



Solving the Problem of Hub Position (Center) for the Comprehensive Supply Chain of Medical Goods under Conditions of Uncertainty with a Robust Approach Using Torabi-Hassini Method, Benders Decomposition Algorithm and Meta-Heuristic Algorithms

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Abstract: Environmental concerns have caused the attention of researchers to study in the field of green supply chain. In this study, the mathematical model is developed with different purpose functions. The target functions are: (a) The amount of coverage demand, (b) the cost due to changes in the structure of the hubs, (c) maximize the level of green hub location. For solving problems with low dimensions with the GAMS solver, the time to solve the problem is not much different from using Benders decomposition algorithm. Finally, to evaluate the model after designing the problem and generating random data, the designed problem was investigated using of MOPSO and NSGA II. The computational time obtained from solving the sample problem with the MOPSO algorithm is less than the NSGA II. Therefore, according to the conditions at each point of the network, facilities can have different characteristics.

Key words: Benders algorithm, Green supply chain, Hub, MOPSO and NSGAII

1 Introduction

The issue of location of activity centers is raised when we are faced with systems consisting of many departments (such as transportation system, etc.), and the need to transfer goods, information or people between two of these departments and since then, it has been noticed that some flow should be sent from a set of origin points to a set of destination points. But



this is if it is not possible to create a direct link between each pair of points due to its high cost. Therefore, a network was designed under the title of wheel and hub, so that the communication between them is done by a series of intermediary centers. Hubs are facilities that play the role of separation, integration and exchange of flows in a transportation system, in such a way that they play a significant role in improving the performance of the system in terms of economy and time budget (Mahmoodjanloo et al. 2020). The problem of locating the hub is one of the new concepts in the science of positioning, as the concept of the hub was first proposed by Hakimi. Also, the first mathematical model in this field was presented by Eckley for the first time. One of the important issues that should be considered in the initial design of industrial systems is the location and deployment of the required equipment. Industrial companies face such a problem in order to evaluate the location of their factories, facilities and distribution centers for their products. The limitation in resources causes the optimal investigation of them to be in a special investigation, so that the funds are carried out in the most appropriate process (Dey et al. 2017). Failure to properly use the investment process not only causes the investor lost opportunities, but also causes irreparable losses. In order to prevent such implementation problems and the proper use of capital, every investment plan is analyzed before implementation, with the help of analysis and review of logical criteria (Raut et al. 2017). Locating operational centers is one of the most important classifications of issues that should be implemented and operationalized based on basic criteria (Kutlu Gündoğdu and Kahraman 2019). In the 17th century, Fermat introduced the issue of location in industrial units as follows: Suppose three points are given in one space, locate the fourth point in such a way that the sum of its distances to the three given points is minimized. In 1909, the first modern location theory was formed with Weber's paper. Hakimi (1964) divided the objective function into two forms, which are minimum sum and minimum and maximum, and the problem of location on the network will be addressed. Therefore, the location theory was created and the attention of researchers was drawn to this direction, so that today it is considered one of the main components of operational research sciences (Li et al. 2020). The issue of location is at strategic levels in the decision making process and will have a basic strategy in its success. Choosing to build a suitable operational center has an important role in the competitiveness of a company in the market and should be chosen in a way that achieves organizational competitive and functional advantages compared to other organizations (Szczepeński et al. 2019). The evaluation and analysis of the location of the operation center is one of the most important steps in the establishment of routes and roads and places of the supply chain, because the results of this decision have appeared in the long term, and has significant effects on the economic, environmental, social issues, etc. (Mokhtarzadeh et al. 2021). One of the aspects of intra-organizational effects will be its direct effect on profitability in distribution, and from the extra-organizational aspect, the construction of large centers in a region can affect various economic, social, cultural,



environmental, and other conditions. Determining the appropriate location economically plays an important role in the amount of initial investment during their establishment (Shang et al. 2021). Also, during the operation of the plan, this decision has a key effect on the cost of maintaining the goods/services. Building one or more centers in the right places and in the best possible condition, not only improves the circulation of raw materials and customer service, but also puts warehouses in a favorable condition (Gergin and Peker 2019). It is a robust solution that does not change the model's answer by making small changes in its initial data. Robust optimization integrates an objective function with scenario-based input data and includes two separate constraints: structural constraint and control constraint. Structural constraints are formulated using linear programming and its input data have no disturbances. While the control constraints are considered as auxiliary constraints that deal with uncertain data (Yu 2019). In the meantime, the studies related to the location of medical centers and facilities are among the most important and interesting topics that are discussed about the physical arrangement of equipment, manpower required to maximize the production efficiency of a product, shorten the total production time, minimize the production cost and the reduction of transportation costs. Therefore, presenting the hub location model for the green supply chain of medical and pharmaceutical equipment can examine the current state of the facilities and significantly improve the demand coverage by spending acceptable costs. Therefore, in the research, a network of facilities for locating the hub in the health field, considering existing facilities and new facilities under possible scenarios is presented. Therefore, after presenting the mathematical model, validation was done in small dimensions and then sensitivity analysis was done on the main parameters of the model.

2 Research Literature

Jafari and Hadianpour (2018) in their research with a MOHLAP hub location single-objective model, after examining the definitions of hub location and types of location models and their solution algorithms, solve a case study example of the classical model algorithm. Locating Hub has added a fixed annual fee to determine the best place to ship goods from one country to another among the 5 most requested cities, with the aim of minimizing shipping and distribution costs. Shang et al. (2021) introduced a multi-model non-stochastic hub location problem in a many-to-many transportation and distribution system and formulated it by considering the transportation cost and uncertain demand and it was solved by Benders algorithm using data from Turkey's transport network and AP data. Ghaffarinasab (2022) in a research investigated the problem of random hub location with Bernoulli demand (HLPBD) and proposed single and multiple allocation variables for the MIP model and solved and developed it with Benders method and Lagrange release. Ghaffarinasab et al. (2022) presented a new MILP formula in a multi-objective problem of random allocation location with risk measurement and average beta approach and solved it with Benders



algorithm.

Momayezi et al. (2021) modeled and solved single-capacity hub locating algorithm with hub distribution capability and considered a small number of capacity transportation devices as a parameter and solved it as a two-step random problem with meta-algorithms. Hosseinzadeh Kashan et al. (2021) in their study entitled Pareto-Based Grouping Metabalization Algorithm for Humanitarian Relief Logistics with Multi-mode Reliability of a Routing Problem - Multipurpose Locating to Submit Uncertainty Resources with Transport Infrastructure and Travel Time, and compared their models to other solutions.

Mohtashami et al. (2020) designed a green closed-loop supply chain model using a queuing system to reduce environmental impacts and energy consumption. Due to the increase in environmental impacts and their important role in human life, the reduction of human-caused impacts has recently attracted more attention. Green supply chains are among the most influential topics related to environmental impacts, and the increasing number of studies in this field confirms this opinion. Transportation fleets move products between supply chain centers and are one of the most important factors that increase environmental impacts. The transportation fleet that transports products between supply chain centers is one of the most important factors that increase environmental impacts while transporting products between centers and waiting in loading queues. The reduction of environmental impacts caused by the transport fleet, from this point of view, is not comprehensively investigated in forward and reverse logistics supply chains. In order to deal with this gap, in this article, a green supply chain is designed with forward and reverse logistics in mind, and the queuing system is used to optimize the transportation and waiting time of the transportation fleet network. This optimization model leads to the reduction of environmental impacts. Our network includes supplier, production system, and distribution center, repair center, recycling center, disposal center and collection center. Returned products from customers are collected in the collection center and transferred to other centers based on their type. The transport fleet in the network is customers of the loading system at each center, each of which has a multi-server queuing system with limited resources. It is assumed that there are enough servers available at the drop-off centers, so there will be no queues there. The proposed model reduces the environmental impact and energy consumption of the transportation fleet by determining the loading, unloading and production rates, which affect the waiting and transportation times. A numerical example for a small size NLP model is discussed and solved by exact methods. In addition, a meta-analytical method is used to solve the large problem. Finally, sensitivity analysis is performed to investigate the effects of changing parameters on model decision variables and target performance. Chen et al. (2018) stated in his research that the tea market in different countries is witnessing expansion and development. In order to identify the most logical development paths, the choice of sales location should consider contradictory criteria. Therefore, a multi-criteria approach is



required to effectively address the location selection problem. In this paper, a multi-criteria framework for tea house selection is presented and applied in Lithuania. In addition, to ensure the validity of the results, two methods of multi-criteria decision-making (MORA) and evaluation of the weighted product with normalization (WASPAS-N) have been used. The weights of the criteria were determined based on the survey of experts. In addition, a Monte Carlo simulation was used to investigate the sensitivity in changing the benchmark weight. The experimental program has shown the validity of the proposed method in choosing the optimal location of a tea house. Pham et al. (2017) stated in his research that logistics centers have emerged as an important logistics infrastructure in supply chains. Therefore, the issue of locating logistics centers plays a fundamental role in the design and practice of logistics and supply chain management. Acknowledging the importance of logistics centers, Vietnam has adopted a comprehensive plan to develop a logistics center system. However, this plan has been difficult to implement due to the lack of prioritization of the determined factors used to locate logistics centers. This study aims to create a suitable framework for selecting the location of logistics centers based on the residual using a combination of the fuzzy method and the technique of preference order by similarity to the ideal solution (TOPSIS), both of which have been used. The results indicate that transportation demand, proximity to the market, production area, customer, and transportation cost are the most important factors in deciding the location of waste logistics centers. Furthermore, among the three locations considered, the northeastern provinces of Ho Chi Minh City were the best locations for logistics centers, followed by North Hanoi and Da Nang City. The findings of this research have a significant contribution in terms of scientific and practical aspects of the location of logistics centers.

3 Research Problem

In this proposed model, it is assumed that there are a number of facilities at different levels in the investigated area. Due to the fact that the existing facilities have been established over a long period of several years and due to the fact that during these years there have been changes in the amount and focus of the level of demand of subscribers for services, therefore it is necessary that all hubs be examined in an integrated way. In this section, at first, the hubs in the area for the transfer of medical equipment are classified into three levels, and then the amount of coverage of demand by these facilities is checked and if there is a change in the capacity or location of the facility, which in terms of amount the cost is also justified; these changes are made in the location and capacity of existing facilities. Finally, a new structure of locating hubs is presented, in which the location and capacity of some hubs have been changed, but on the other hand, the coverage of the demand for medical goods has increased. Improving demand coverage is not done only by checking the capacity level of existing hubs and modifying the location of these hubs. In this article, three capacity levels are also



considered for hubs, and if needed, the hub will be upgraded to a higher capacity level or a new hub will be located. In order to improve the coverage quality of medical equipment customers' demand, coverage radius is also included for each hub. The coverage radius helps to allocate the demand to the hubs in such a way that the distance between the demand point and the hub is optimal and acceptable. In this article, the coverage radius is considered as a variable. Considering that the amount of demand coverage has a direct relationship with the distance between the demand point and the hub as well as the variable cost of coverage; it is tried to minimize the coverage radius of each hub. In improving the existing situation, not only changes in the existing hubs are addressed, but if there is a need to locate a new hub; a new hub is being established. In this issue, we are looking for the optimality of existing hubs in terms of coverage capacity and locating new hubs with the aim of reaching a higher demand coverage level, reducing costs and maximizing the level of greenness of hub location. For this purpose, in our review, considering economic efficiency, we will consider these measures: establishing a new hub, removing the previous or current hub, increasing and reducing the capacity of hubs.

4 Assumptions of Mathematical Modeling

The study area consists of a number of areas. Areas are also a subset of a larger area called area, which are covered by hubs. In this research, three hub levels are considered: W-level hub, which is the lowest capacity level and is responsible for providing basic services of medical equipment. C-level hub, that provides additional medical services. And finally, h-level hub, which provides the most advanced medical equipment services, and also provides c-level hub services.

Each demand point must be covered by a w-level hub and a c-level hub as well as an h-level hub, but h-level hubs can also provide c-level hub services; Therefore, if a demand point is covered by the h-level hub, there is no need for that demand point to be covered by the c hub. In each region, there must be at least one hub at level w.

In one region, you cannot have both a c-level hub and an h-level hub at the same time. No more than one level h hub can be placed in one area. Demand is considered as a point.

5 Suggested Model

In this proposed model, it is assumed that there are a number of hubs at different levels in the investigated area and the amount of demand coverage by these hubs should be checked. And if there is a change in the capacity or location of the facility that is justified in terms of cost; these changes are made in the location and capacity of existing hubs. Finally, a new structure of hubs is presented in which the location and capacity of some hubs have been changed, but the amount of demand coverage has increased.



Ranges, collections, indices		Limitations	
N	Set of grid points	1	$Max \sum_{i,s} p^s(h_i z_i^s)$
B	The set of regions in the model	2	$Min \sum_{l,j,t} FC_{j,t}^l x_{j,t}^l$
R	The set of areas in the model		$+ \sum_j r_j(\varphi)$
S	The set of available scenarios		
Fr	The set of nodes in the area r	3	$Max \sum_{l,j,t} G_{j,t}^l x_{j,t}^l$
i	Index showing demand points $i \in N$	4	$\forall i \in N_b, b \in B, l \in \{w, h\}, s \in S$
j	Index showing candidate points for establishment of facilitation $j \in N$	5	$\forall i \in Fr, r \in R, s \in S$
l	The index related to the level of facilitation $l \in \{w, h, c\}$	6	$\forall b \in B$
t	Index related to hub capacity level	7	$\forall b \in B$
b	Index related to the region		
Nb	nodes in the region b, $Nb \in N$		
r	Area index	8	$\forall r \in R$
s	Index for each scenario	9	$\forall j$
parameters		10	$\forall j, l, s \in S$
h_i	Point i demand		
$FC_{j,t}^l$	The fixed cost of establishing a hub at the level l at the point j with the capacity level t		
$G_{j,t}^l$	The green level of the hub at the level l at the point j with the capacity level t	11	$\forall i, l, j, s \in S$
$level_t$	The amount of capacity related to the surface t	12	$\forall j$



$Q_{j,t}^l$	1 If the hub already exists on the level l at the point j and capacity level t ; 0 otherwise	13	$\forall l \in L$	$\sum_{j,t} y_{j,t}^l \leq P^l$
p^l	The number of hubs in the level l that should be at the end	14	$\forall l, j, t$	$x_{j,t}^l = y_{j,t}^l - Q_{j,t}^l $
$d_{i,j}$	The distance between two points i and j	15	$\forall i, j, l, t, s$	$z_i^s, x_{j,t}^l, a_{i,j}^{l,s}, y_{j,t}^l \in \{0,1\}$
$RMax$	Maximum acceptable coverage radius			
p^s	The probability of the scenario s occurring	16	$\forall j$	$r_j \geq 0$
u_j^s	1 if the existing hub at the point j in the scenario is available; 0 if the existing hub at the point j in the scenario s is disturbed and out of reach			
Decision variables				
z_i^s	1 if the point i demand is covered; 0 otherwise			
$x_{j,t}^l$	1 If the hub at the level l at the point j in the capacity level t does not exist before; 0 otherwise			
r_j	The coverage radius of the existing facility at the point j			
$a_{i,j}^l$	1 if the demand point i is assigned to the hub j at the level l ; 0 otherwise			

The objective function (1) shows the amount of covered demand. The objective function (2) shows the cost caused by changes in the structure of the hubs. In this way, in case of establishing a new hub or changing the capacity of a hub or moving a hub, the binary variable will take the value of one and the cost of this change will be calculated. In the second part of the equation, the cost of covering the radius of each hub is calculated as a function of the radius. Each demand point is covered under the condition that it is assigned to at least one hub in the level and to one hub in the level in its own area; this condition is considered in equations (3). The third objective function (4) is to maximize the level of greenness of the



hubs. According to the assumptions of the model, the hub at the level can eliminate the demand point from being assigned to the hub at the level; Therefore, in equations (5), the assumption is applied that by assigning a demand point to a surface hub in another area, the assignment of that point is not required to facilitate the surface. Constraint category (6) states the requirement to establish at least one level hub in each region. The assumption of prohibiting the establishment of two hubs at the same time has been considered in the levels and in equations (7). Equations (8) limit the establishment of more than one hub on the surface in one area. Equations (9) limit the establishment of more than one hub at one point. Equation (10) is the limitation of the hub capacity and prevents the allocation of more than the optimal capacity level for the hub Constraints (11) and (12) determine the coverage radius of each hub. Constraints (13) show the allowed number of final network hubs at each level. Constraint (14) recognizes that the hub has changed or remained unchanged during the location-relocation.

5.1 Stabilization of the Mathematical Model

Considering that the presented model is a non-linear model. In this part, uncertainty in demand will be added to the model with the help of robust planning and Bertsimas and Sim's approach. Therefore, the first objective function and constraint number (10) will be modified as a Bertsimas model. Investigations in this research show that the customer demand parameter is one of the important parameters whose values may exceed the nominal values. Therefore, considering this parameter in uncertain conditions can bring the proposed model closer to the reality of the problem. To consider the uncertainty in the demand, as mentioned, the robust planning and Bertsimas and Sim approach will be used. The robust optimization method looks for optimal or near-optimal solutions that are justified with a high probability. Bertsimas and Sim approach is one of the four main approaches to consider uncertainty in robust planning. In this section, we will briefly mention to this approach. For this purpose, we consider the following linear programming model:

$$\begin{aligned} & \text{Min} \sum_j c_j x_j \\ & \text{s.t} \\ & \text{Ax} \leq \text{b} \end{aligned}$$

In this model, we assume that only the coefficients on the right side in the constraints, i.e. the matrix A, have non-deterministic values and the terms of this matrix a_{ij} fluctuate in the range $[\tilde{a}_{ij} - \hat{a}_{ij}, \tilde{a}_{ij} + \hat{a}_{ij}]$ that \tilde{a}_{ij} and \hat{a}_{ij} are the nominal value and the maximum deviation of the parameter a_{ij} respectively.

The robust model proposed by Bertsimas and Sim is as follows:



$$\begin{aligned}
 & \text{Min} \sum_j c_j x_j \\
 \text{s.t.} \quad & \sum_j \tilde{a}_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} \mu_{ij} \leq b_i \quad \forall i \\
 & z_i + \mu_{ij} \geq \hat{a}_{ij} x_{ij} \quad \forall i, j \\
 & z_i, \mu_{ij} \geq 0 \quad \forall i, j
 \end{aligned}$$

In these relationships z_i and μ_{ij} auxiliary variables are dual and the parameter Γ_i called uncertainty budget shows the level of conservatism that is chosen according to the importance of the constraint and the risk-taking of the decision-maker.

5.2 Solution Method

The second proposed model is a mixed integer two-objective model based on the scenario. To solve this proposed model, we first convert the three-objective model into an equivalent single-objective model by using the Torabi-Hassini method; then, we solve the single-objective model by using Benders decomposition algorithm.

5.2.1 Torabi-Hassini Method

This method was presented by Torabi and Hassini (2008) and is called the TH method. The efficiency of the answers produced by this method can be proven if we consider the following general model:

$$[f_1(x), f_2(x)] \quad (17)$$

$$\text{s. t.} \quad (18)$$

$$x \in Fx \quad (19)$$

The general shape of the model in the TH method is as follows:

$$\text{Max } \gamma \beta_0 + (1 - \gamma) \sum_k w(x) \quad (20)$$

$$\text{s. t.}$$

$$(x) \geq \beta_0; \forall k \quad (21)$$

$$x \in Fx \quad (22)$$

$$\beta_0 \in [0, 1] \quad (23)$$

In this model, it is considered as a compromise coefficient between goals. It represents the minimum satisfaction level of the objective functions (). The values represent the weight of the objective functions. The values that represent the level of satisfaction of each of the objective functions are also calculated in the following order:

$$\mu_1 = \begin{cases} 1 & \text{If } z_1 \leq z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & \text{If } z_1^{PIS} \leq z_1 \leq z_1^{NIS} \\ 0 & \text{If } z_1^{NIS} \leq z_1 \end{cases} \quad (24)$$



$$\mu_2 = \begin{cases} 1 & \text{If } z_2 \leq z_2^{PIS} \\ \frac{z_2^{NIS} - z_2}{z_2^{NIS} - z_2^{PIS}} & \text{If } z_2^{PIS} \leq z_2 \leq z_2^{NIS} \\ 0 & \text{If } z_2^{NIS} \leq z_2 \end{cases} \quad (25)$$

To determine the values $z_{(i \beta)}^{PIS}$ and $z_{(i \beta)}^{NIS}$ that are the optimistic and pessimistic i values of the target function respectively; we act as follows:

To obtain the optimistic value of an objective function, we solve the model for the same objective function and consider the obtained value as the optimistic value. When we solve the model based on an objective function; the value obtained for the other objective function can be considered as the pessimistic value of that objective function. Consider the following equations:

$$z_1^{NIS} = Z_1(x^{PIS}) \quad (26)$$

$1 \beta \qquad 2 \beta$

$$z_2^{NIS} = Z_2(x^{PIS}) \quad (27)$$

$2 \beta \qquad 1 \beta$

5.2.2 Benders Analysis Method

Benders decomposition algorithm is a mixed integer programming problem decomposition method that has been used in various fields. This algorithm is a method of resource allocation in large-scale problems that was proposed by Benders in 1962 and has been used in network, transportation, location, etc. problems so far. The mentioned method can be used independently as well as in combination with other methods such as Lagrange release and branch-and-bound algorithm in solving problems. In short, it tries to provide a redefinition of the problem under investigation by applying concepts such as duality and using primal-dual relations.

$$\text{Min } z = c^T x + b^T y \quad (28)$$

$$\text{s. t. } Ax \geq d, \quad (29)$$

$$Bx + Dy \geq h, \quad (30)$$

$$x \in X, y \geq 0. \quad (31)$$

Benders method divides the variables of the model into two groups (x, y), so that in each iteration of the algorithm, problems involving either x or y (and not both) are solved. The components y are assumed continuous, but the components x can be continuous or discrete. In addition, there are two groups of adverbs, the first group is only related to variables x and the second group is related to variables x and y . Of course, the constraints of the first group may not exist in the problem, in this case we can consider a suitable limit on the variable x and use it as the constraints of the first group. The model may be computationally complex



with both groups of variables x and y . While considering the problem with variables x or y is simpler. For example, if the vector of variables x is integer, this classification leads to a linear programming model according to y and an integer programming model according to x or there may be special structures in the problem, such as the structure of the transportation problem, that facilitate its solution. In the model, we consider x as a complex variable that makes problem solving difficult. For example, x can be a vector or integer components, or a variable that perturbs the particular structure of the problem, and by considering a fixed value for it, the problem acquires a particular structure, such as the structure of the transportation problem, or it is divided into several smaller problems. We rewrite the model as model 2.

$$\text{Min } z = c^T x + \Phi(x) \quad (32)$$

$$\text{s. t. } Ax \geq d, \quad (33)$$

$$x \in X \quad (34)$$

Model 2

Which $\Phi(x)$ is defined according to model 3 and is called Benders sub-problem.

$$\Phi(x) = \text{Min } b^T y \quad (35)$$

$$Dy \geq h - Bx, \quad (36)$$

$$y \geq 0. \quad (37)$$

Model 3

For a fixed value of model 3-3 is a linear programming problem, in fact, model 3-3 can be considered a parameter problem where the parameter x is on the right side of the constraint. According to Dugan's theorem, model 3-3 can be written as model 4-3, where the vector π corresponds to Dugan's variables of model 4-3.

$$\text{Max } (h - Bx) \quad (38)$$

$$\pi^T D \geq b, \quad (39)$$

$$\pi \geq 0. \quad (40)$$

Model 4

Without losing generality, suppose that the problem corresponding to model 3-3 is feasible for every x , in fact, by considering the variables of deficiency and excess, the constraints of model 3-3 can be written in such a way that it is possible to violate the constraints. And this error in the objective function should be penalized with a sufficiently large positive coefficient. Also suppose that model 3-3 has a finite optimal solution for every x , because if this model becomes infinite, model 3-1 will also become infinite. Considering these two assumptions on model 3-3, according to the duality theorem, model 3-4 will have a finite optimal solution for each x . We denote the feasible region of model 3-4 by \mathcal{A} , which is independent of the choice and the maximum occurs at one of the vertices of \mathcal{A} . Without loss of generality, we can assume that \mathcal{A} is bounded, because if \mathcal{A} is unbounded,



assuming π is an m -component column matrix of the form $\pi^T = [\pi_1 \pi_2 \dots \pi_m]$ by adding the following constraint, we bound the solvability of the argument where M is considered a large positive number.

$$\sum_{i=1}^m \pi_i \leq M \quad (41)$$

Therefore, suppose that $\{Q_1, Q_2, \dots, Q_k\}$ is the vertices of the area of \mathcal{A} , in this case model 3-4 and model 3-4 are equivalent to each other.

$$\text{Max } (h - Bx) \quad (42)$$

$$\pi \in \{Q_1, Q_2, \dots, Q_k\} \quad (43)$$

Model 5

Thus, model 6 will be equivalent to the initial problem (Model 1).

$$\text{Min } z = c^T x + (\max \pi^T (h - Bx)) \quad (44)$$

$$s. t. Ax \geq d, \quad (45)$$

$$\pi \in \{Q_1, Q_2, \dots, Q_k\}, \quad (46)$$

$$x \in X \quad (47)$$

Model 6

We rewrite model 3 as model 7 and describe Benders algorithm based on it.

$$\text{Min } z = c^T x + \varphi \quad (48)$$

$$(h - Bx) \leq \varphi, \pi \in \{Q_1, Q_2, \dots, Q_k\}, \quad (49)$$

$$Ax \geq d, \quad (50)$$

$$x \in X, \varphi \in R \quad (51)$$

Model 7

In the above problem, Φ is a free variable in terms of sign. By solving this problem, the optimal solution to the initial problem X can be obtained. The question may be raised whether is it necessary to identify all the vertices

$\{Q_1, Q_2, \dots, Q_k\}$ of the region F ? No, to determine the optimal solution of the problem, an iteration method is used, and in each iteration, one of the vertices is identified and the corresponding constraint is added to the problem. The general idea is to set appropriate upper ($z_{UB} = c^T x + b^T y$) and lower bounds ($z_{LB} = c^T x + \varphi$) on the optimal value of the objective function z^* from model 1-3. So that $z_{LB} \leq z^* \leq z_{UB}$ the upper and lower bounds are changed successively, so that each new bound is obtained by identifying a new vertex point of F and finally, at the vertex point for which $z_{LB} = z_{UB}$ the optimal solution will be reached. Therefore, this process will end after identifying a maximum of K vertices from F region.



5.2.3 Mathematical Model Validation

In this model, the TH method is used to convert the three-objective model into a single-objective model. Figure 1 shows the result of solving the model for the sum of the first and third objective functions and the second objective function. In this figure, the triangles represent w-level facilities, the hexagons represent h-level facilities, the blue rectangles represent the coverage area of w-level facilities, and the red rectangles represent the coverage area of h-level facilities. In the first case, which shows the optimal solution considering the first and third objective function; considering that there is no limit for the cost and the function of the second objective is cost minimization; is not considered; Facilitation has been established in all candidate points. According to the nature of the limitations of the model, two facilitations at h level have been established at points 3 and 8, and at the rest of the points, facilitation at w level has been established. The allocation of demand is also such that the demand of points 1, 2 and 3 in level h is allocated to the facility established in point 3 and the demand of other points in level h is allocated to the facility established in point 8. The demand of level w is also allocated to these points due to the establishment of facilitation at level-W in all points except points 3 and 8. Demand level w points 3 and 8 are assigned to facilities 2 and 7, respectively. In the second case, which shows the optimal solution considering the second objective function; Considering that only cost minimization is considered; Only facilities have been established that are mandatory based on the limitations of the model. According to the limitations of the model, there should be at least one facility in the level w in each region. Considering that in the existing network, there are two facilities at h and w levels, respectively, at points 1 and 7 of the network; after solving the model, only two facilities have been established at the level of w at points 2 and 4. But since considering the coverage radius for facilities requires variable cost, the coverage radius of all facilities is zero in the case of considering the second objective function. Therefore, no demand point is covered and the demand coverage becomes zero in this case. In the third case, showing the answer by solving the single-objective model is equivalent to the two-objective model; In addition to the existing h-level facility at point 1, another h-level facility is also established at point 8. At level w, two facilities are considered at points 2 and 4. In terms of assigning demand points to the facilities, the points in the first area (points 1, 2 and 3) are assigned to facility 2 at level w and facility 1 at level h, in the second area at level w to facility at point 4 and in the third area at level w, they are assigned to facilitate in point 7. At level h, the entire second area has been allocated to the facility in point 8.

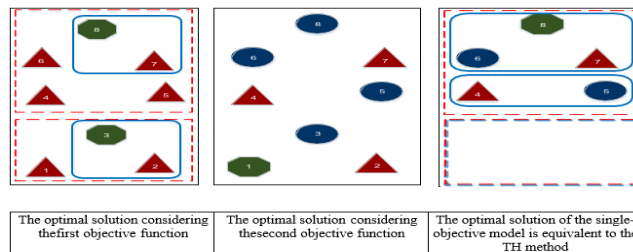


Figure 1 The result of solving the model for the first and third objective function with the second objective function and the equivalent single-objective model using the TH method

In Table 1, the values of both objective functions can be seen in the three stated states:

As can be seen, the amount of covered demand in the first case is equal to the third case. However, the amount of cost in these two cases is significantly different.

Table 1 Values of objective functions in three modes

	Solution of the single-objective model is equivalent to the TH method	Considering the second objective function	Solution considering the first and third objective function
The value of the First and third objective function	375	0	375
The value of the second objective function	580396.178	270942	1958114.916

5.2.4 Sensitivity Analysis of the Proposed Model

Figure 2 shows the changes in demand coverage with changes in demand. As can be seen in the diagram; the amount of covered demand increases up to 1.5 times the amount of demand, and this upward trend is because the empty capacity of the established facilities has been used to cover the increased demand. However, in the continuation of the trend of the graph, there is no significant increase or decrease, and the reason is that there is no empty capacity to cover the high demand without coverage, and covering the demand requires the establishment of a new facility, and this is in conflict with the second objective function, which is cost minimization. The result does not have an upward or downward trend from the demand level of 1.75 times. Figure 3 shows the changes of the second objective function, i.e. the amount of cost in relation to the increase in demand. As can be seen, there are no



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significant changes or special trends in the graph and the reasons that the facility structure has changed very little with the increase in demand. That is, the changes have been to the extent of changing the facility level or changing the coverage radius, and the overall structure has had few changes, which is very desirable because the goal is to increase the amount of demand coverage with the least amount of changes in the current structure of the facility.

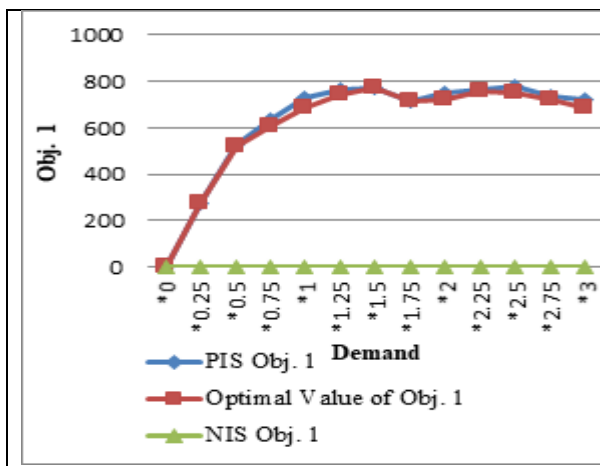


Figure 2 Changes of the first objective function in relation to the increase in demand

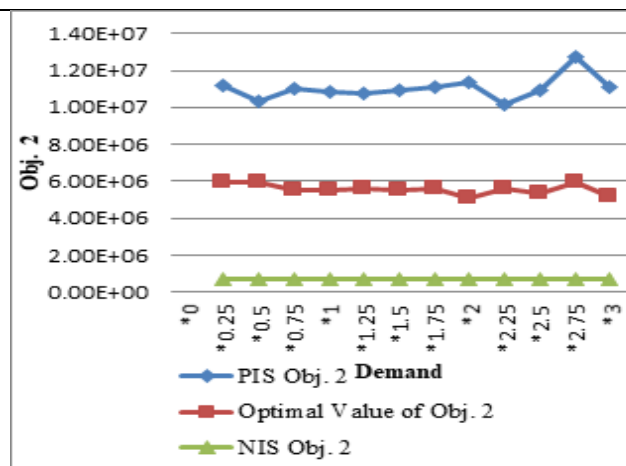


Figure 3 Changes of the Second Objective Function In Relation To the Increase in Demand

Figure 4 shows the amount of changes in covered demand because of increasing the level of facility capacity. As can be seen, the chart has an upward trend until the threshold of 2.25 times the facility capacity level, which is predictable. But after 2.5 times the capacity levels, the upward trend almost stops; This is because either all the demand in the area is covered or the distance of the demand point to the facility is so great that it is not optimal coverage based on the second objective function. Figure 5 shows the changes of the second objective function, i.e. the total cost, and as can be seen, the cost does not change or have a significant trend, and the reason is that the cost of establishing the facility at different levels has been kept constant and only the capacity of the facility has changed; therefore, the changes in the cost are within the limits of the costs caused by the change in the coverage radius of the facilities, which are variable costs and are insignificant. However, in the third point of the graph, there is a small jump, the reason for which is the establishment of a new facility; after this jump, the graph shows a decrease, which is due to the decrease in the coverage radius of the facility due to the increase in the facility.

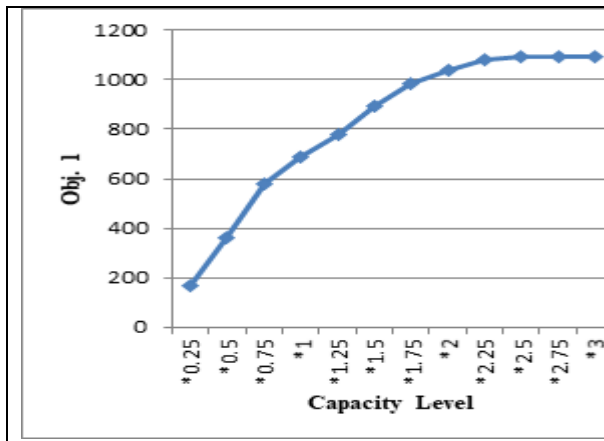


Figure 4 Covered demand changes due to increase in facility capacity level

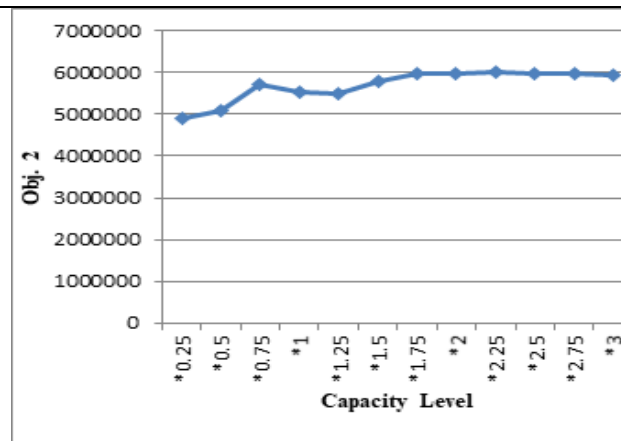


Figure 5 Cost changes due to the increase in facility capacity

Figure 6 shows the amount of covered demand changes because of increasing the maximum number of facilities in the network. With the increase in the maximum number of facilities, the amount of covered demand tends to rise until either all the demand is covered or the establishment of a new facility is not optimal according to the second objective function, or the increase in the desired parameter is so high that it is not possible to make improvements. Figure 7 shows the amount of changes in the second objective function because of changing the maximum number of facilities that can be established at each level. As can be seen, by increasing the maximum number of facilities, the total cost decreases, and the reason for this decrease is the increase of the feasible area and, as a result, better solutions. In Figure 8, the trend of changes for demand against the increase of the maximum coverage radius of the facility, as you can see, with the increase of the maximum coverage radius of the facility, the amount of covered demand also increases. This increase has a higher slope at the beginning, but the slope decreases later on. The reason for this decrease in slope is that the number of uncovered demand points decreases and therefore the slope of the graph decreases. At the end, the upward slope becomes downward and the reason for that is the balance between the objective functions. This means that the reduction in coverage is equivalent to a significant and economical reduction in cost. Figure 9 shows the changes of the total cost with the increase of the maximum coverage radius. As it can be seen, at first, with the increase of the maximum radius of coverage, the total cost increases significantly, because with the increase of the maximum radius of coverage, it is possible to cover the wide demand by establishing new facilities, so new facilities are built, and the coverage of demand and the total cost both increase.

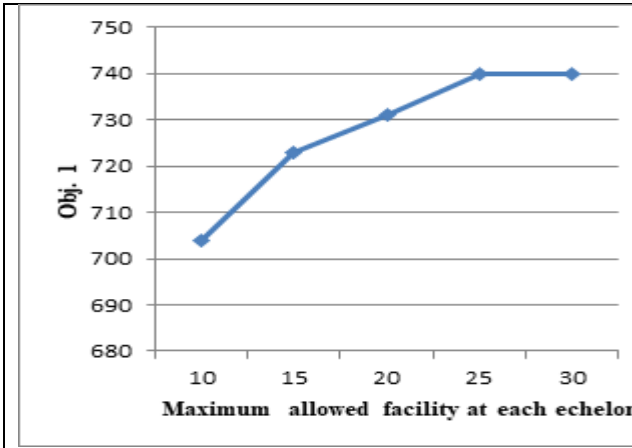


Figure 6 Changes in covered demand as a result of increasing the maximum number of facilities

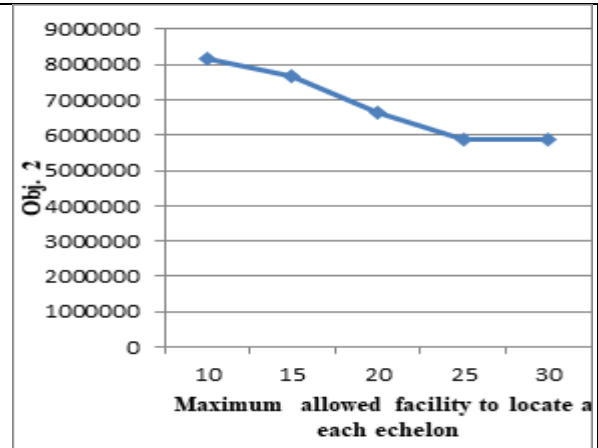


Figure 7 Changes of the second objective function because of changing the maximum number of facilities that can be established at each level

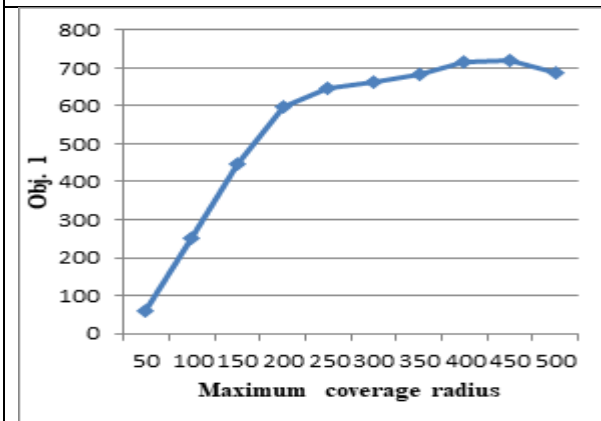


Figure 8 Changes in the amount of covered demand versus the increase of the maximum coverage radius

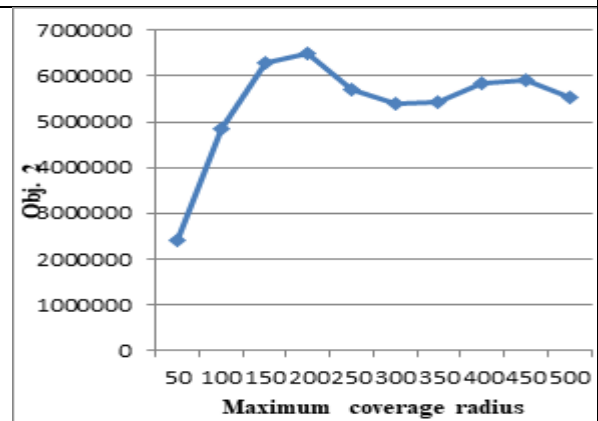
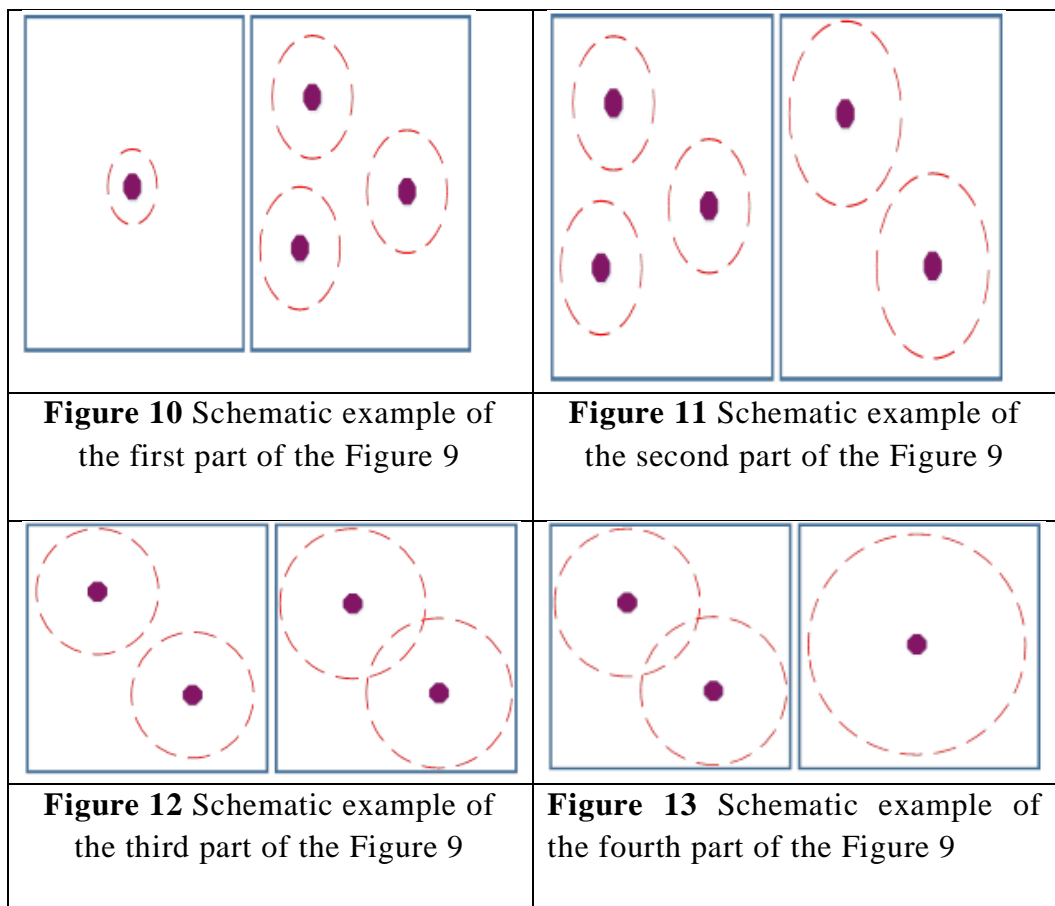


Figure 9 Total cost changes with increasing maximum coverage radius

In the continuation of the diagram, by increasing the maximum radius of coverage, it is possible to improve allocations and reduce the need to establish facilities, which results in a reduction of the total cost. That is, increasing the coverage radius makes it possible to cover more demand with less facilitation. In the following, the cost increases and the reason is that the number of facilities does not decrease anymore, but with the increase of the



maximum radius of cost coverage and demand coverage, it increases. After this increase, a decrease occurs again, and the reason for this decrease is the balance between the two objective functions, in this way, with the decrease in facilitation and the consequent decrease in the amount of demand coverage, the cost of coverage is reduced to the amount of expenses (Figures 10 to 13).



6 Calculation results

In this section, the effectiveness of the linearization of the model's nonlinear constraints and the effectiveness of the release of binary variables on the model's solution time are evaluated. In this way, during the solution of 8 numerical examples that were generated; the solution time and the value of the obtained objective functions are recorded and randomly compared with each other. In the following, we will evaluate the performance of the Benders analysis algorithm both in terms of the quality of the solution and in terms of the solution time by solving sample problems that were randomly generated. Figures 14 and 15 show the solution time and the value of the objective function resulting from the solution of the single-objective equivalent model obtained by the TH method.

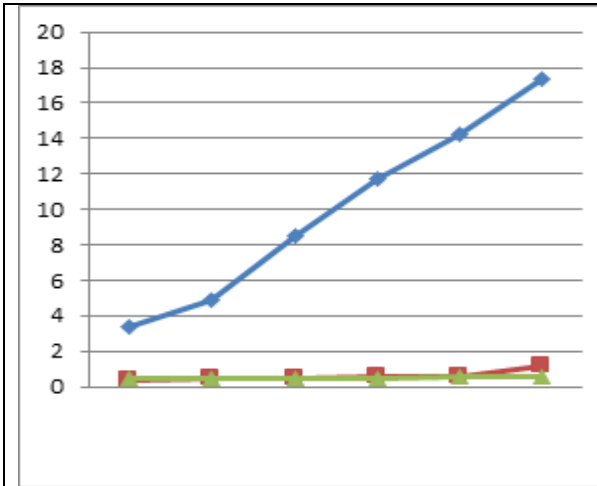


Figure 14 The solution time and the value of the objective function resulting from the solution of the equivalent single-objective equivalent model by the TH method

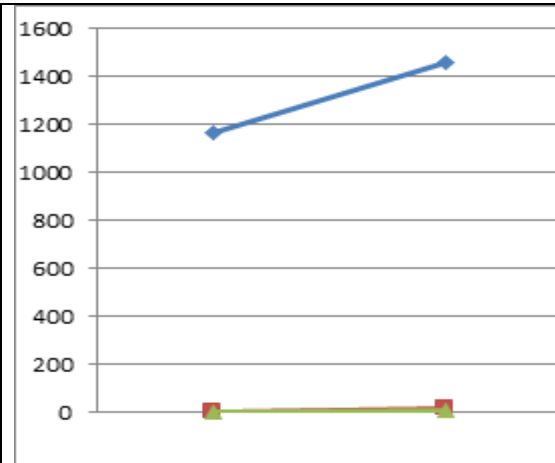


Figure 15 Continuation of the solution time and the value of the objective function resulting from the solution of the equivalent single-objective equivalent model by the TH method

As can be seen, the time to solve the mixed integer model is significantly less than the time to solve the nonlinear mixed integer model. By releasing the binary variables, the time of solving the model is reduced again, but this reduction is very small, which is depicted in Figure 14 and Figure 15. These graphs show a huge difference in the solution time of MIP and MINLP models. As the dimension of the examples increase, the solution time difference between the MIP and MINLP modes increases greatly. The increase in solving time in MINLP mode is exponential.

In Table 2, the values of the model solving time, the first objective function and the second objective function obtained for solving the model in the mode of applying the Benders algorithm and the mode of solving with the GAMS solver are displayed. As can be seen in the table, for solving problems with low dimensions, the time to solve the problem does not differ much with the use of Bander's decomposition algorithm. Even in some cases, the time to solve using the Benders algorithm is longer than the time to solve with the GAMS solver. However, with the increase in the dimensions of the problems, the solving time with the GAMS solver increases significantly and the time difference of the solution increases exponentially. Since the values obtained for the objective functions from the Benders decomposition algorithm have very little difference with the values obtained from the GAMS solver, it can be said that the Benders decomposition algorithm has a suitable efficiency for solving this problem.



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Table 2 Comparison of solution time and values of objective functions per solution with gams solver and benders decomposition algorithm

Math problem number	Dimensions of the problem				GAMS		Benders Decomposition			
	n	1	s	t	Solving time	Objective 1&3	Objective 2	Solving time	Objective 1&3	Objective 2
1	8	3	2	2	0.48	114	1257878	0.49	114	1257878
2	8	3	2	3	0.45	133	884013	0.48	133	884013
3	8	3	2	4	0.45	118	1432853	0.49	118	1432853
4	8	3	3	4	0.47	142	1432853	0.52	142	1432853
5	8	3	4	4	0.59	169	1451249	0.49	169	1451249
6	8	3	5	4	0.7	161	1308914	0.48	160	1308917
7	16	3	5	4	1.73	188	1194150	0.74	185	1194161
8	24	3	5	4	3.62	191	543187	1.619	190	543195
9	30	3	5	4	6.42	197	901783	1.985	195	901783
10	40	3	5	4	7.92	184	840753	2.93	180	840760
11	60	3	5	4	17.73	187	1287222	7.944	181	1287227
12	90	3	5	4	108.83	190.6	1456815	23.4	185	1456827
13	120	3	5	4	643.66	198	1296209	56.97	195	1296217
14	150	3	5	4	1814.9	184	1140060	115.16	173	1140160
15	200	3	5	4	14341.4	187	1027115	287.82	180	1027220



7 Mathematical Model Validation

According to the presented mathematical model, in order to evaluate the presented mathematical model, it has been evaluated and analyzed using MOPSO and NSGAI algorithm. Hence, in this section, in order to solve sample problems in a larger size, meta-heuristic NSGA II and MOPSO algorithms with priority-based encryption have been used. Therefore, at the beginning of this section, the initial solution used in problem solving and the operators of meta-heuristic algorithms have been discussed. At the end of this section, the parameter setting of meta-heuristic algorithms with Taguchi method has been discussed.

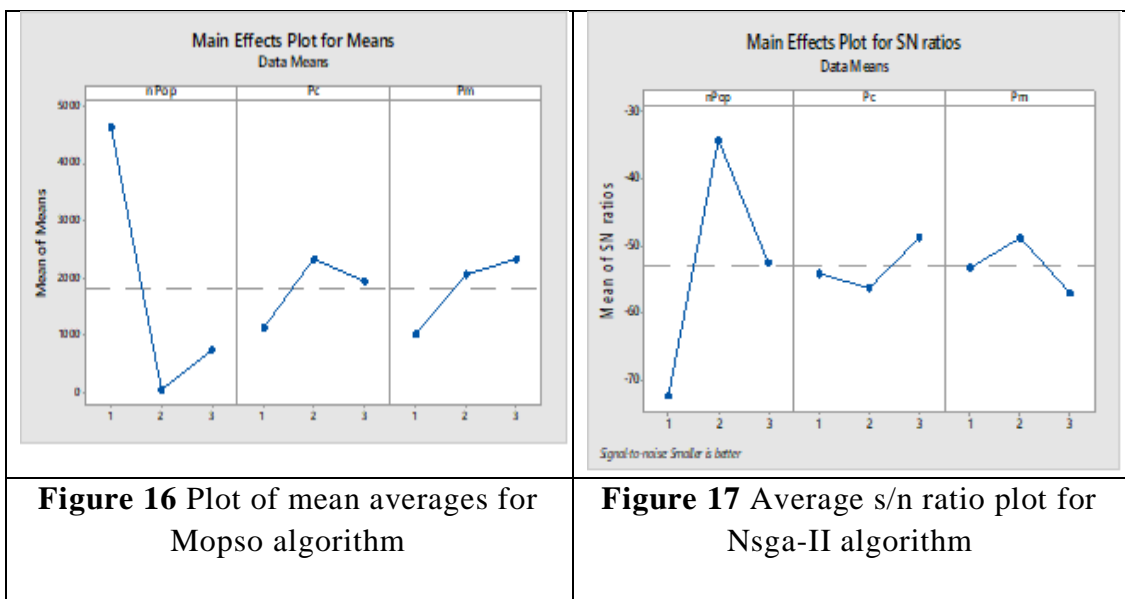
7.1 Parameter Setting of Meta-Heuristic Algorithms

The response variable is used to set the parameter. This response variable is a combination of the 5 criteria presented and its value is calculated using the following formula. Considering that the criteria do not have the same importance, the weight coefficients used for them are determined.

$$R_i = \frac{NPF_1 + MSI_2 + SM_3 + CPU-time}{w_1 + w_2 + w_3 + w_4} \quad (52)$$

7.2 Factors and Levels Related to NSGA-II Algorithm

From the implementation of the Taguchi test, the results, the average means and the average S/N ratio for each level of factors in the NSGA-II algorithm for the model presented in Figures 16 and 17 are shown.



According to the graphs obtained, the optimal level of NSGA-II algorithm factors is equal to (Table 3).



Table 3 Optimal levels of the agent used for the Nsga-II algorithm

Optimal levels of factors	Agent levels			
	Parameters	1	2	3
70	nPop	50	70	100
2/0	pc	2/0	5/0	8/0
2/0	pm	2/0	3/0	4/0

7.3 Factors and Levels Related to the MOPSO Algorithm

After the execution of the Taguchi test, the results of the average means and the average S/N ratio for each level of factors in the MOPSO algorithm for the model presented in Figures 18 and 19 are shown.

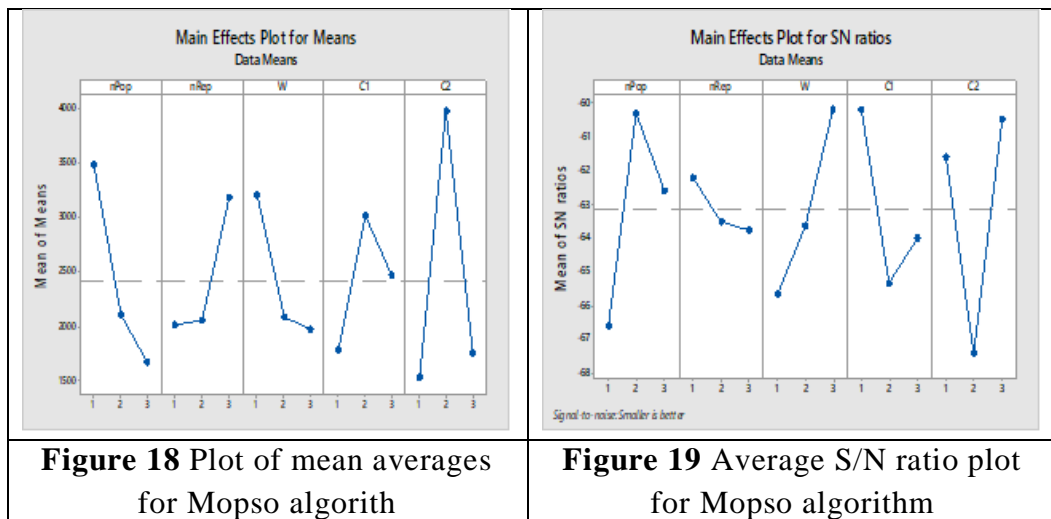


Figure 18 Plot of mean averages for Mopso algorithm

Figure 19 Average S/N ratio plot for Mopso algorithm

According to the above graphs, the optimal level of the factors has been obtained as described in Table 4.

Table 4 Factor levels used for mopso algorithm

Agent levels	The optimal level of factors			
Parameters	1	2	3	
nPop	50	75	100	100
nRep	70	100	150	70
W	5/0	6/0	7/0	7/0
C1	1	25/1	5/1	1
C2	1	25/1	5/1	1

7.4 Evaluation of NSGA II and MOPSO algorithm

To evaluate and determine the correctness of the coding performed in the MATLAB



software, a sample problem is shown in other small ones for the proposed design algorithms and the output variables resulting from the first effective solution of each algorithm; Therefore, the size of the problem determined in the initial validation and based on the random parameters generated is based on a uniform distribution. After designing the problem and generating random data, the problem designed using meta-heuristic algorithms was performed in 100 repetitions and the comparison indices of multi-objective meta-heuristic algorithms were determined for each algorithm. Table 5 shows the average and indicators of the results obtained from the implementation of NSGA II and MOPSO algorithms.

According to the results of Table 5, the computational time obtained from solving the sample problem with the MOPSO algorithm is less than the NSGA II algorithm. While the NSGA II algorithm has performed better than the MOPSO algorithm in finding the number of efficient solutions; therefore, the Pareto front obtained from solving the sample problem with the NSGA II and MOPSO algorithms is shown in Figure 20.

Table 5 Comparison indices of meta-heuristic algorithms in solving the sample problem

Indicator	MOPSO algorithm	NSGA II algorithm
Computational time	6.64	18.88
The average of the first and third objective function	569563.94	573954.21
Average of the second objective function	49371.86	49622.11
NPF	9	10
MSI	35751.92	36643.30
SM	0.381	0.476

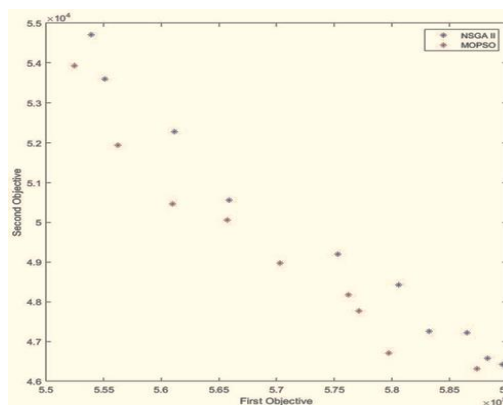


Figure 20 Pareto front resulting from solving the small size problem with Nsga ii and Mopso algorithm



8 Conclusions

In the current research, modeling and solving the multi-objective problem of hub location for medical and pharmaceutical equipment under uncertainty conditions was presented with a robust approach. Considering that one of the most important goals to be considered is to minimize the total costs of hubs in the relevant network, in this research, the total maximum coverage, the cost of using different hubs, the costs of opening, re-opening and activating the hubs in the centers were minimized. One of the challenges in the issue of location of facilities is to consider facilities in different places in the same way and with similar services and capacity. This is despite the fact that the characteristics of the located points, the limitations, the type and amount of needs in each area are not necessarily similar to each other and these differences make the same facilities not efficient enough. In this research, the facilities can be different from various aspects. The facilities are considered hierarchically and at three different levels. The facility capacity is not fixed and the capacity of each facility can be determined from three different levels by solving the model. The coverage radius of each facility is also different and is considered as a decision variable. Therefore, according to the conditions at each point of the network, facilities can have different characteristics. Below are suggestions for future research in order to develop and improve the research: Considering the fact that considering the distance as a factor to improve the quality of demand coverage alone is not enough; therefore, considering limitations such as time, chain disturbances, along with paying attention to the distance criterion, can bring the model closer to the real world.

- Due to the fact that the facilities considered in this research are health facilities and also due to the importance of time to handle the demand in the health field, it can be useful to consider the queuing theory in this model.
- Given that the important goals in improving a network of health facilities are not limited to considering the cost and the amount of covered demand. It is appropriate that other goals of maximizing the quality of receiving service or reducing the concentration of demand coverage by a facility are also considered.
- Considering the backup facility, in such a way that every time a disruption occurs, a backup facility is considered for the disrupted facility.
- Taking into account the limits and goals that will not worsen the situation of demand coverage in any region or region after the implementation of the model. Currently, there may be a facility at level w in an area, but due to the non-establishment of facilities of other levels and the continued lack of full coverage of that area, the existing facility will also be closed or moved, which will worsen the conditions of the said area.



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