In their 2010 paper, Ghorbani and Hosseinzadeh [17] presented the fourth iteration of the atomic-bond connectivity index (ABC_4) and fifth iteration of the geometric arithmeticindex (GA_5) for graph as follows,

$$ABC_4 = \sum_{u_i,v_i \in E(G)} \sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}, \tag{2}$$

$$GA_5 = \sum_{u_i,v_i \in E(G)} \frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}. \tag{3}$$

Definition 2.3 In 2017, Hosamani introduced the Sanskruti index [6] for a molecular graph G, denoted by S(G) and defined as,

$$S(G) = \sum_{u_i,v_i \in E(G)} \left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3 \tag{4}$$

Shannon first introduced the concept of entropy in his seminal work [13] in 1948. Entropy quantifies the unpredictability or uncertainty of information content within a system, often measured through the entropy of a probability distribution. Over time, this notion found applications in graph theory and chemical networks, offering insights into their structural characteristics. The utilization of graph entropies has seen a surge in various fields including biology, chemistry, ecology, and sociology. Within these domains, the degree of each atom holds significant importance. Both graph theory and network theory have extensively researched invariants, serving as functional information measures in scientific investigations. Also Zakir et al., computed Weighted Entropy Based on Topological Indices for Triazine-Based Dendrimer and worked on Exploring the topological aspects of the chemical structure of the nanotube [23,24].

In this article, we focus on exploring graph entropy measures applied to germanium phosphide and computed, the 4^{th} atomic bond connectivity entropy, the 5^{th} geometric arithmetic entropy, and the Sanskruti entropy, utilizing their respective indices. Our approach draws inspiration from the concept of entropy elucidated in Manzoor et al., [14].

3. Applications of Entropy

In the context of a bilayer germanium phosphide structure, graph entropy plays a pivotal role in analyzing the structural information inherent in the system. Specifically, distance-based entropy measures are instrumental in various aspects, ranging from mathematical analyses to

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investigations in biology, chemical graph theory, and organic chemistry, tailored to the unique characteristics of the germanium phosphide bilayer. Drawing on Shannon's entropy concept, the structural complexity of the bilayer structure is enhanced by incorporating topological indices. These indices serve as invaluable molecular descriptors, particularly pertinent in studies concerning the properties and behaviours of the germanium phosphide bilayer, such as its electronic structure, vibrational properties, and interlayer interactions [15].

Shannon's seminal work in 1948 laid the groundwork for modern information theory, which found early applications in linguistics and electrical engineering before branching out into biology and chemistry. Early studies, like those in 1953, [16] paved the way for applying information theory to address structural complexities in materials such as the bilayer germanium phosphide [20].

Further advancements in understanding the structural information content of the bilayer germanium phosphide emerged in 2004, leveraging Shannon's entropy formulas. Additionally, contributions from researchers like Rashevsky and Trucco in the 1950s [22] are pertinent to these applications, providing foundational insights that continue to inform contemporary analyses of the bilayer structure.

Graph entropy measures are widely employed in biology, computer science, and structural chemistry, and tailored applications in studying the bilayer germanium phosphide structure. These measures facilitate a comprehensive understanding of the system's properties and behaviours, aiding in tasks such as characterizing its electronic band structure, predicting its optical properties, and exploring its potential applications in nano-electronics and optoelectronics. Overall, the applications of graph entropy in the context of a bilayer germanium phosphide structure encompass a broad spectrum of analyses, ranging from fundamental structural characterization to predictive modeling and materials design, with implications for advancing various technological frontiers.

4. Neighborhood Degree-Based Entropy

In 2015, Chen et al., [24, 25] introduced the concept of entropy for an edge-weighted graph G denoted as $G = \left(V_G,; E_G; (u_i, v_j)\right)$

Here, $V_G$represents the set of vertices, E_G represents the set of edges, and (u_i, v_j) represents the weight associated with the edge (u_i, v_j). The entropy of an edge-weighted graph is defined as follows, Chen et al., (2021) introduced the probability density function of a simple connected graph G given by,

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$$P_{ij} = \frac{w(xy)}{\sum W(xy)} \tag{5}$$

Cao et al., (2017) introduced degree base entropy for any graph G which is denoted and defined as,

$$I(G, w) = -\sum P_{ij} \log(P_{ij}) \,. \tag{6}$$

Chen et al., (2014) proposed the definition of entropy of an edge-weighted graph.

$$ENT_{I(G)} = \log I(G) - \frac{1}{I(G)} log \left\{ \prod_{u_i, v_j \in E_G} \left[\frac{N_{u_i} + N_{v_j}}{N_{u_i} \times N_{v_j}} \right]^{\left[\frac{N_{u_i} + N_{v_j}}{N_{u_i} \times N_{v_j}}\right]} \right\}. \tag{7}$$

Where I(G) represents the value of the topological index used to calculate the entropy of graph G.Manzoor et al., (2020) introduced the following entropies by using the above formula.

Entropy with ABC_4 Index:

$$ENT_{ABC_4}(G) = log(ABC_4(G)) - \frac{1}{ABC_4(G)} log \left\{ \prod_{u_i, v_j \in E_G} \left[\sqrt{\frac{N_{u_i} + N_{v_j} - 2}{N_{u_i} N_{v_j}}} \right]^{\left[\sqrt{\frac{N_{u_i} + N_{v_j} - 2}{N_{u_i} N_{v_j}}}\right]} \right\},$$

(8)

Entropy with GA_5 Index:

$$ENT_{GA_5}(G) = \log(GA_5(G)) - \frac{1}{GA_5(G))} log \left\{ \prod_{u_i, v_j \in E_G} \left[\frac{2\sqrt{N_{u_i} N_{v_j}}}{N_{u_i} + N_{v_j}} \right] \right\}^{\left[\frac{2\sqrt{N_{u_i} N_{v_j}}}{N_{u_i} + N_{v_j}}\right]}, \tag{9}$$

Entropy with Sansakruti Index:

$$ENT_{S(G)} = \log(S(G)) - \frac{1}{S(G)} log \left\{ \prod_{u_i, v_j \in E_G} \left\{ \frac{N_{u_i} \times N_{v_j}}{N_{u_i} + N_{v_j} - 2} \right\}^{3\left\{\frac{N_{u_i} \times N_{v_j}}{N_{u_i} + N_{v_j} - 2}\right\}^3} \right\}$$

(10)

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5. Germanium Phosphide

The emergence of graphene signalled a breakthrough in the _field of two-dimensional (2D) crystals, leading to a broad spectrum of materials with critical electronic characteristics required for nano-electronics. Semiconductors like transition metal dichalcogenides (TMDCs) and Dirac semimetals like graphene, silicene, and germanene have found useful uses in nanotechnology, especially in optoelectronics, where the first applications have been reported based on TMDCs. Every element in Group 14 of the periodic table has semi-metallic characteristics. In electronics, combining Group 14 elements with Phosphorene, an allotropic form of Phosphorus (P), has produced positive outcomes. In 1970, a layered substance called Germanium Phosphide (GeP_3) with a stoichiometric of Phosphorene (P) and Germanene (a single-layer material composed of Germanium atoms, Ge) was initially identified. GePx is frequently found in three forms. Structure of $2LGeP_3$ This article uses geometrical forms such as triangles and rhombus, to illustrate the structure of GeP_3 in two distinct ways. designating as rectangular and circumcoronene, the corresponding forms of these formations. These structural forms are widely recognized in the field of nanotechnology. In the rectangular, $2LGeP_3^n$ configuration, the bilayer structure of germanium phosphide ($2LGeP_3$) is depicted with a rectangular shape. This configuration encompasses a total of $12n^2$ edges, where n represents the number of hexagons present in the base of the monolayer 1LGeP3 structure, with n being greater than 2.

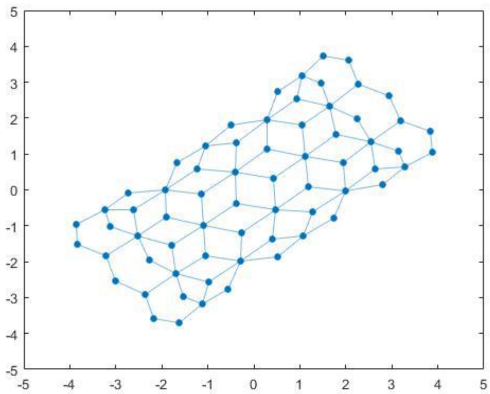

Figure 1: Rectangular Bilayer Germanium Phosphide for n = 4

Neighborhood Degree-Based Partition

Now, utilizing the definition (1) of N_u, we propose a distinct edge partition for the edge set of the rectangular bilayer Germanium Phosphide. This partition is based on the sum degree of neighborhood vertices associated with each edge. We have determined that for (n > 3), the

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edge partition based on neighborhood characteristics for the edge set of $(Rec2LGeP_3^n)$ comprises 12 sub-partite sets, as described below.

$$\eta_1 = (u,v)|\, N_u = 5 \wedge N_v = 6, \qquad \eta_2 = (u,v)|\, N_u = 5 \wedge N_v = 10$$

$$\eta_3 = (u,v)|\, N_u = 6 \wedge N_v = 10, \qquad \eta_4 = (u,v)|\, N_u = 6 \wedge N_v = 9$$

$$\eta_5 = (u,v)|\, N_u = 10 \wedge N_v = 9, \qquad \eta_6 = (u,v)|\, N_u = 10 \wedge N_v = 10$$

$$\eta_7 = (u,v)|\, N_u = 10 \wedge N_v = 16, \qquad \eta_8 = (u,v)|\, N_u = 12 \wedge N_v = 16$$

$$\eta_9 = (u,v)|\, N_u = 9 \wedge N_v = 16, \qquad \eta_{10} = (u,v)|\, N_u = 16 \wedge N_v = 16$$

$$\eta_{11} = (u,v)|\, N_u = 16 \wedge N_v = 18, \qquad \eta_{12} = (u,v)|\, N_u = 18 \wedge N_v = 18$$

with cardinalities as,

$|\eta_1| = |\eta_2| = |\eta_4| = |\eta_9| = 4, |\eta_3| = |\eta_6| = |\eta_8| = 4n - 8, |\eta_{10}| = 4n, |\eta_7| = 10n - 10, |\eta_5| = 8, |\eta_{11}| = 14n - 22 |\eta_{12}| = 12n^2 - 40n + 32$

Theorem: 5.1: For $n \geq 3$ the fourth version of the atomic bond connectivity index for $(Rec2LGeP_3^n)$ is given by $\boldsymbol{ABC_4(Rec2LGeP_3^n) = 3.887n^2 + 2.052n + 0.226}$.

Proof: By applying the neighborhood partition for rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ in Equation (2) we have,

$$ABC_4(Rec2LGeP_3^n) = \frac{2\times\sqrt{34}}{3}n^2 + \left[4\times\sqrt{\frac{14}{60}} + 4\times\sqrt{\frac{18}{100}} + 10\times\sqrt{\frac{24}{160}}\right.$$
$$+4\times\sqrt{\frac{26}{192}} + 4\times\sqrt{\frac{30}{256}} + 14\times\sqrt{\frac{32}{288}} - 40\times\left.\sqrt{\frac{34}{324}}\right]n + \left[4\times\sqrt{\frac{9}{30}}\right.$$
$$+4\times\sqrt{\frac{13}{50}} - 8\times\sqrt{\frac{14}{60}} + 4\times\sqrt{\frac{13}{54}} + 8\times\sqrt{\frac{17}{90}} - 8\times\sqrt{\frac{18}{100}}$$
$$\left.-10\times\sqrt{\frac{24}{160}} - 8\times\sqrt{\frac{26}{192}} + 4\times\sqrt{\frac{23}{144}} - 22\times\sqrt{\frac{32}{288}} + 32\times\sqrt{\frac{34}{324}}\right],$$

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after simplifying the above expression we have

$$\boldsymbol{ABC_4(Rec2LGeP_3^n) = 3.887n^2 + 2.052n + 0.226.}$$

Theorem: 5.2: For $n \geq 3$ the fifth version of geometric index GA_5 for rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ is given by,

$$\boldsymbol{GA_5(Rec2LGeP_3^n = 12n^2 - 0.46220n + 23.4800.}$$

Proof: By applying the neighborhood partition for rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ in Equation (3) we have,

$$GA_5(Rec2LGeP_3^n) = 12n^2 + \left[\frac{8\times\sqrt{60}}{16} + \frac{8\times\sqrt{100}}{20} + \frac{20\times\sqrt{160}}{26} + \frac{8\times\sqrt{192}}{28}\right.$$
$$\left. + \frac{8\times\sqrt{256}}{32} + \frac{28\times\sqrt{288}}{34} - \frac{80\times\sqrt{324}}{36}\right]n + \left[\frac{8\times\sqrt{30}}{11} + \frac{8\times\sqrt{50}}{15}\right.$$
$$- \frac{16\times\sqrt{60}}{26} + \frac{8\times\sqrt{54}}{15} + \frac{16\times\sqrt{90}}{19} - \frac{16\times\sqrt{100}}{20} - \frac{20\times\sqrt{160}}{26}$$
$$\left. - \frac{16\times\sqrt{192}}{28} + \frac{8\times\sqrt{144}}{25} - \frac{44\times\sqrt{288}}{34} + \frac{64\times\sqrt{324}}{36}\right],$$

In simplifying the above expression we have ,

$$\boldsymbol{GA_5(Rec2LGeP_3^n = 12n^2 - 0.46220n + 23.4800}\,.$$

Theorem: 5.3: For $n \geq 3$ the sansakruti index for rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ is given by,

$$\boldsymbol{S(Rec2LGeP_3^n = 10384.35n^2 - 10893.97n + 22138.697.}$$

Proof: By applying the neighborhood partition for rectangular bilayer germanium phosphide in Equation (4) we have,

$$S(Rec2LGeP_3^n) = 4\times\left(\frac{30}{9}\right)^3 + (4n-8)\left(\frac{60}{14}\right)^3 + 4\times\left(\frac{50}{13}\right)^3 + 4\times\left(\frac{54}{13}\right)^3 + 8\times\left(\frac{90}{17}\right)^3$$

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$$+(4n-8)\left(\frac{100}{18}\right)^3+(10n-10)\left(\frac{160}{24}\right)^3+(4n-8)\left(\frac{192}{26}\right)^3+4\times\left(\frac{144}{23}\right)^3$$

$$+4n\left(\frac{256}{30}\right)^3+(14n-22)\left(\frac{288}{32}\right)^3+(12n^2-40n+32)\left(\frac{324}{34}\right),^3$$

In simplifying the above expression we have,

$$\boldsymbol{S(Rec2LGeP_3^n = 10384.35n^2 - 10893.97n + 22138.697.}$$

Theorem: 5.4: For $n \geq 3$ the entropy of rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ with ABC_4 index is given by,

$$\boldsymbol{ENT_{ABC4}(Rec2LGeP_3^n) = \log(3.887n^2 + 2.052n + 0.226)}$$

$$\boldsymbol{-\frac{1}{3.887n^2+2.052n+0.226}[0.191n^2 + 0.434n - 0.063].}$$

Proof: By using the neighborhood partition for rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ in Equation (8) we have,

$$ENT_{ABC4}(Rec2LGeP_3^n)$$

$$=\left[\log(ABC_4)-\frac{1}{(ABC_4)}\right]log\left[\prod_{N(5,6)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}\right.$$

$$+\prod_{(5,10)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(6,10)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(6,9)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(10,9)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

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$$+\prod_{N(10,10)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(10,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(12,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(9,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(16,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(16,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$\left.+\prod_{N(18,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}\right],$$

$$ENT_{ABC4}(Rec2LGeP_3^n)=\left[\log(ABC_4)-\frac{1}{(ABC_4)}\right]log\left[\left(4\times\sqrt{\frac{9}{30}}\right)^{\sqrt{\frac{9}{30}}}\right.$$

$$\times\left(4\times\sqrt{\frac{13}{50}}\right)^{\sqrt{\frac{13}{50}}}\times\left((4n-8)\times\sqrt{\frac{14}{60}}\right)^{\sqrt{\frac{14}{60}}}\times\left(4\times\sqrt{\frac{13}{54}}\right)^{\sqrt{\frac{13}{54}}}$$

$$\times\left(4\times\sqrt{\frac{23}{144}}\right)^{\sqrt{\frac{23}{144}}}\times\left(8\times\sqrt{\frac{17}{90}}\right)^{\sqrt{\frac{17}{90}}}\times\left((4n-8)\times\sqrt{\frac{18}{100}}\right)^{\sqrt{\frac{18}{100}}}$$

$$\times\left((10n-10)\times\sqrt{\frac{24}{160}}\right)^{\sqrt{\frac{24}{160}}}\times\left((4n-8)\sqrt{\frac{26}{192}}\right)^{\sqrt{\frac{26}{192}}}\times\left(4n\times\sqrt{\frac{30}{256}}\right)^{\sqrt{\frac{30}{256}}}$$

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$$\times\left((14n-22)\times\sqrt{\frac{32}{288}}\right)^{\sqrt{\frac{32}{288}}}\times\left((12n^2-40n+32)\times\sqrt{\frac{34}{324}}\right)^{\sqrt{\frac{34}{324}}}$$

On putting the value of ABC_4 index for rectangular bilayer germanium phosphide $Rec2LGeP_3^n$ and simplifying the above expression we have entropy of rectangular bilayer germanium phosphide with ABC_4 index as,

$$\boldsymbol{ENT_{ABC4}(Rec2LGeP_3^n)} = \log \boldsymbol{3.887n^2 + 2.052n + 0.226}$$

$$-\frac{1}{\boldsymbol{3.887n^2+2.052n+0.226}}\left[\boldsymbol{0.191n^2 \quad + 0.434n - 0.063}\right]\blacksquare$$

Theorem: 5.5: For $n \geq 3$ the entropy of rectangular bilayer germanium phosphide

($RecLGeP_3^n$) ith GA_5 index is given by,

$$\boldsymbol{ENT_{GA_5}(Rec2LGeP_3^n) = \log(12n^2 - 0.46220n + 23.4800)}$$

$$-\frac{\boldsymbol{1}}{\boldsymbol{12n^2-0.46220n+23.4800}}\left[\boldsymbol{1.0791n^2+3.0432n-0.2642}\right].$$

Proof: By using the neighborhood partition for rectangular bilayer germanium phosphide in Equation (9) we have,

$$ENT_{GA_5}(Rec2LGeP_3^n) = \log(GA_5) - \frac{1}{GA_5}\log\left[\prod_{N(5,6)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}\right.$$

$$+\prod_{(5,10)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(6,10)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(6,9)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(10,9)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

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$$+\prod_{N(10,10)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(10,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(12,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(9,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(16,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(16,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$\left.+\prod_{N(18,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}\right],$$

$$ENT_{GA_5}(Rec2LGeP_3^n)$$

$$=\log(GA_5)-\frac{1}{GA_5}log\left[\left(4\times\frac{2\sqrt{30}}{11}\right)^{\frac{2\sqrt{30}}{11}}\times\left(4n\times\frac{2\sqrt{256}}{32}\right)^{\frac{2\sqrt{256}}{32}}\right.$$

$$\times\left(4\times\frac{2\sqrt{50}}{15}\right)^{\frac{2\sqrt{50}}{15}}\times\left((4n-8)\times\frac{2\sqrt{60}}{16}\right)^{\frac{2\sqrt{60}}{16}}\times\left(4\times\frac{2\sqrt{54}}{15}\right)^{\frac{2\sqrt{54}}{15}}$$

$$\times\left(4\times\frac{2\sqrt{144}}{25}\right)^{\frac{2\sqrt{144}}{25}}\times\left(8\times\frac{2\sqrt{90}}{19}\right)^{\frac{2\sqrt{90}}{19}}\times\left((4n-8)\times\frac{2\sqrt{100}}{20}\right)^{\frac{2\sqrt{100}}{20}}$$

$$\times\left((10n-10)\times\frac{2\sqrt{160}}{26}\right)^{\frac{2\sqrt{160}}{26}}\times\left((4n-8)\times\frac{2\sqrt{192}}{28}\right)^{\frac{2\sqrt{192}}{28}}$$

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$$\times\left((14n-22)\times\frac{2\sqrt{288}}{34}\right)^{\frac{2\sqrt{288}}{34}}\times\left((12n^2-40n+32)\times\frac{2\sqrt{324}}{36}\right)^{\frac{2\sqrt{324}}{36}},$$

In simplifying the above expression we have

$$\boldsymbol{ENT_{GA_5}(Rec2LGeP_3^n)=\log(12n^2-0.46220n+23.4800)}$$

$$\boldsymbol{-\frac{1}{12n^2-0.46220n+23.4800}[1.0791n^2+3.0432n-0.2642]}\ \blacksquare$$

Theorem: 5.6: For $n \geq 3$ the entropy of rectangular bilayer germanium phosphide $(Rec2LGeP_3^n)$ with Sansakruti index is given by,

$$\boldsymbol{ENT_S(Rec2LGeP_3^n)=\log(10384.35n^2-10893.97n+22138.697)-\frac{1}{10384.35n^2-10893.97n+22138.697}[3476n^2+832n+82]}$$

Proof: By using the neighborhood partition for rectangular bilayer germanium phosphide in Equation (8) we have,

$$ENT_S(Rec2LGeP_3^n)=\log(S)-\frac{1}{S}\log\left[\prod_{N(5,6)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}\right.$$

$$+\prod_{(5,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(6,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(6,9)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(10,9)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

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$$+\prod_{N(10,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(10,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(12,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(9,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(16,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(16,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$\left.+\prod_{N(18,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}\right],$$

$$ENT_{GA_5}(Rec2LGeP_3^n)=\log(GA_5)-\frac{1}{GA_5}\left[\left(4\times\left(\frac{30}{9}\right)^3\right)^{\left(\frac{30}{9}\right)^3}\times\left(4\times\left(\frac{50}{13}\right)^3\right)^{\left(\frac{50}{13}\right)^3}\right.$$

$$\times\left((4n-8)\times\left(\frac{60}{14}\right)^3\right)^{\left(\frac{60}{14}\right)^3}\times\left(4\times\left(\frac{54}{13}\right)^3\right)^{\left(\frac{54}{13}\right)^3}\times\left(4\times\left(\frac{144}{23}\right)^3\right)^{\left(\frac{144}{23}\right)^3}$$

$$\times\left(8\times\left(\frac{90}{17}\right)^3\right)^{\left(\frac{90}{17}\right)^3}\times\left((4n-8)\times\left(\frac{100}{18}\right)^3\right)^{\left(\frac{100}{18}\right)^3}\times\left((4n-8)\left(\frac{192}{26}\right)^3\right)^{\left(\frac{192}{26}\right)^3}$$

$$\times\left((10n-10)\times\left(\frac{160}{24}\right)^3\right)^{\left(\frac{160}{24}\right)^3}\times\left(4n\times\left(\frac{256}{30}\right)^3\right)^{\left(\frac{256}{30}\right)^3}$$

$$\left.\times\left((14n-22)\times\left(\frac{288}{32}\right)^3\right)^{\left(\frac{288}{32}\right)^3}\times\left((12n^2-40n+32)\times\left(\frac{324}{34}\right)^3\right)^{\left(\frac{324}{34}\right)^3}\right],$$

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In simplifying the above expression we have

$$\boldsymbol{ENT_S(Rec2LGeP_3^n) = \log(10384.35n^2 - 10893.97n + 22138.697)}$$

$$\boldsymbol{-\frac{1}{10384.35n^2 - 10893.97n + 22138.697}[3476n^2 + 832n + 82]} \blacksquare$$

6. Circumcoronene Bilayer Germanium Phosphide

In circumcoronene $(C2LGeP_3^n)$ configuration, the bilayer structure of Germanium Phosphide is depicted with the influence of concentric circles. This configuration encompasses a total of $(18n^2 - 6n - 2)$ edges, where (n) represents the count of hexagons on either side of the monolayer $1LGeF$

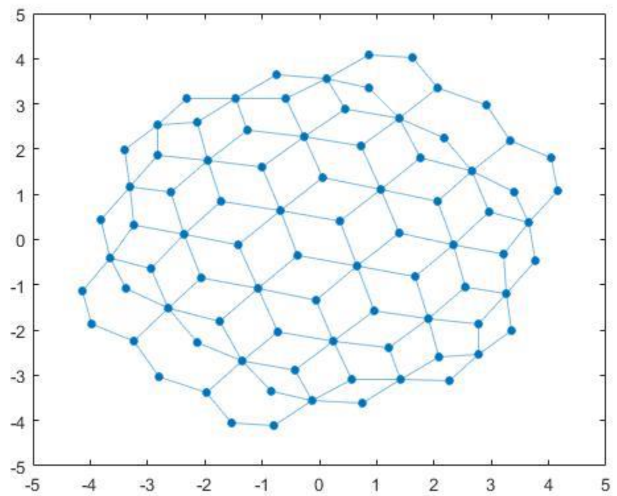

Figure 2: Circumcoronene Bilayer Germanium Phosphide C2LGePn for n = 4

Neighborhood Partition Now, utilizing the definition (1) of N_u we propose another distinct edge partition for the edge set of the circumcoronene bilayer Germanium Phosphide. This partition is based on the sum degree of neighborhood vertices associated with each edge. We have determined that for $n \geq 3$, the edge partition is based on neighborhood.

$$\eta_1 = (u,v)|\, N_u = 5 \wedge N_v 7, \qquad \eta_2 = (u,v)|\, N_u = 5 \wedge N_v = 10$$

$$\eta_3 = (u,v)|\, N_u = 6 \wedge N_v = 10, \qquad \eta_4 = (u,v)|\, N_u = 9 \wedge N_v = 10$$

$$\eta_5 = (u,v)|\, N_u = 7 \wedge N_v = 12, \qquad \eta_6 = (u,v)|\, N_u = 11 \wedge N_v = 12$$

$$\eta_7 = (u,v)|\, N_u = 10 \wedge N_v = 12, \qquad \eta_8 = (u,v)|\, N_u = 10 \wedge N_v = 13$$

$$\eta_9 = (u,v)|\, N_u = 9 \wedge N_v = 13, \qquad \eta_{10} = (u,v)|\, N_u = 11 \wedge N_v = 16$$

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$$\eta_{11} = (u,v)|\ N_u = 12\ \wedge N_v = 16, \qquad \eta_{12} = (u,v)|\ N_u = 10\ \wedge N_v = 15$$

$$\eta_{13} = (u,v)|\ N_u = 12\ \wedge N_v = 17, \qquad \eta_{14} = (u,v)|\ N_u = 13\ \wedge N_v = 16$$

$$\eta_{15} = (u,v)|\ N_u = 13\ \wedge N_v = 17, \qquad \eta_{16} = (u,v)|\ N_u = 13\ \wedge N_v = 15$$

$$\eta_{17} = (u,v)|\ N_u = 15\ \wedge N_v = 18, \qquad \eta_{18} = (u,v)|\ N_u = 17\ \wedge N_v = 18$$

$$\eta_{19} = (u,v)|\ N_u = 18\ \wedge N_v = 18, \qquad \eta_{20} = (u,v)|\ N_u = 16\ \wedge N_v = 17$$

$$\eta_{21} = (u,v)|\ N_u = 10\ \wedge N_v = 16, \qquad \eta_{22} = (u,v)|\ N_u = 16\ \wedge N_v = 18,$$

with cardinalities as, $|\eta_1| = |\eta_2| = |\eta_4| = |\eta_5| = |\eta_6| = |\eta_7| = |\eta_9| = |\eta_{10}| = |\eta_{12}| = |\eta_{13}| = |\eta_{16}| = |\eta_{17}| = |\eta_{20}| = 4$, $|\eta_3| = \eta_{15}| = 4n-8$, $|\eta_8| = |\eta_{14}| = 8n-20$, $|\eta_{11}| = 4n-4$ $|\eta_{18}| = 8n-12$, $|\eta_{21}| = 2n-2$, $|\eta_{22}| = 10n-18$, $|\eta_{19}| = 18n^2-54n+40$

Theorem: 6.1: For $n \geq 3$ the fourth version of atomic bond connectivity index circumcoronene bilayer germanium phosphide $(C2LGeP_3^n)$ is given by

$$\boldsymbol{ABC_4(C2LGeP_3^n) = 5.830n^2 + 1.991n - 0.498}$$

Proof: By applying the neighborhood partition for Circumcoronene bilayer germanium phosphide in Equation (2) we have,

$$ABC_4(C2LGeP_3^n) = (\sqrt{34})n^2 + \left(4\times\sqrt{\frac{14}{60}} + 8\times\sqrt{\frac{21}{130}} + 4\times\sqrt{\frac{26}{192}} + 8\times\sqrt{\frac{27}{208}}\right.$$

$$\left. +4\times\sqrt{\frac{28}{221}} + 8\times\sqrt{\frac{33}{306}} - 54\times\sqrt{\frac{34}{324}} + 2\times\sqrt{\frac{29}{160}} + 10\times\sqrt{\frac{32}{288}}\right)n$$

$$+4\times\sqrt{\frac{10}{35}} + 4\times\sqrt{\frac{13}{50}} - 8\times\sqrt{\frac{14}{60}} + 4\times\sqrt{\frac{17}{90}} + 4\times\sqrt{\frac{17}{84}} + 4\times\sqrt{\frac{21}{132}}$$

$$+4\times\sqrt{\frac{20}{120}} - 20\times\sqrt{\frac{21}{130}} + 4\times\sqrt{\frac{20}{117}} + 4\times\sqrt{\frac{25}{176}} - 4\times\sqrt{\frac{26}{192}}$$

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$$+4 \times \sqrt{\frac{23}{150}} + 4 \times \sqrt{\frac{27}{204}} - 20 \times \sqrt{\frac{27}{208}} - 8 \times \sqrt{\frac{28}{221}} + 4 \times \sqrt{\frac{26}{195}}$$

$$+4 \times \sqrt{\frac{31}{270}} - 12 \times \sqrt{\frac{33}{306}} + 40 \times \sqrt{\frac{34}{324}} + 4 \times \sqrt{\frac{31}{272}} - 2 \times \sqrt{\frac{24}{160}}$$

$$\left. -18 \times \sqrt{\frac{32}{288}} \right),$$

In simplifying the above expression we have

$$\boldsymbol{ABC_4(C2LGeP_3^n) = 5.830n^2 + 1.991n - 0.498.}$$

Theorem: 6.2: For $n \geq 3$ the fifth version of geometric index GA_5 for $(C2LGeP_3^n)$ is given by

$$\boldsymbol{GA_5(C2LGeP_3^n) = 18n^2 + 1.293n - 0.551.}$$

Proof: By applying the neighborhood partition for rectangular bilayer germanium phosphide in Equation (3) we have,

$$GA_5(C2LGeP_3^n) = 18n^2 + \left(\frac{8 \times \sqrt{60}}{16} + \frac{16 \times \sqrt{130}}{23} + \frac{8 \times \sqrt{192}}{28} + \frac{16 \times \sqrt{208}}{29} + \frac{24\sqrt{34}}{35} \right.$$

$$+ \frac{8 \times \sqrt{221}}{30} + \frac{16 \times \sqrt{306}}{35} - \frac{108 \times \sqrt{324}}{36} + \frac{8 \times \sqrt{272}}{33} + \frac{4 \times \sqrt{160}}{31}$$

$$\left. + \frac{20 \times \sqrt{288}}{34} \right) n + \left(\frac{8 \times \sqrt{35}}{12} + \frac{8 \times \sqrt{50}}{15} - \frac{16 \times \sqrt{60}}{16} + \frac{8 \times \sqrt{90}}{19} \right.$$

$$+ \frac{8 \times \sqrt{84}}{19} + \frac{8 \times \sqrt{132}}{23} + \frac{8 \times \sqrt{120}}{22} - \frac{40 \times \sqrt{130}}{23} + \frac{8 \times \sqrt{117}}{22}$$

$$+ \frac{8 \times \sqrt{176}}{27} - \frac{8 \times \sqrt{192}}{28} + \frac{8 \times \sqrt{150}}{25} + \frac{8 \times \sqrt{204}}{29} - \frac{40 \times \sqrt{208}}{29}$$

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$$-\frac{16\times\sqrt{221}}{30}+\frac{8\times\sqrt{195}}{28}+\frac{8\times\sqrt{270}}{33}-\frac{24\times\sqrt{306}}{35}+\frac{80\times\sqrt{324}}{36}$$
$$+\frac{8\times\sqrt{272}}{33}-\frac{4\times\sqrt{160}}{31}-\frac{36\times\sqrt{288}}{34}\Bigg),$$

On simplifying the above expression we have the value of GA_5 index for Circumcoronene bilayer germanium phosphide as,

$$\boldsymbol{GA_5(C2LGeP_3^n) = 18n^2 + 1.293n - 0.551.}$$

Theorem: 6.3: For $n \geq 3$ the sansakruti index $S(G)$ for $C2LGeP_3^n$ is given by,

$$\boldsymbol{S(C2LGeP_3^n = 15576.75n^2 - 23592.24n + 6420.}$$.

Proof: By applying the neighborhood partition for Circumcoronene bilayer germanium phosphide in Equation (4) we have,

$$S(C2LGeP_3^n) = 18\times\left(\frac{324}{34}\right)^3 n^2 + \Bigg(4\times\left(\frac{14}{60}\right)^3 + 8\times\left(\frac{130}{21}\right)^3 + 4\times\left(\frac{192}{26}\right)^3 + 8\times$$
$$\left(\frac{208}{27}\right)^3 + 4\times\left(\frac{221}{28}\right)^3 + 8\times\left(\frac{306}{33}\right)^3 - 54\times\left(\frac{324}{34}\right)^3 + 2\times\left(\frac{160}{29}\right)^3$$
$$+10\times\left(\frac{288}{32}\right)^3\Bigg)n + \Bigg(4\times\left(\frac{35}{10}\right)^3 + 4\times\left(\frac{50}{13}\right)^3 - 8\times\left(\frac{60}{14}\right)^3 + 4\times\left(\frac{90}{17}\right)^3$$
$$+4\times\left(\frac{84}{17}\right)^3 + 4\times\left(\frac{132}{21}\right)^3 + 4\times\left(\frac{120}{20}\right)^3 - 20\times\left(\frac{130}{21}\right)^3 + 4\times\left(\frac{117}{20}\right)^3$$
$$+4\times\left(\frac{176}{25}\right)^3 - 4\times\left(\frac{192}{26}\right)^3 + 4\times\left(\frac{150}{23}\right)^3 + 4\times\left(\frac{204}{27}\right)^3 - 20\times\left(\frac{208}{27}\right)^3$$
$$-8\times\left(\frac{221}{28}\right)^3 + 4\times\left(\frac{195}{26}\right)^3 + 4\times\left(\frac{270}{31}\right)^3 - 12\times\left(\frac{306}{33}\right)^3 + 40\times\left(\frac{324}{34}\right)^3$$
$$+4\times\left(\frac{272}{33}\right)^3 - 2\times\left(\frac{160}{29}\right)^3 - 18\times\left(\frac{288}{32}\right)^3\Bigg),$$

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In simplifying the above expression we have

$$\boldsymbol{S(C2LGeP_3^n = 15576.75 - 23592.24n + 6420.}$$

Theorem: 6.4: For $n \geq 3$ the entropy of Circumcoronene bilayer germanium phosphide

$(\,C2LGeP_3^n)$ with ABC_4 index is given by,

$$\boldsymbol{ENT_{ABC4}(C2LGeP_3^n) = \log(5.830n^2 + 1.991n - 0.498) - \frac{1}{5.830n^2 + 1.991n - 0.498} \times [0.248n^2 + 0.5308n + 0.1320].}$$

Proof: By using the neighborhood partition for circumcoronene bilayer germanium phosphide in Equation (8) we have,

$$ENT_{ABC4}(C2LGeP_3^n) = \left[\log(ABC_4) - \frac{1}{(ABC_4)}\right] log \left[\prod_{N(5,7)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{(5,10)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{N(6,10)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{N(9,10)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{N(7,12)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{N(11,12)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}} + \prod_{N(10,12)} \left(\sqrt{\frac{N_{u_i} + N_{v_i} - 2}{N_{u_i} \times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

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$$+\prod_{N(10,13)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(9,13)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(11,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(12,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(10,15)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(12,17)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$,+\prod_{N(13,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(13,17)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(13,15)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(15,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(17,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(18,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$+\prod_{N(16,17)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}+\prod_{N(10,16)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}$$

$$\left.+\prod_{N(16,18)}\left(\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}\right)^{\sqrt{\frac{N_{u_i}+N_{v_i}-2}{N_{u_i}\times N_{v_i}}}}\right],$$

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$$
\begin{aligned}
ENT_{ABC4}(C2LGeP_3^n) = {} & \left[\log(ABC_4) - \frac{1}{(ABC_4)}\right] log\left[\left(4 \times \sqrt{\frac{10}{35}}\right)^{\sqrt{\frac{10}{35}}} \times \left(4 \times \sqrt{\frac{13}{60}}\right)^{\sqrt{\frac{13}{60}}}\right. \\
& \times \left((4n-8) \times \sqrt{\frac{14}{60}}\right)^{\sqrt{\frac{14}{60}}} \times \left(4 \times \sqrt{\frac{17}{90}}\right)^{\sqrt{\frac{17}{90}}} \left(4 \times \sqrt{\frac{17}{84}}\right)^{\sqrt{\frac{17}{84}}} \\
& \times \left(4 \times \sqrt{\frac{21}{132}}\right)^{\sqrt{\frac{21}{132}}} \times \left(4 \times \sqrt{\frac{20}{120}}\right)^{\sqrt{\frac{20}{120}}} \times \left((4n-20) \times \sqrt{\frac{21}{130}}\right)^{\sqrt{\frac{21}{130}}} \\
& \times \left(4 \times \sqrt{\frac{20}{117}}\right)^{\sqrt{\frac{20}{117}}} \times \left((4n-4) \times \sqrt{\frac{26}{192}}\right)^{\sqrt{\frac{26}{192}}} \times \left(4 \times \sqrt{\frac{25}{176}}\right)^{\sqrt{\frac{25}{176}}} \\
& \times \left(4 \times \sqrt{\frac{23}{150}}\right)^{\sqrt{\frac{23}{150}}} \times \left(4 \times \sqrt{\frac{27}{204}}\right)^{\sqrt{\frac{27}{204}}} \times \left((8n-20) \times \sqrt{\frac{27}{208}}\right)^{\sqrt{\frac{27}{208}}} \\
& \times \left(4 \times \sqrt{\frac{26}{195}}\right)^{\sqrt{\frac{26}{195}}} \times \left((4n-8) \times \sqrt{\frac{28}{221}}\right)^{\sqrt{\frac{28}{218}}} \times \left(4 \times \sqrt{\frac{31}{270}}\right)^{\sqrt{\frac{31}{270}}} \\
& \times \left((8n-12) \times \sqrt{\frac{33}{306}}\right)^{\sqrt{\frac{34}{306}}} \times \left((18n^2-54n+40) \times \sqrt{\frac{34}{324}}\right)^{\sqrt{\frac{34}{324}}} \\
& \left. \times \left(4 \times \sqrt{\frac{31}{172}}\right)^{\sqrt{\frac{31}{172}}} \times \left(4 \times \sqrt{\frac{29}{160}}\right)^{\sqrt{\frac{29}{160}}} \times \left((10n-18) \times \sqrt{\frac{32}{288}}\right)^{\sqrt{\frac{32}{288}}}\right],
\end{aligned}
$$

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On putting the value of ABC_4 index for circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ and simplifying the above expression we have entropy of Circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ with ABC_4 index as,

$$\boldsymbol{ENT_{ABC4}(C2LGeP_3^n)} = \mathbf{log(5.830n^2 + 1.991n - 0.498) - \frac{1}{5.830n^2 + 1.991n - 0.498}}$$

$$\times [\mathbf{0.248n^2 + 0.5308n + 0.1320}] \blacksquare$$

Theorem: 6.5: For $n \geq 3$ the entropy of Circumcoronene bilayer germanium phosphide

($C2LGeP_3^n$) with ABC_4 index is given by

$$\boldsymbol{ENT_{GA_5}(C2LGeP_3^n)} = \mathbf{log(18n^2 + 1.293n - 0.551) - \frac{1}{18n^2 + 1.293n - 0.551}} \times$$

$$[\mathbf{1.2552n^2 + 3.9845n + 2.2596}]$$

Proof: By using the neighborhood partition for Circumcoronene bilayer germanium phosphide in Equation (6) we have

$$ENT_{GA_5}(C2LGeP_3^n) = \left[\log(GA_5) - \frac{1}{GA_5}\right] log \left[\prod_{N(5,7)} \left(\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i} + N_{v_i}} \right)^{\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i}+N_{v_i}}} \right.$$

$$+ \prod_{(5,10)} \left(\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i} + N_{v_i}} \right)^{\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i}+N_{v_i}}} + \prod_{N(6,10)} \left(\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i} + N_{v_i}} \right)^{\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+ \prod_{N(9,10)} \left(\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i} + N_{v_i}} \right)^{\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i}+N_{v_i}}} + \prod_{N(7,12)} \left(\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i} + N_{v_i}} \right)^{\frac{2\sqrt{N_{u_i} \times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

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$$+\prod_{N(11,12)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(10,12)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(10,13)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(9,13)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(11,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(12,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(10,15)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(12,17)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$,+\prod_{N(13,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(13,17)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(13,15)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(15,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(17,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(18,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

$$+\prod_{N(16,17)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}+\prod_{N(10,16)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}$$

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$$+\prod_{N(16,18)}\left(\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}\right)^{\frac{2\sqrt{N_{u_i}\times N_{v_i}}}{N_{u_i}+N_{v_i}}}\Bigg],$$

$$ENT_{GA_5}(C2LGeP_3^n)=\left[\log(GA_5)-\frac{1}{GA_5}\right]log\Bigg[\left(\frac{4\times 2\sqrt{35}}{12}\right)^{\frac{2\sqrt{35}}{12}}\times\left(\frac{4\times 2\sqrt{50}}{15}\right)^{\frac{2\sqrt{35}}{12}}$$

$$\times\left(\frac{(4n-8)\times 2\sqrt{60}}{16}\right)^{\frac{2\sqrt{60}}{16}}\times\left(\frac{4\times 2\sqrt{90}}{19}\right)^{\frac{2\sqrt{90}}{19}}\times\left(\frac{4\times 2\sqrt{84}}{19}\right)^{\frac{2\sqrt{84}}{19}}$$

$$\times\left(\frac{4\times 2\sqrt{132}}{23}\right)^{\frac{2\sqrt{132}}{23}}\times\left(\frac{4\times 2\sqrt{120}}{20}\right)^{\frac{2\sqrt{120}}{20}}\times\left(\frac{4\times 2\sqrt{117}}{22}\right)^{\frac{2\sqrt{117}}{22}}$$

$$\times\left(\frac{(8n-20)\times 2\sqrt{130}}{23}\right)^{\frac{2\sqrt{130}}{23}}\times\left(\frac{4\times 2\sqrt{176}}{27}\right)^{\frac{2\sqrt{176}}{27}}\times\left(\frac{4\times 2\sqrt{150}}{25}\right)^{\frac{2\sqrt{150}}{25}}$$

$$\times\left(\frac{(4n-4)\times 2\sqrt{192}}{28}\right)^{\frac{2\sqrt{192}}{28}}\times\left(\frac{4\times 2\sqrt{204}}{29}\right)^{\frac{2\sqrt{204}}{29}}\times\left(\frac{4\times 2\sqrt{195}}{28}\right)^{\frac{2\sqrt{195}}{28}}$$

$$\times\left(\frac{(8n-20)\times 2\sqrt{208}}{29}\right)^{\frac{2\sqrt{208}}{29}}\times\left(\frac{(4n-8)\times 2\sqrt{221}}{30}\right)^{\frac{2\sqrt{221}}{30}}$$

$$\times\left(\frac{4\times 2\sqrt{270}}{33}\right)^{\frac{2\sqrt{270}}{33}}\times\left(\frac{(8n-12)\times 2\sqrt{306}}{35}\right)^{\frac{2\sqrt{306}}{35}}\times\left(\frac{4\times 2\sqrt{272}}{33}\right)^{\frac{2\sqrt{272}}{33}}$$

$$\times\left(\frac{(2n-2)\times 2\sqrt{160}}{26}\right)^{\frac{2\sqrt{160}}{26}}\times\left(\frac{(10n-18)\times 2\sqrt{288}}{34}\right)^{\frac{2\sqrt{288}}{34}}$$

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$$\times\left(\frac{(18n^2-54n+40)\times 2\sqrt{324}}{36}\right)^{\frac{2\sqrt{324}}{36}}\Bigg],$$

On putting the value of GA_5 index for Circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ and simplifying the above expression we have entropy of Circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ with GA_5 index as,

$$\boldsymbol{ENT_{GA_5}(C2LGeP_3^n)=\log(18n^2+1.293n-0.551)-\frac{1}{18n^2+1.293n-0.551}\times}$$
$$\boldsymbol{[1.2552n^2+3.9845n+2.2596]}\blacksquare$$

Theorem: 6.6: For $n\geq 3$ the entropy of Circumcoronene bilayer germanium phosphide

($C2LGeP_3^n$) with Sanskruti index is given by

$$\boldsymbol{ENT_S(C2LGeP_3^n)=\log(15577n^2-23592n+6420)-\frac{1}{15577n^2-23592n+6420}}$$
$$\times\boldsymbol{[3628n^2+1072n+1522]}.$$

Proof: By using the neighborhood partition for Circumcoronene bilayer germanium phosphide in Equation (8) we have

$$ENT_S(C2LGeP_3^n)=\left[\log(S)-\frac{1}{S}\right]log\left[\prod_{N(5,7)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}\right.$$

$$+\prod_{(5,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(6,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(9,10)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(7,12)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

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$$+\prod_{N(11,12)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(10,12)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(10,13)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(9,13)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(11,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(12,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(10,15)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(12,17)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$,+\prod_{N(13,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(13,17)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(13,15)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(15,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(17,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(18,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$+\prod_{N(16,17)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}+\prod_{N(10,16)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}$$

$$\left.+\prod_{N(16,18)}\left(\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3\right)^{\left(\frac{N_{u_i}\times N_{v_i}}{N_{u_i}+N_{v_i}-2}\right)^3}\right],$$

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$$ENT_S(C2LGeP_3^n) = \left[\log(S) - \frac{1}{S}\right] log \left[\left(4 \times \left(\frac{35}{10}\right)^3\right)^{\left(\frac{35}{10}\right)^3} \times \left(4 \times \left(\frac{50}{13}\right)^3\right)^{\left(\frac{50}{13}\right)^3} \right.$$

$$\times \left((4n-8) \times \left(\frac{60}{14}\right)^3\right)^{\left(\frac{60}{14}\right)^3} \times \left(4 \times \left(\frac{90}{17}\right)^3\right)^{\left(\frac{90}{17}\right)^3} \left(4 \times \left(\frac{84}{17}\right)^3\right)^{\left(\frac{84}{17}\right)^3}$$

$$\times \left(4 \times \left(\frac{132}{21}\right)^3\right)^{\left(\frac{132}{21}\right)^3} \times \left(4 \times \left(\frac{120}{20}\right)^3\right)^{\left(\frac{120}{20}\right)^3} \left((8n-20) \times \left(\frac{130}{21}\right)^3\right)^{\left(\frac{130}{21}\right)^3}$$

$$\times \left(4 \times \left(\frac{117}{20}\right)^3\right)^{\left(\frac{117}{20}\right)^3} \times \left(4 \times \left(\frac{176}{25}\right)^3\right)^{\left(\frac{176}{25}\right)^3} \times \left((4n-4) \times \left(\frac{192}{26}\right)^3\right)^{\left(\frac{192}{26}\right)^3}$$

$$\times \left(4 \times \left(\frac{150}{23}\right)^3\right)^{\left(\frac{150}{23}\right)^3} \times \left(4 \times \left(\frac{204}{27}\right)^3\right)^{\left(\frac{204}{27}\right)^3} \times \left((8n-20) \times \left(\frac{208}{27}\right)^3\right)^{\left(\frac{208}{27}\right)^3}$$

$$\times \left((4n-8) \times \left(\frac{221}{28}\right)^3\right)^{\left(\frac{221}{28}\right)^3} \times \left(4 \times \left(\frac{196}{26}\right)^3\right)^{\left(\frac{196}{26}\right)^3} \times \left(4 \times \left(\frac{272}{31}\right)^3\right)^{\left(\frac{272}{31}\right)^3}$$

$$\times \left((8n-12) \times \left(\frac{306}{33}\right)^3\right)^{\left(\frac{306}{33}\right)^3} \times \left((2n-2) \times \left(\frac{160}{24}\right)^3\right)^{\left(\frac{160}{24}\right)^3} \times \left(4 \times \left(\frac{272}{31}\right)^3\right)^{\left(\frac{272}{31}\right)^3}$$

$$\left. \times \left((18n^2-54n+40) \times \left(\frac{324}{34}\right)^3\right)^{\left(\frac{324}{34}\right)^3} \times \left((10n-18) \times \left(\frac{288}{32}\right)^3\right)^{\left(\frac{288}{32}\right)^3} \right],$$

On putting the value of the sanskruti index for Circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ and simplifying the above expression we have entropy of Circumcoronene bilayer germanium phosphide $C2LGeP_3^n$ with the sanskruti index as,

$$\boldsymbol{ENT_S(C2LGeP_3^n) = \log(15577n^2 - 23592n + 6420) - \frac{1}{15577n^2 - 23592n + 6420} \times [3628n^2 + 1072n + 1522]} \blacksquare$$

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7. Comparison of Topological Indices and Entropies

This section presents four succinct and informative tables. Tables 1 and 3 show the numerical values of topological indices (ABC_4,GA_5, and Sansakruti index) for rectangular and Circumcoronene bilayer germanium phosphide respectively, at different values of n. The values of these indices increase with increasing n, indicating a growth in the complexity and connectivity of the structures.

Tables 2 and 4 show the numerical values of entropies concerning these topological indices for both structures. The entropy values also increase with increasing n, suggesting a higher degree of disorder or randomness in the structures as n increases. The calculated topological indices and entropies for rectangular and circumcoronene bilayer germanium phosphide reveal a consistent trend of increasing values with increasing n. This suggests that both structures exhibit growing complexity, connectivity, and disorder as the number of layers (n) increases. The increasing entropy values indicate a higher degree of randomness in the structures, which may have implications for their physical and chemical properties.

n	ABC_4 Index	GA_5 Index	Sanskruti Index
3	41.338	130.093	82912.76
4	70.578	213.631	144706.79
5	107.586	321.169	227268.82
6	152.362	452.706	330598.85
7	204.906	608.244	454696.88
8	265.218	786.782	599562.91
9	333.298	991.320	765196.94
10	409.146	1218.858	951598.97

Table: 1 Variation of n in ABC_4, GA_5 & Sanskruti index for rectangular bilayer germanium phosphide $Rec2LGeP_3^n$.

n	$ENT(Rec2)_{ABC_4}$	$ENT(Rec2)_{GA_5}$	$ENT(Rec2)_S$
3	2.814	1.971	4.798
4	3.478	2.193	4.988
5	4.094	2.376	5.142
6	4.679	2.530	5.273
7	5.241	2.662	5.388
8	5.786	2.778	5.490
9	6.319	2.880	5.583

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10	6.842	2.973	6.669

Table: 2 Variation of n in entropies with ABC_4, GA_5 & Sanskruti index for rectangular bilayer germanium phosphide $Rec2LGeP_3^n$.

n	ABC_4 Index	GA_5 Index	Sanskruti Index
3	57.945	130.093	82915.24
4	100.746	213.631	144711.72
5	155.207	321.169	227276.9
6	221.328	452.706	330610.78
7	299.109	608.244	454713.36
8	388.55	787.782	599584.64
9	489.651	991.320	765224.62
10	604.412	1218.858	951633.3

Table: 3 Variation of n in ABC_4, GA_5 & Sanskruti index for circumcoronene bilayer germanium phosphide $C2LGeP_3^n$.

n	$ENT(C2)_{ABC_4}$	$ENT(C2)_{GA_5}$	$ENT(C2)_S$
3	1.694	2.064	4.386
4	1.941	2.335	4.811
5	2.133	2.541	5.092
6	2.289	2.707	5.303
7	2.422	2.846	5.472
8	2.537	2.966	5.613
9	2.638	3.071	5.734
10	2.400	3.165	5.840

Table: 4 Variation of n in entropies with ABC_4, GA_5 & Sanskruti index for circumcoronene bilayer germanium phosphide $C2LGeP_3^n$.

7. Graphical Representation:

This section features a meticulously crafted visual representation, designed for precision and clarity, to underscore our in-depth exploration of entropy via various topological indices.

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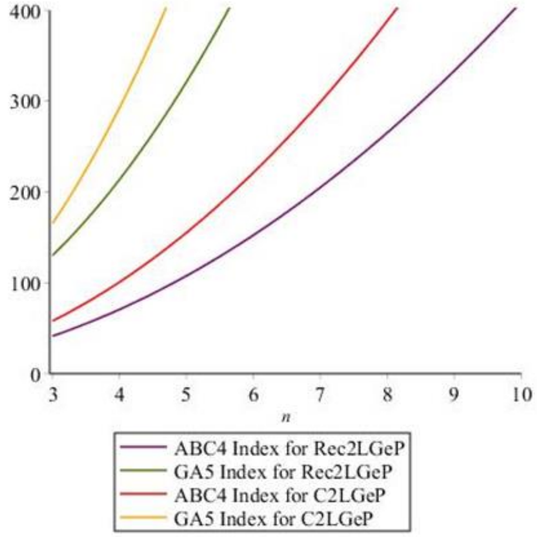

Figure: 3 The ABC_4 & GA_5 indices for $(Rec2LGeP_3^n)$ and $(C2LGeP_3^n)$

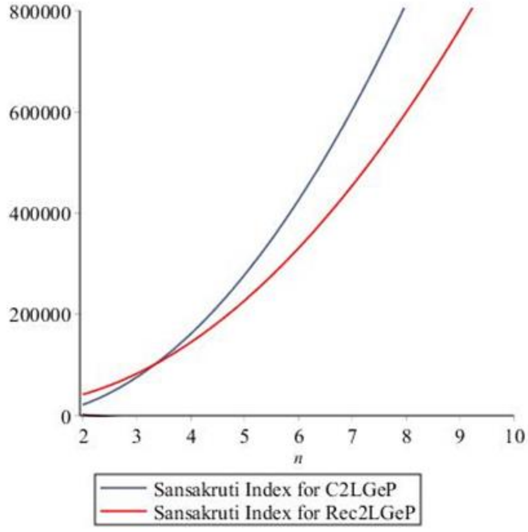

Figure: 4 The Sansakruit index for $(Rec2LGeP_3^n)$ &$(C2LGeP_3^n)$

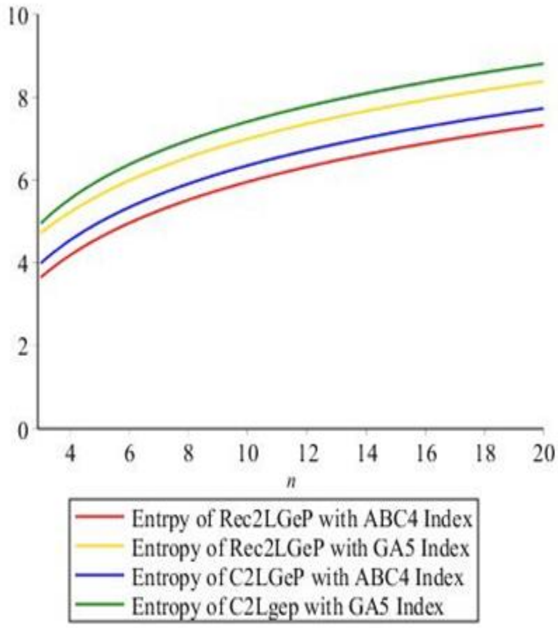

Figure: 5 presents visuals of the entropy with ABC_4 index, GA_5 index for for $T2LGeP_3^n$ & $(R2LGeP_3^n)$

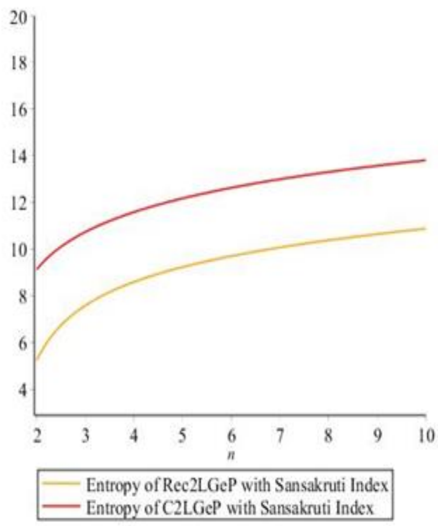

Figure: 6 presents visuals of the entropy with sansakruti index for $(T2LGeP_3^n)$ & $(R2LGeP_3^n)$

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Every data point and curve in these visuals stems from a thorough examination, reflecting the depth and insights garnered during our investigative journey. Our goal with these illustrations is to offer readers a clear and quick understanding of our comprehensive exploration of the intricacies of topological indices and entropy.

8. Exploring Future Directions and Final Thoughts

In conclusion, our comprehensive analysis of neighborhood degree-based topological indices and entropies has revealed insightful patterns and relationships. Our findings demonstrate the potential of these mathematical constructs in capturing the particulars of complex systems. The entropies associated with these indices further underscore the importance of considering both structural and informational aspects. We have studied the topological diversity in rectangular and circumcoronene phases of bilayer germanium phosphide and also its entropic nature. Also we have analyzed our finding through numerical tables and graphical representation. It has been observed that steadily increase in complexity make this material suitable for sustainable material productions.

Our work contributes to the ongoing effort to bridge the gap between mathematical theory and practical applications. Future research directions include investigating the applicability of these topological indices and entropies in materials science, nanotechnology, and other domains, as well as exploring the potential of machine learning algorithms in predicting properties and behavior based on these mathematical constructs. Additionally, extending this research to other neighborhood degree-based topological indices and entropies will further elucidate their relationships and implications. Developing new mathematical tools and techniques will also be crucial in better capturing the complexity of real-world systems. Finally, collaborating with experimentalists will be essential in validating theoretical predictions and exploring potential practical applications, paving the way for ground-breaking discoveries and innovations in various fields.

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