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A Novel Class of Ratio Estimators Incorporating Auxiliary Variables in Simple Random Sampling for Estimating Population Mean: An Application to Real-Life Data

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Abstract

This paper presents a novel class of ratio estimators using non-conventional measures as auxiliary information to estimate the population mean of the study variable under simple random sampling without replacement. Three special cases are discussed, utilizing the coefficient of variation, median, and quartile deviation as auxiliary variables. The bias and mean square error (MSE) of the proposed estimators are derived up to the first order of approximation. Furthermore, theoretical conditions are established to compare the proposed class with existing estimators, supported by real-life data. Numerical results demonstrate that the proposed class of estimators is more efficient than traditional and other ratio-type estimators.

Keywords: Coefficient of variation, median, quartile deviation, bias, mean square error."

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Introduction

It is usual practice to use sampling techniques when the population is very large, or it is difficult to obtain information about each and every unit of the population. The sampling technique is the best alternative to study the behavior of the study variable. In sampling techniques, we take a small portion of a population and draw inferences about the whole population. It is very common to estimate the population parameter by its sample statistic, for example, to estimate the population mean, an appropriate estimator is a sample mean which is an unbiased estimator but hasa large variance. Lots of researches have been done to derive such an estimator for the population mean which is more efficient than the existing ones. These research areas are concerned to make use of auxiliary information to improve the sample estimate of the population parameter. For example, Kadilar and Cingi [1, 2] produced a ratio estimator in stratified random sampling scheme, Subramani and Kumarapandiyan [3] worked on the modified ratio estimator by using the quartiles as auxiliary variables, Yadav et.al [4] improved the estimation of the population mean by using non-Conventional measures of dispersion, Subzar et.al [5] worked on the new class of efficient ratio estimators, Sisodia and Dwivedi [6] improved the classical ratio estimator by use of a coefficient of variation as an auxiliary variable.Jeelani et.al [7] presented a Modified ratio estimator by using the linear combination of the coefficient of skewness and quartile deviation, for other estimators, we refer to see [8-20]. The most recent contributions to this area of research are presented byKumar et.al [21], Unal and Kadilar [22], Singhand Usman [23], and Priam [24].

The basic aim of the paper is to produce a new class of ratio estimators which improved the precision of estimators by using the auxiliary information. We obtain MSE of the class of proposed estimators. The class of proposed estimators and other existing estimators were compared at both theoretical conditions and numerical illustrations.

Notation

The following notation would be of interest in the research work.

N – Size of the population, n – Size of the sample, Y – Study variable

X – Auxiliary variable, $\bar{X}, \bar{Y}$ - Population means, $\bar{x}, \bar{y}$ - Sample means

S_x, S_y - Population standard deviation, C_x, C_y - Coefficient of variation

ρ_{xy} - Population Correlation coefficient between X and Y, $\beta_{2(x)}$ - Kurtosis of auxiliary variable

M_d - Median, Q.D – Quartile deviation, MSE – Mean square error

SRSWOR – Simple random sampling without replacement.

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Review of existing Estimators

The concept of an auxiliary variable was suggested by Cochron [10] for the first time to estimate the population mean of the variable of interest. The classical estimator proposed by [10] is given by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \quad (1)$$

The MSE of the above proposed estimator up to the first degree of approximation is

$$MSE(\bar{y}_r) \cong \frac{1-f}{n}(R^2 S_x^2 - 2RS_{xy} + S_y^2) \quad (2)$$

where $f = n/N$, and $R = \bar{Y}/\bar{X}$ is the population ratio.

Sisodia and Dwivedi [6] presented a new modified ratio estimator. The estimator is defined by

$$\bar{y}_{SD} = \bar{y}\frac{\bar{X}+C_x}{\bar{x}+C_x} = \frac{\bar{y}}{\bar{x}_{SD}}\bar{X}_{SD} = \hat{R}_{SD}\bar{X}_{SD} \quad (3)$$

The MSE to the first degree of approximation is given as

$$MSE(\bar{y}_{SD}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\alpha(\alpha-2k)\right] \quad (4)$$

Where C_x is the coefficient of variation of the auxiliary variable, C_y is the coefficient of variation of the study variable, $k = \rho\frac{C_y}{C_x}$ (ρ is correlation coefficient between Y and X) and $\alpha = \frac{\bar{X}}{\bar{X}+C_X}$.

Singh and Kakran [8] defined modified ratio estimator by using the coefficient of Kurtosis as auxiliary variable. The modified estimator is expressed as

$$\bar{y}_{Sk} = \bar{y}\frac{\bar{X}+\beta_{2(x)}}{\bar{x}+\beta_{2(x)}} = \frac{\bar{y}}{\bar{x}_{Sk}}\bar{X}_{Sk} = \hat{R}_{Sk}\bar{X}_{Sk} \quad (5)$$

MSE corresponding to the above expression up to the first degree of approximation is given by

$$MSE(\bar{y}_{SK}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\delta(\delta-2k)\right] \quad (6)$$

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where $\delta = \frac{\bar{X}}{\bar{X}+\beta_2}$ and $k = \rho \frac{C_y}{C_x}$ (ρ is the coefficient of correlation between Y and X)

The estimator suggested by Upadhyaya and Singh [11] is defined by

$$\bar{y}_{US1} = \bar{y}\frac{\bar{X}\beta_{2(x)} + C_x}{\bar{x}\beta_{2(x)} + C_x} = \frac{\bar{y}}{\bar{x}_{US1}}\bar{X}_{US1} = \hat{R}_{US1}\bar{X}_{US1} \quad (7)$$

$$\bar{y}_{US2} = \bar{y}\frac{\bar{X}C_x + \beta_{2(x)}}{\bar{x}C_x + \beta_{2(x)}} = \frac{\bar{y}}{\bar{x}_{US2}}\bar{X}_{US2} = \hat{R}_{US2}\bar{X}_{US2} \quad (8)$$

The related MSE up to the first degree of approximation is

$$MSE(\bar{y}_{US1}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\omega(\omega - 2k)\right] \quad (9)$$

$$MSE(\bar{y}_{US2}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \vartheta(\vartheta - 2k)C_x^2\right] \quad (10)$$

where w and ϑ are the real numbers which is defined by

$w = \frac{\bar{X}\beta_{2(x)}}{\bar{x}\beta_{2(x)} + C_x}$, $\vartheta = \frac{\bar{X}C_x}{\bar{x}C_x + \beta_{2(x)}}$ and $k = \rho\frac{c_y}{c_x}$ (ρ is the coefficient of correlation between Y and X)

Proposed estimators

We propose a class of efficient ratio estimator and presents three of its special cases by using coefficient of variation, median, and quartile deviation as auxiliary variables. The suggested class of ratio estimators is defined by

$$y_{bi} = \bar{y}\left[\frac{\bar{X} + \beta_{2(x)}/G(x)}{\bar{x} + \beta_{2(x)}/G(x)}\right] \quad (11)$$

where $G(x)$ is the general auxiliary variable which would be chosen by choice and $\beta_{2(x)}$ is the kurtosis of the auxiliary variable.

The special cases of the above general class of estimatorsrespectively then take the following form

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$$y_{b1} = \bar{y}\left[\frac{\bar{X} + \beta_{2(x)}/C_x}{\bar{x} + \beta_{2(x)}/C_x}\right] \quad (12)$$

where C_x is the coefficient of variation of the auxiliary variable X.

$$y_{b2} = \bar{y}\left[\frac{\bar{X} + \beta_{2(x)}/M_d}{\bar{x} + \beta_{2(x)}/M_d}\right] \quad (13)$$

where M_d is the median of the auxiliary variable X.

$$y_{b3} = \bar{y}\left[\frac{\bar{X} + \beta_{2(x)}/QD}{\bar{x} + \beta_{2(x)}/QD}\right] \quad (14)$$

where QD is the quartile deviation of the auxiliary variable X.

The bias and mean square error of the proposed estimators up to the first order of approximation is given by

$$Bias(\bar{y}_{bi}) \cong \bar{Y}\left(\frac{1-f}{n}\right)\left(\frac{\theta^2}{2!}C_x^2 - \theta_i \rho_{xy} C_x C_y\right) \quad (15)$$

$$\text{MSE}(\bar{y}_{bi}) = \bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i \rho_{xy} C_x C_y\right) \quad (16)$$

where i = 1, 2, 3

and $\theta_1 = \dfrac{\bar{X}}{\bar{X} + \beta_{2(x)}/C_x}, \theta_2 = \dfrac{\bar{X}}{\bar{X} + \beta_{2(x)}/M_d}, \theta_3 = \dfrac{\bar{X}}{\bar{X} + \beta_{2(x)}/QD}.$

Efficiency Comparison

In this section, the proposed class of estimators is compared at theoretical conditions with other existing estimators of the population mean under simple random sampling without replacement scheme.

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Comparison with usual ratio estimator

We compare the MSE of the proposed class of estimators given in Eq (16) with the MSE of the classical ratio estimator given in Eq (2). The proposed estimator performs better than the classical ratio estimator if,

$$\mathrm{MSE}(\bar{y}_{bi}) < MSE(\bar{y}_r)$$
$$\bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right) < \left(\frac{1-f}{n}\right)(R^2S_x^2-2RS_{xy}+S_y^2)$$
$$\bar{Y}^2\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right) < (R^2S_x^2-2RS_{xy}+S_y^2)$$
$$\bar{Y}^2\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right) < \left(\bar{Y}^2C_x^2-2\bar{Y}^2\rho_{xy}C_xC_y+\bar{Y}^2C_y^2\right)$$
$$\bar{Y}^2\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right) < \bar{Y}^2\left(C_x^2-2\rho_{xy}C_xC_y+C_y^2\right)$$
$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y < C_x^2-2\rho_{xy}C_xC_y$$
$$\theta_i^2C_x^2-C_x^2 < -2\rho_{xy}C_xC_y+2\theta_i\rho_{xy}C_xC_y$$

$$C_x^2\left(\theta_i^2-1\right) < 2\rho_{xy}C_xC_y\left(-1+\theta_i\right)$$
$$C_x\left(\theta_i-1\right)\left(\theta_i+1\right) < 2\rho_{xy}C_y\left(\theta_i-1\right)$$
$$2\rho_{xy}C_y > C_x\left(\theta_i+1\right)$$

$$\rho_{xy} > \frac{C_x\left(\theta_i+1\right)}{2C_y} \tag{17}$$

where, i=1,2,3 and $\theta_1=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/C_x}, \theta_2=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/M_d}, \theta_3=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/QD}$.

Comparison with Sisodia and Dwivedi 1981 estimator

We compare the MSE of the proposed class of estimators given in Eq (16) with the MSE given in Eq (4). The proposed estimator performs better than the Sisodia and Dwivedi estimator if,

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$$\text{MSE}(\bar{y}_{bi}) < MSE(\bar{y}_{SD})$$

$$\bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right)<\left(\frac{1-f}{n}\right)\bar{Y}^2\left[C_y^2+C_x^2\alpha(\alpha-2k)\right]$$

$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y<C_x^2\alpha(\alpha-2k)$$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\alpha(\alpha-2k)$$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\alpha^2-2C_x\alpha k$$

By putting $k=\rho\dfrac{C_y}{C_x}$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\alpha^2-2\alpha\rho_{xy}C_y$$

$$\theta_i^2C_x-C_x\alpha^2<-2\alpha\rho C_y+2\theta_i\rho_{xy}C_y$$

$$C_x\left(\theta_i^2-\alpha^2\right)<2\rho_{xy}C_y\left(-\alpha+\theta_i\right)$$

$$C_x\left(\theta_i^2-\alpha^2\right)<2\rho_{xy}C_y\left(\theta_i-\alpha\right)$$

$$C_x\left(\theta_i-\alpha\right)\left(\theta_i+\alpha\right)<2\rho_{xy}C_y\left(\theta_i-\alpha\right)$$

$$C_x\left(\theta_i+\alpha\right)<2\rho_{xy}C_y$$

$$2\rho_{xy}C_y>C_x\left(\theta_i+\alpha\right)$$

$$\rho_{xy}>\frac{C_x}{2C_y}\left(\theta_i+\alpha\right) \tag{18}$$

where, i=1,2,3 and $\theta_1=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/C_x},\theta_2=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/M_d},\theta_3=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/QD},\alpha=\dfrac{\bar{X}}{\bar{X}+C_X}$.

Comparison with Singh and Kakran 1993 estimator

We compare the MSE of the proposed class of estimators given in Eq (16) with the MSE given in Eq (6). The proposed estimator performs better than the Singh and Kakran estimator if,

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$$\text{MSE}(\bar{y}_{bi}) < MSE(\bar{y}_{SK})$$
$$\bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right) < \left(\frac{1-f}{n}\right)\bar{Y}^2\left[C_y^2+C_x^2\delta(\delta-2k)\right]$$
$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y < C_y^2+C_x^2\delta(\delta-2k)$$
$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y < C_x^2\delta^2-2C_x^2\delta k$$
$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y < C_x\delta^2-2C_x\delta k$$

By putting $k=\rho\frac{C_y}{C_x}$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y < C_x\delta^2-2C_y\rho_{xy}\delta$$
$$\theta_i^2C_x-C_x\delta^2 < -2C_y\rho_{xy}\delta+2\theta_i\rho_{xy}C_y$$
$$C_x\left(\theta_i^2-\delta^2\right) < 2C_y\rho_{xy}\left(-\delta+\theta_i\right)$$
$$C_x\left(\theta_i^2-\delta^2\right) < 2C_y\rho_{xy}\left(\theta_i-\delta\right)$$
$$C_x\left(\theta_i-\delta\right)\left(\theta_i+\delta\right) < 2C_y\rho_{xy}\left(\theta_i-\delta\right)$$
$$C_x\left(\theta_i+\delta\right) < 2C_y\rho_{xy}$$
$$2C_y\rho_{xy} > C_x\left(\theta_i+\delta\right)$$

$$\rho_{xy} > \frac{C_x}{2C_y}\left(\theta_i+\delta\right) \tag{19}$$

where, i=1,2,3 and $\theta_1=\frac{\bar{X}}{\bar{X}+\beta_{2(x)}/C_x}, \theta_2=\frac{\bar{X}}{\bar{X}+\beta_{2(x)}/M_d}, \theta_3=\frac{\bar{X}}{\bar{X}+\beta_{2(x)}/QD}$, $\delta=\frac{\bar{X}}{\bar{X}+\beta_2}$

Comparison with Upadhyaya and Singh 1999 estimator

We compare the MSE of the proposed class of estimators given in Eq (16) with the MSE given in Eq (8)and (10). The proposed estimator performs better than the Upadhyaya and Singh estimator if,

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$$\text{MSE}(\bar{y}_{bi}) < MSE(\bar{y}_{US1})$$

$$\bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right)<\left(\frac{1-f}{n}\right)\bar{Y}^2\left[C_y^2+C_x^2\omega(\omega-2k)\right]$$

$$\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right)<\left[C_y^2+C_x^2\omega(\omega-2k)\right]$$

$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y<C_x^2\omega(\omega-2k)$$

$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y<C_x^2\omega^2-2\omega C_x^2k$$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\omega^2-2C_x\omega k$$

By putting $k=\rho\dfrac{C_y}{C_x}$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\omega^2-2C_y\rho_{xy}\omega$$

$$\theta_i^2C_x-C_x\omega^2<-2C_y\rho_{xy}\omega+2\theta_i\rho_{xy}C_y$$

$$C_x\left(\theta_i^2-\omega^2\right)<2C_y\rho_{xy}\left(-\omega+\theta_i\right)$$

$$C_x\left(\theta_i-\omega\right)\left(\theta_i+\omega\right)<2C_y\rho_{xy}\left(\theta_i-\omega\right)$$

$$C_x\left(\theta_i+\omega\right)<2C_y\rho_{xy}$$

$$2C_y\rho_{xy}>C_x\left(\theta_i+\omega\right)$$

$$\rho_{xy}>\frac{C_x}{2C_y}\left(\theta_i+\omega\right) \tag{20}$$

where, i=1,2,3 and $\theta_1=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/C_x}$, $\theta_2=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/M_d}$, $\theta_3=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}/QD}$, $w=\dfrac{\bar{X}\beta_{2(x)}}{\bar{x}\beta_{2(x)}+C_x}$

Comparison with Upadhyaya and Singh 1999estimator

$$\text{MSE}(\bar{y}_{bi}) < MSE(\bar{y}_{US2})$$

$$\bar{Y}^2\left(\frac{1-f}{n}\right)\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right)<\left(\frac{1-f}{n}\right)\bar{Y}^2\left[C_y^2+C_x^2\vartheta(\vartheta-2k)\right]$$

$$\left(C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y\right)<\left[C_y^2+C_x^2\vartheta(\vartheta-2k)\right]$$

$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y<C_x^2\vartheta(\vartheta-2k)$$

$$\theta_i^2C_x^2-2\theta_i\rho_{xy}C_xC_y<C_x^2\vartheta^2-2\vartheta C_x^2k$$

$$\theta_i^2C_x-2\theta_i\rho_{xy}C_y<C_x\vartheta^2-2C_x\vartheta k$$

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By putting $k=\rho\frac{C_y}{C_x}$

$$\theta_i^2 C_x - 2\theta_i \rho_{xy} C_y < C_x \vartheta^2 - 2C_y \rho_{xy} \vartheta$$
$$\theta_i^2 C_x - C_x \vartheta^2 < -2C_y \rho_{xy} \vartheta + 2\theta_i \rho_{xy} C_y$$
$$C_x\left(\theta_i^2 - \vartheta^2\right) < 2C_y \rho_{xy}\left(-\vartheta + \theta_i\right)$$
$$C_x\left(\theta_i - \vartheta\right)\left(\theta_i + \vartheta\right) < 2C_y \rho_{xy}\left(\theta_i - \vartheta\right)$$
$$C_x\left(\theta_i + \vartheta\right) < 2C_y \rho_{xy}$$
$$2C_y \rho_{xy} > C_x\left(\theta_i + \vartheta\right)$$

$$\rho_{xy} > \frac{C_x}{2C_y}\left(\theta_i + \vartheta\right) \tag{21}$$

where, i=1,2,3 and $\theta_1 = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}/C_x}, \theta_2 = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}/M_d}, \theta_3 = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}/QD}$ $\vartheta = \frac{\bar{X}C_x}{\bar{x}C_x + \beta_{2(x)}}$

Numerical illustration to applied data

For numerical illustration, the data set is taken from Kadilar and Cingi [2]. The data set represents the amount of apple production (as a variable of interest) and the number of apple trees (as an auxiliary variable) in 106 villages of the Aegean Region in 1999. The statistics about the population parameters is given below in table 1. It is noted that the correlation between study and the auxiliary variable is 86% and we take a sample of size n=20. It is of keen interest that the efficiency of the estimators is independent of a sample size n.

MSE of the proposed class of estimators as well as other existing estimators are computed and is given in table 2.As a general rule, the estimator with the fever value of MSE leads to a more precise result. It has been noted that the proposed estimators have fever MSE than others and hence we obtain satisfactory results.

Table 1. Auxiliary information

N=106	n=20	$\bar{Y}$=2212.59	$\bar{X}$=27421.70	S_{yx}=568176176.10	S_y=11551.53
C_y=5.22	M_d=7297.50	QD=12156.25	S_x=57460.61	C_x=2.10	$\beta_{2(x)}$=34.57

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α=0.9999	δ =0.9987	k=2.1327	ω=0.9999	ϑ =0.9994	$\rho = 0.86$
R=0.0807	R_{SD}=0.0807	R_{SK}=0.0806	R_{US1}=0.0023	R_{us2}=0.0385	

Table 2. constant and MSE of the proposed and other estimators.

Estimator	Constant	MSE	PRE
$\bar{y}_r$	------	2565510.488	100
$\bar{y}_{SD}$	α=0.9999 k=2.1327	2551970.93	100.53
$\bar{y}_{Sk}$	α=0.9999 δ =0.9987	2554353.47	100.437
$\bar{y}_{US1}$	ω=0.9999 k=2.1327	2551970.931	100.531
$\bar{y}_{US2}$	ϑ =0.9994 k=2.1327	2552963.345	100.491
y_{b1}	θ_1 =0.999400032	**2544185.217**	**100.838**
y_{b2}	θ_2 =0.9999998	**2542989.056**	**100.886**
y_{b3}	θ_3 =0.9999999	**2542988.851**	**100.886**

Table 3. Efficiency Comparison of equation (17),(18),(19),(20),(21)

Equation(17)	0.86>0.4022	0.86>0.4023	0.86>0.402298
Equation(18)	0.86>0.40216	0.86>0.4023	0.86>0.402279
Equation(19)	0.86>0.401917	0.86>0.40204	0.86>0.402037
Equation(20)	0.86>0.4022	0.86>0.4023	0.86>0.402298
Equation(21)	0.86>0.40206	0.86>0.40218	0.86>0.40218

Table 4: PRE of the Proposed Estimators with the Estimators in Literature for this population

Estimators	$\bar{y}_r$	$\bar{y}_{SD}$	$\bar{y}_{Sk}$	$\bar{y}_{US1}$	$\bar{y}_{US2}$
y_{b1}	100.838	100.306	100.399	100.306	100.345
y_{b2}	100.886	100.353	100.447	100.353	100.392

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y_{b3}	100.886	100.353	100.447	100.353	100.392

Result and Conclusion

The paper presents an improved class of ratio estimators for estimating the population mean by having auxiliary information. The class of estimators has been proposed under simple random sampling without replacement scheme. The equations for bias and mean square error have been obtainedup to the first order of approximation.For illustration, a real data set have been used and the results reveal that the proposed class of estimators improves the efficiency of estimation thanother existing estimators. Moreover, among the proposed estimators, the estimator y_{b3} performs better than rest of the two estimators.

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