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Analytic Classes based on Bohr Inequality Using Salagean Operator

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Abstract:- Geometric Function Theory (GFT) primarily concerns itself with the theory of analytic functions, particularly focusing on analytic univalent functions that are normalized within this framework. The study defines analytic classes based on the Bohr inequality and explores various geometric properties of these classes using Bohr-type radii and inequalities within the unit disc via Salagean operator. It includes comparisons between established and recent findings, and examines alternating series of Bohr-type radii for the Taylor series of analytic functions.

Keywords*: Analytic function; Radius of Bohr inequality; Bohr type inequality; Salagean operator.*

1. Introduction

Geometric Function Theory (GFT) is a relatively modern field within complex analysis that investigates the geometric properties of functions in the unit disk, initially proposed towards the end of the 18th century, Function Theory gained momentum with the contributions of Nilsson and Koebe [1], who introduced concepts of univalent and multivalent functions, marking a significant advancement. Bohr [2], notably discovered crucial equivalences in this area, contributing to its foundational development.

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$$f(z)=\sum_{m=0}^{\infty}a_m z^m ,$$

(1)

Where,

$$|f(z)|<1,$$

$for\, all\, z \in U, then$

$$f(z)=\sum_{m=0}^{\infty}a_m z^m \leq 1. \qquad (2)$$

Geometric Function Theory (GFT) primarily concerns itself with the theory of analytic functions, particularly focusing on analytic univalent functions that are normalized within this framework. The study defines analytic classes based on the Bohr inequality and explores various geometric properties of these classes using Bohr-type radii and inequalities within the unit disc. It includes comparisons between established and recent findings, and examines alternating series of Bohr-type radii for the Taylor series of analytic functions Bohr originally established the inequality for $\frac{1}{6}$, later demonstrated to be sharp at $\frac{1}{3}$ by Wiener, Riesz, and Schur [3],[4]. Further supporting evidence can be found in [5, 6]. Dixon [7] established a connection between this inequality and its characterization in Banach algebras satisfying Von Neumann's inequality, which drew attention from operator algebraists. Recently, extensive research has focused on generalizing Bohr's theorem across various types of analytic functions [9]. For example, Aytuna and Djakov examined the Bohr phenomenon [10], while Aizenberg et al [11] explored its implications for holomorphic functions, and Ali et al [12] discovered the Bohr radius in 2016 for star-like logarithmic mappings. Additionally, Ali and Ng [13] expanded the classical Bohr inequality in the Poincaré disc model of the hyperbolic plane. Bayart et al [14] discovered an inequality concerning bounded analytic functions on the closed unit disc D in the complex plane, marking a significant development in the field. This inequality was subsequently modified by Wiener, Riesz, and Schur. Muhanna et al [15] explored the Bohr radius in relation to star-like and harmonic mappings within D, underscoring ongoing interest in this classical result. The Bohr radius continues to be a focal point across various branches of mathematics, as evidenced by the extensive body of work in recent years [16-20].

The exact Bohr-type radius was shown by Liu et al [21], and when we substitute $f(z)$ or its higher order derivatives for the coefficient of Bohr's inequality, we can deduce that the Bohr-type radius that results is less than the Bohr radius. This article's primary goal is to

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investigate the Bohr inequality for the class of Analytic functions defined in the complex plain that are generally simply connected domains.

Salagean [22] introduced a distinct operator, referred to as the Salagean operator. Janowski [23] introduced additional classes. Kuroki, Owa, and Srivastava [24, 25-35] discussed the extension of Janowski function. We study a new family of analytic functions in the open unit disc using the Salagean operator, deriving several inclusion relations from the Hadamard product. This operator allows us to create new classes of analytic functions based on various lemmas. Several features of these classes are explored, along with numerous sharp results.

Suppose that $G(z)$ is Halomophic function and is defined by Salagean operator as

$$G(z)=(1-\lambda)D^nG(z)+\lambda D^{n+1}G(z), \tag{3}$$

where $0 \leq \lambda \leq 1$ and using by,

$$D^nG(z)=z+\sum_{m=2}^{\infty}m^na_mz^m, n\in N,$$

(4)

$$D^{n+1}G(z)=z+\sum_{m=2}^{\infty}m^{n+1}a_mz^m.$$

(5)

By using the equation (3),

$$G(z)=(1-\lambda)\left(z+\sum_{m=2}^{\infty}m^na_mz^m\right)+\lambda\left(z+\sum_{m=2}^{\infty}m^{n+1}a_mz^m\right), \tag{6}$$

$$=z+\sum_{m+2}^{\infty}m^na_mz^m-\lambda z-\lambda\sum_{m=2}^{\infty}m^na_mz^m+\lambda z+\lambda\sum_{m=2}^{\infty}m^{n+1}a_mz^m, \tag{7}$$

$$=z+\sum_{m=2}^{\infty}m^na_mz^m-\lambda\sum_{m=2}^{\infty}m^na_mz^m+\lambda\sum_{m=2}^{\infty}m^{n+1}a_mz^m, \tag{8}$$

$$=z+\sum_{m=2}^{\infty}m^n(1-\lambda+\lambda m)a_mz^m, \tag{9}$$

Where

$$A_m=m^n(1-\lambda+\lambda m)a_m, m\in N.$$

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$$G(z) = z + \sum_{m=2}^{\infty} A_m z^m \ . \tag{10}$$

So

$$G'(z) = 1 + \sum_{m=2}^{\infty} mA_m z^{m-1}. \tag{11}$$

$$zG'(z) = z + \sum_{m=2}^{\infty} mA_m z^m. \tag{12}$$

2. Main Results

We initially present a result involving the Bohr-type radius. Equation (10), (11) and (12) represents Salagean operator equations which are utilized in Analytic functions, yielding the Bohr-type radius.

In this context, the Salagean operator is employed to determine the Bohr-type radius of certain Analytic functions.

Theorem 2.1 Suppose $G(z) = \sum_{m=0}^{\infty} A_m z^m$ is analytic in U and $|G(z)| \le 1$ in U.

Then

$$|G(z)| + |G'(z)| + \sum_{m=1}^{\infty} |a_m||z|^m \le 1 \ for \ |z| = r \le \frac{-74 - \sqrt[2]{1262}}{-107}. \tag{1}$$

By applying Salagean operator

$$\left| z + \sum_{m=2}^{\infty} A_m z^m \right| + \left| z + \sum_{m=2}^{\infty} mA_m z^m \right| + \sum_{m=1}^{\infty} |a_m||z|^m \le 1 \tag{2}$$

$$|z| + |z| + \left| \sum_{m=2}^{\infty} A_m z^m \right| + \left| \sum_{m=2}^{\infty} mA_m z^m \right| + \sum_{m=1}^{\infty} |a_m||z|^m \le 1$$

(3)

$$2|z| + \left| \sum_{m=2}^{\infty} A_m z^m + \sum_{m=2}^{\infty} mA_m z^m \right| + \sum_{m=1}^{\infty} |a_m||z|^m \le 1$$

(4)

Since

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$$|z| = r \tag{5}$$

$$2r + \sum_{m=2}^{\infty}(1+|m|)\sum_{m=2}^{\infty}|A_m||z|^m + \sum_{m=2}^{\infty}|a_m||z|^m \leq 1 \tag{6}$$

By using sum of series,

$$2r + (1+2+\cdots\infty)\sum_{m=2}^{\infty}|A_m||z|^m + \sum_{m=1}^{\infty}|a_m||z|^m \leq 1\,. \tag{7}$$

Using by lemma,

$$\sum_{m=2}^{\infty}|a_m||z|^m = \left(1-|\bar{a}|^2\right)\frac{r^2}{1-r^2}. \tag{8}$$

$$2r + \left(\tfrac{-1}{12}\right)\left(1-|\bar{a}|^2\right)r^2\frac{r^2}{1-r} + \left(1-|\bar{a}|^2\right)r^2\frac{r^2}{1-r} \leq 1 \tag{9}$$

$$\frac{24r - 24r^2 - 1 + r + |\bar{a}|^2 r^2 + 12 - 12|\bar{a}|^2 r^2}{12(1-r)} \leq 1. \tag{10}$$

Where,

$$|\bar{a}| < 1 \Rightarrow 2|\bar{a}| - 1 = 0 \Rightarrow |\bar{a}| = \frac{1}{2} \tag{11}$$

and,

$$2|\bar{a}| + 1 = 0 \Rightarrow |\bar{a}| = \frac{-1}{2} \tag{12}$$

$$25r - 24r^2 + 11 + r^2\left(\frac{-1}{2}\right)^2 - 12\left(\frac{-1}{2}\right)^2 r^2 \leq 12(1-r) \tag{13}$$

$$\frac{-107}{4}r^2 + 37r - 1 = 0\,. \tag{14}$$

Given radius of inequality, $\left(r - \frac{-74+2\sqrt{1262}}{-107}\right)\left(r + \frac{-74-2\sqrt{1262}}{-107}\right)$.

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Corollary 2.1 Consider the function,

$$G(z)=\frac{a-z}{1-az}=a-(1-a^2)\sum_{m=1}^{\infty}a^{m-1}z^m, z\in U \tag{15}$$

For this function,

$$|G(-r)|+|G'(-r)|+\sum_{m=1}^{\infty}|a_m|r^m=\frac{a+r}{(1+ar)^2}r+(1-a^2)\frac{ar^2}{1-ar}$$
(16)

The expression >1 if and only if,

$$(1-a)(-1+(2+a)r+a^2r^2+a^2(2a+1)r^3+a^3(1+a)r^4)\geq 0$$
(17)

After elementary calculation, we found that

$$=r+2ar^2+6a^2r^3+2ar^3+3a^2r^4+a^3r^4\geq 0$$
(18)

For any $r\in[0,1)$, so equation implies that,

$$-1+3r+r^2+3r^3+2r^4=2(1+r^2)\left(r-\frac{-74+2\sqrt{1262}}{-107}\right)\left(r+\frac{-74-2\sqrt{1262}}{-107}\right).$$

Therefore, equation (16) is less or equal to 1 for all $a\in[0,1)$, only in case when $r\leq\frac{-74-\sqrt[2]{1262}}{-107}$.

Theorem 2.2 Suppose $G(z)=z+\sum_{m=0}^{\infty}A_m z^m$ is analytic in U and $|G(z)|\leq 1$ in U. Then using Salagean operator.

$$|G(z)|+\sum_{m=1}^{\infty}|a_{2m}||z|^{2m}\leq 1 \quad for\ |z|=r\leq -1-\sqrt{3}..$$
(20)

By using equations $(10) and (8) to (20)$ yields the corresponding equality.

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$$\left|z+\sum_{m=2}^{\infty}A_m z^m\right|+\left(1-\left|\bar{a}\right|^2\right)\frac{r^2}{1-r^2}\leq 1 \tag{21}$$

$$|z|+\sum_{m=2}^{\infty}|A_m||z|^m+\left(1-\left|\bar{a}\right|^2\right)\frac{r^2}{1-r^2}\leq 1. \tag{22}$$

Since

$$|z|=r.$$

$$r+\left(1-\left|\bar{a}\right|^2\right)\frac{r^2}{1-r^2}+\left(1-\left|\bar{a}\right|^2\right)\frac{r^2}{1-r^2}\leq 1 \tag{23}$$

$$r+\frac{r^2}{1-r^2}-\frac{r^2\left|\bar{a}\right|^2}{1-r^2}+\frac{r^2}{1-r^2}-\frac{r^2\left|\bar{a}\right|^2}{1-r^2}\leq 1$$
(24)

$$\frac{r\left(1-r^2\right)+2r^2-2r^2\left|\bar{a}\right|^2}{1-r^2}\leq 1. \tag{25}$$

$$\left|\bar{a}\right|=\frac{1}{2} \tag{26}$$

$$r-r^3+2r^2-2r^2\left(\frac{1}{2}\right)^2\leq 1-r^2 \tag{27}$$

$$-r^3+3r^2-\frac{1}{2}r^2+r-1\leq 0 \tag{28}$$

$$=-r^3+\frac{5}{2}r^2+r-1, \tag{29}$$

$$=-\frac{1}{2}(2r-1)\left(r^2-2r-2\right). \tag{30}$$

$$=-\frac{1}{2}(2r-1)\left(r-1+\sqrt{3}\right)\left(r-1-\sqrt{3}\right).$$

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This demonstrates that the radius $r=\sqrt{3}-1$ is optimally feasible.

The corresponding alternating series (1.1) was defined by Ali et al [25], as:

$$\left||G(z)|+\sum_{m=2}^{\infty}(-1)^{m}|a_m||z|^m\right|\leq 1$$

Corollary 2.2 Suppose $G(z)=z+\sum_{m=0}^{\infty}A_m z^m$ is analytic in U and $|G(z)|\leq 1$ in U. Then applying Salagean operator.

$$\left||G(z)|+\sum_{m=2}^{\infty}(-1)^{m}|a_m||z|^m\right|\leq 1 \tag{31}$$

$$|G(z)|+\sum_{m=2}^{\infty}(-1)^{m}|a_m||z|^m\leq 1 \tag{32}$$

By applying Salagean operator

$$\left|z+\sum_{m=2}^{\infty}A_m z^m\right|+\sum_{m=2}^{\infty}|a_{2m}|r^{2m}-\sum_{m=2}^{\infty}|a_{2m-1}|r^{2m-1}\leq 1 \tag{33}$$

$$|z|+\sum_{m=2}^{\infty}|A_m||z|^m+\sum_{m=2}^{\infty}|a_{2m}||z|^{2m}\leq 1 \tag{34}$$

since

$$|z|=r.$$

The given equality is equal to (22) that is smaller than or equal to 1, which holds for $r\leq\sqrt{3}-1$.

So consider the following sequence of relations:

$$R(z)=|G(z)|+\sum_{m=2}^{\infty}|a_{2m}|r^{2m}-\sum_{m=2}^{\infty}|a_{2m-1}|r^{2m-1}, \tag{35}$$

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$$\geq -\sum_{m=2}^{\infty}|a_{2m-1}|r^{2m-1} = \left(\left|z+\sum_{m=2}^{\infty}A_m z^m\right| + \sum_{m=2}^{\infty}|a_{2m+1}|r^{2m+1}\right), \tag{36}$$

$$\geq -\left(\left|z+\sum_{m=2}^{\infty}A_m z^m\right| + \sum_{m=2}^{\infty}|a_{2m+1}|r^{2m}\right) \tag{37}$$

$$\geq -\left(G(z)+\sum_{m=2}^{\infty}|a_{2m+1}|r^{2m}\right). \tag{38}$$

Where the given inequality obtained by simple calculation.

Theorem 2.3 Suppose $G(z)=\sum_{m=0}^{\infty}A_m z^m$ is analytic in U and $|G(z)|\leq 1$ in unit disc U .

$$|G(z)|^2+\sum_{m=1}^{\infty}|a_{2m}||z|^{2m}\leq 1 \;\; for \;\; |z|=r\leq \frac{\sqrt{17}-1}{8}. \tag{39}$$

Inserting the Salagean operator (10) and lemma (8) into (39), we obtain;

$$\left|z+\sum_{m=2}^{\infty}A_m z^m\right|^2+\sum_{m=1}^{\infty}|a_m||z|^m\leq 1 \tag{40}$$

$$|z|^2+\sum_{m=2}^{\infty}|A_{2m}||z|^{2m}+2|z|\sum_{m=2}^{\infty}|A_m||z|^m+\sum_{m=1}^{\infty}|a_m||z|^m\leq 1. \tag{41}$$

$$|z|=r. \tag{42}$$

$$r^2+\sum_{m=2}^{\infty}|A_{2m}||z|^{2m}+2r\sum_{m=2}^{\infty}|A_m||z|^m+\sum_{m=1}^{\infty}|a_m||z|^m\leq 1 \tag{43}$$

$$r^2+\left(1-|\bar{a}|^2\right)\frac{r^2}{1-r}+2r\left(1-|\bar{a}|^2\right)\frac{r^2}{1-r}+\left(1-|\bar{a}|^2\right)\frac{r^2}{1-r}\leq 1 \tag{44}$$

$$\frac{r^2(1-r)+(1-r)-|\bar{a}|^2r^2+2r(1-r)-|\bar{a}|^2r^2+1-r-|\bar{a}|^2r^2}{1-r}\leq 1 \tag{45}$$

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$$=r^2-r^3+1-r-\left|\bar{a}\right|^2r^2+2r-2r^2-\left|\bar{a}\right|^2r^2+1-r-\left|\bar{a}\right|^2r^2-1+r, \tag{46}$$

$$=-r^3-3\left|\bar{a}\right|^2r^2-r^2+r+1. \tag{47}$$

$$=-r^3-\left(3\left(\frac{1}{4}\right)+1\right)r^2+r+1, \tag{48}$$

$$=-r^3-\frac{7}{4}r^2+r+1, \tag{50}$$

$$=-\frac{1}{4}(2+r)\left(4r^2-r-2\right). \tag{51}$$

The radius of the given inequality is $r=-\frac{1}{4}(2+r)\left(r+\frac{\sqrt{33}+1}{8}\right)\left(r-\frac{\sqrt{33}-1}{8}\right).$

Theorem 2.4 Suppose $G(z)=z+\sum_{m=0}^{\infty}A_m z^m$ is analytic in unit disc U and $|G(z)|\le 1$ in unit disc U. Then prove $G(z)=\bar{a}$ is positive then by applying Salagean operator.

$$|G(z)|+\sum_{m=1}^{\infty}|a_{nm}||z|^{nm}\le 1 \;\; for \;\; |z|=r\le R_N, \tag{53}$$

Where R_N is the best possible.

Applying the provided lemma and Salagean operator (10) in the context of (53).

$$\sum_{m=2}^{\infty}|a_{nm}||z|^{nm} \;=2(1-\bar{a})\frac{r^n}{1-r^n},$$

Then,

$$\left|z+\sum_{m=2}^{\infty}A_m z^m\right|+2(1-\bar{a})\frac{r^n}{1-r^n}\le 1 \tag{54}$$

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$$|z|+\sum_{m=2}^{\infty}|A_m||z|^m+2\left(1-\bar{a}\right)\frac{r^n}{1-r^n}\leq 1. \tag{55}$$

$$\because |z|=r \tag{56}$$

$$r+\left(1-\left|\bar{a}\right|^2\frac{r^2}{1-r}\right)+2\left(1-\bar{a}\right)\frac{r^n}{1-r^n}\leq 1 \tag{57}$$

$$\frac{r(1-r)(1-r^n)+(1-r)(1-r^n)-\left|\bar{a}\right|^2 r^2(1-r^n)+2(1-r)(1-a)r^n}{(1-r)(1-r^n)}\leq 1 \tag{58}$$

$$r(1-r)(1-r^n)+(1-r)(1-r^n)-\left|\bar{a}\right|^2 r^2(1-r^n)+2(1-r)(1-a)r^n-(1-r)(1-r^n)\leq 1 \tag{59}$$

$$r-r^2-\left|\bar{a}\right|^2 r^2+2r^n-2ar^n-3r^{1+n}+2ar^{1+n}+r^{2+n}+a^2r^{2+n}\leq 1, \tag{60}$$

by simplifying,

$$r-\left(1+a^2\right)r^2-2(-1+a)r^n+(-3+2a)r^{1+n}+\left(1+a^2\right)r^{2+n}. \tag{61}$$

Equation (61) is greater then zero. This prove is sharpness and $r>R_N$. This prove represents sharpness.

In the presented paper, the majority of outcomes derived from the specified equality exhibit a high degree of precision.

We extracted most of the approximately sharp results with Bohr Inequality by applying Salagean operator.

3. Application

The Bohr inequality, named after Danish mathematician Harald Bohr, is a significant result in complex analysis and number theory. It provides bounds on the absolute values of Dirichlet series within specific regions. Specifically, the Bohr inequality addresses the convergence of Dirichlet series and power series in the context of bounded holomorphic functions. Bohr inequality has several important applications. The Bohr inequality is utilized to study the behavior of holomorphic functions (complex functions differentiable in a neighborhood of every point in their domain) and Dirichlet series. It aids in understanding uniform

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convergence and in bounding the values of these series within specific regions of the complex plane. In functional analysis, the Bohr inequality helps analyze the properties of various functional spaces. It aids in bounding function norms and understanding the compactness and boundedness of operators acting on these spaces. In complex analysis, the Bohr inequality aids in investigating the properties of power series. It can be used to derive results concerning the radius of convergence and the maximum modulus of these series within specific disks in the complex plane.

The Salagean operator is a pivotal concept within geometric function theory, particularly in the examination of univalent and analytic functions. Named after Gheorghe S. Salagean, this operator finds extensive applications across different realms of mathematical analysis. It plays a crucial role in areas such as Univalent Function Theory, Subordination, Differential Subordinations, Geometric Properties, Coefficient Problems, and Operator Theory. In the study of univalent functions, the Salagean operator is instrumental in generating new classes of functions that are injective within the unit disk and other domains. It facilitates the construction of functions with specific geometric and analytical properties. Moreover, in operator theory, the Salagean operator is utilized to define and explore other operators acting on spaces of analytic functions, contributing to a deeper understanding of these function spaces' structural characteristics. By harnessing the Salagean operator, researchers can systematically investigate various facets of analytic functions, leading to novel insights and advancements in complex analysis.

4. Conclusion

By strategically employing the Salagean operator, we achieve a precise determination of the Bohr-type radius. Our thorough analysis not only outlines the method for accurately calculating this radius but also emphasizes the importance of integrating various lemmas into this approach. This multifaceted strategy enhances our comprehension and establishes a robust framework for investigating the Bohr-type radius with enhanced accuracy and depth.

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