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A Study of Diverse Modifications in Particle Swarm Optimization

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Abstract:- This paper presents a comprehensive study of the diverse modifications to Particle Swarm Optimization (PSO) for solving science and engineering-related optimization problems. PSO is a powerful, easy-to-implement, gradient-free optimization technique, and its versatility has led researchers to apply it to both simple and complex systems. In most optimization problems, we want to find the global optimal solution, so we need an optimizer which can successfully do this job, but PSO, in its basic form has the problem of premature convergence, due to which, sometimes, it traps in local optima and does not reach to the global solution. In this point of view, many researchers have made their efforts to eliminate this problem. If we go through the literature about improving the performance of PSO in diverse directions, then we will find four major directions of improving the PSO, which are new formulation for the basic parameters of PSO, Adding new parameters, alteration in the topology of basic PSO and hybridization with other algorithms. This study concludes that the PSO is a dynamic optimizer, which can efficiently solve the problems of numerous areas of science and engineering.

***Keywords**:* Optimization, Algorithm, Metaheuristic, Particle Swarm Optimization (PSO), Global Optimization, Computational Mathematics

INTRODUCTION

A wide range of science and engineering problems are, by definition, inverse problems, where the output is known, but the input or the system itself is unknown. Most of these problems include the optimization of multimodal cost functions with continuous and/or discrete variables. Currently, there are a lot of traditional deterministic optimization techniques, but in most cases, they fail to find the optimal solution for multimodal and high-dimension real-world problems. For this purpose, different stochastic evolutionary algorithms have been developed, including, but not limited to Genetic algorithm, differential evolution, ant colony algorithm and PSO. PSO is one of the most powerful, easy to implement and new techniques, developed by Kennedy and Eberhart in 1995 [1]. The PSO's original idea has been taken from the natural behaviour of birds and animals when they try to find their food in the form of a swarm. Due to the meta-heuristic features of PSO, it is well suited for solving complex and multidimensional problems. Accordingly, different variants of PSOs have been proposed and have been found useful in many research areas.

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It is a reality that no single optimization algorithm is capable of finding the optimal solutions to all the engineering and scientific problems, so the same case is with PSO, as it has the problem of premature convergence because of using the constant parameters and inadequate selection of the personal and global best particles. Therefore sometimes it weakens to find the global optimal solution of high dimension real world problems. Now, to handle the problems of diversified areas the researchers have made efforts to improve the performance of PSO.

BASIC PARTICLE SWARM OPTIMIZATION

Swarm intelligence is based on the social interaction and collective performance of the distributed and self-organized systems, presented by Gerardo Beni and his fellow Jing Wang in 1989 [2], from the perspective of cellular robotic systems. Today there are several optimization algorithms originated from swarm intelligence, i.e. Artificial Bee Colony (ABC), Artificial Immune System (AIS), Particle swarm optimization (PSO) and Ant Colony Optimization (ACO).

PSO is a member of swarm intelligence algorithms and an efficient population based evolutionary algorithm, inspired by the collective flying of birds to search the food. In 1995, Dr. Russell C. Eberhart (Electrical engineer) and Dr. James Kennedy (Social Psychologist) [1] proposed the idea of the PSO algorithm after deeply studying the social and cognitive behaviour of birds flying in the form of flocks for searching the food. They observed, that initially, all the birds fly in random directions, but when a bird finds himself closer to the food, it informs the other birds by producing a special sound, this is social interaction and then all the birds follow him. At the same time, each bird also tries to come closer to food by using his natural senses, this is cognitive learning. During this whole process, birds are in continuous movement and change their velocity and position. In light of these observations, the two scientists mentioned above modified the famous equations of motion to fit the above discussed scenario and presented the mathematical model of PSO consisting of two equations, one is for updating the velocity of the particle, while another is for adapting a new position. After presenting the PSO model, they applied it to different optimization problems to test its effectiveness and the results were amazing. Presently the PSO is one of the powerful optimization algorithms for solving the systems of non-linear equations and has been successfully applied to power systems [3][4][5], automatic systems [6][7], image processing techniques [8], magnetic storage devices [9], robotics [10] and many others.

In PSO, a number of random particles, sometimes referred to as agents are generated; all the individual particles represent a solution to the problem. In a second step, the algorithm tries to find the best solution in the entire search space, by evaluating the fitness of the objective function. In case of not finding the required solution, the velocity of particles is updated according to the velocity equation and consequently the position of particles is also updated and the process of searching for the best solution by evaluating the objective function is repeated. This practice is continued until the required criteria occur, which is to find the best solution or maximum number of iterations. The mathematical model of PSO is given below in Eq.1 and Eq. 2. Eq. 1 is for updating the velocity of the individual particles, while Eq. 2 is for updating the positions of the individual particles.

$$v_i^{k+1} = w \times v_i^k + c_1 \times r_1 . \times (p^k{}_{best} - x^k{}_i) + c_2 \times r_2 (g^k{}_{best} - x^k{}_i) \quad 1$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad 2$$

The description of all parameters in equation 1 and 2 is given in Table 2.1.

Table 1 Description of parameters of Basic PSO

Parameter	Description
w	Inertia Weight (Control Parameter)
$c1$	Cognitive component (Self-learning parameter)
$c2$	Social component (Social learning parameter)
$r1, r2$	Random numbers
p_{best}	Personal best position of an individual particle
g_{best}	Global best position
x^k_i	Current Position of a Particle
v^k_i	The velocity of a particle in the current iteration

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Before explaining the role of these parameters, it is mandatory to explain the two important terms related to optimization, which are exploitation and exploration. Exploitation is the property of an optimizer to stick with the local or nearby points in the search space and search for the best solution, while exploration is the ability of an optimizer to leave the local solutions in the search space and search for better solutions in the entire search space. The control parameter "w", also known as inertia weight maintains a proper balance between global and local searches of the particle during the optimization process. Every individual remembers two different positions, the personal best and the global best. The personal best or Pbest position is the particle's own best position achieved so far, while the global best or g_{best} position is the best position in the entire swarm during the current iteration [11].

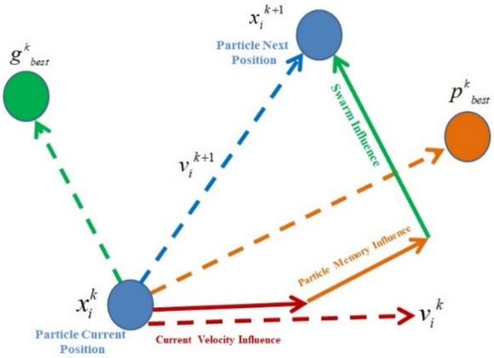

Figure 1 (Demonstration of adopting new position by a particle)

The learning parameters guide the global best particle and personal best particle during the search process. When all the particles participate in searching for the target randomly and independently and share the information, then, as a result, they move towards the target by following diverse paths and directions. Hence, a large search space is explored and there are maximum chances of convergence to the global optima.

In every iteration, the particles change their positions as described above and Figure 1 demonstrates this phenomenon of updating their positions and the factors involved. It is clear from the figure, that each particle regulates its travelling velocity dynamically based on its own learning experience (cognitive learning) and the experience of its colleagues (social learning). In other words, three different factors influence a particle to adopt a new position, its current velocity, the global best position of the whole swarm and the particle's own best position.

LITERATURE REVIEW

In this section, extensive literature has been reviewed about PSO. The PSO can be effectively used to optimize a variety of problems from simple to hard but to tackle the high dimension complex problems the traditional PSO must be properly tuned to avoid premature convergence. In this regard, researchers have modified the basic PSO in four major directions, as given in Table 2. The following subsections cover the literature based on these four directions.

Table 2 Directions of modification in PSO

S. No	Modification
1	By new formulation for basic parameters
2	By adding a new operator
3	By change in topology
4	By hybridization with other Algorithm

a. Modification by new formulation for basic parameters

During the search process, the three main parameters are of great importance in the performance of PSO. Hence, different researchers amended these parameters to enhance the swarm diversity to avoid early convergence and to

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make it a global optimizer with more robustness. The initial few paragraphs of this section present the modifications by the founders of PSO and after that, the modifications made by different researchers have been discussed. Eberhart modified the PSO by including a control parameter in the traditional PSO [12]. Further, they modified the equation for inertia weight [15], given below:

$$\omega = 0.5 + (rand(\ )/2) \quad 3$$

Where rand() is a uniform random number between 0 and 1. To solve a dynamic optimal problem, Eberhart proposed to use a pure random number. A nonlinear decreasing approach was introduced in [13] for adjusting this control parameter to enhance the search abilities that can ignore the local optimal solution and find the global one. There is also a relation between the function value and Inertia weight in [14], and is formulated as

$$w = w_{\max} - (w_{\max} - w_{\min}) \times (f - f_{\min}) / (f^{-} - f_{\min}) \text{ if } f <= f^{-} \quad 4$$

$$w = w_{\max}, \text{ if } f > f^{-} \quad 5$$

where, f represents the objective function value in the current iteration, f - is the average value of the objective function, f_{min} is the lowest value of the objective function. The Wmax is the maximum possible value of the inertia weight, while the Wmin is the minimum inertia weight value.

Eberhart and Shi further made another variation in the inertia weight, i.e. presented a linearly reducing inertia weight [15], [12] starting from Wmax to Wmin by using the following mathematical formula:

$$w_k = Iter_{\max} - k / Iter_{\max}(w_{\max} - w_{\min}) + w_{\min} \quad 6$$

Where Itermax is the total number of iterations. Chatterjee and Siarry proposed a nonlinear declining inertia weight [16]. The nonlinear declining inertia weight is implemented using

$$w_k = f_k = \{(Iter_{\max} - k)^n / Iter_{\max}\}(w_{\max} - w_{\min}) \quad 7$$

In the above expression, n is a nonlinear modulation index and variations in the value of n give diverse variation in the inertia weight. Hui Lu developed a new updating strategy for the inertia weight to enhance the diversity of the particles. According to this strategy, the inertia weight "ω" of every particle is controlled dynamically using the Euclidean distance between a particle and the global best particle [29]. The variation of the inertia weight can be expressed by:

$$w = w_{\min} + d_{ig} / d_{\max} \times w_{\max} - w_{\min} / Iter_{\max} \times (Iter_{\max} - k) \quad 8$$

Where d_{ig} is the distance between a current particle and the global best particle, dmax is the maximum Euclidean distance of particles.

Sun Rui [17] added an adaptive inertia weight into the basic PSO process for keeping a good balance in exploitation and exploration strategies during the search process. A mutation operator was also introduced to the PSO method for pushing the particles to leave the local optimum region and move the candidates towards the global optimal solution. In this way, the diversity of the particle swarm is preserved and avoids premature convergence during the PSO process. The expression for finding the value of the adaptive inertia weight is given below.

$$\omega = \begin{cases} 1 - \sqrt[3]{\dfrac{f(x_i^{(t)})}{f(Pbest_i^t)}}, if\ f(x_i^{(t)}) > f_{\max} \\ 1 - \sqrt[3]{\dfrac{f(Pbest_i^t)}{f(x_i^{(t)})}}, if\ f(x_i^{(t)}) < f_{\min} \end{cases} \quad 9$$

where, $f(Pbest_i^t)$ is the fitness function value for the best position of the ith particle, $f(x_i^{(t)})$ represents the function value for the ith agent, $f_{\max}$ is the maxima of the function, $f_{\min}$ and is the minima of the function.

C. Wang et al [18] modified the conventional PSO process in three different directions. Firstly, the best particle in all the neighboring particles and the global best particle of the whole population participate in the PSO search space. Secondly, the unrestricted mechanism is applied to move the particles inside the search space to control premature convergence of the PSO algorithm. At last chaotic search mechanism is introduced into the basic PSO method at the current iteration index to increase the convergence speed of the optimizer, the neighbours of xi are defined by using the mean Euclidean distance between xi and the rest of solutions. Let (i, j) be the Euclidean

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distance between xi and xj and let md$i$ be the mean Euclidean distance for xi. Then md$i$ can be computed as follows:

$$md_i = \frac{\sum_{j=1}^{M} d(i,j)}{M-1} \quad 10$$

M. Spichakova [19] suggested a novel PSO method. The main idea of this novel algorithm is that the learning coefficients are iteratively updated based on gravitational interactions between the particles during the optimization process. The suggested algorithm solved the two-dimensional Diophantine equation problem to observe the search space and workflow of the algorithm direction in the two-dimensional plane. In the proposed approach the learning parameter values are changed according to the distance between particles under consideration by using gravitational laws and therefore the formulas for inertia weight and velocity have been modified in this new approach.

$$v_d(t) = M_i.v_d(t-1) + a_{p,b}.r1.(P_{best_d} - p_d(t-1)) + a_{g,b}.r1.(G_{best_d} - p_d(t-1)) \quad 11$$

Where M_i (instead of α) is the inertial mass

a_{pb} (instead of β) is acceleration towards Pbest: $a_{pb} = \frac{G.M_b}{R^2(p(t-1), P_{best})}$

and

a_{gb} (instead of β) is acceleration towards Gbest: $a_{gb} = \frac{G.M_g}{R^2(p(t-1), G_{best})}$

Huang et al [20] addressed the main problem of stagnation in the traditional PSO process when solving hard optimization problems. In this point of view, a novel PSO using multi-information properties of all the personal-best particles is developed. According to their proposal, two positions for personal-best particles are described. Secondly, an improved cognition term was introduced in the PSO process having three positions for enhancing the searching ability of the particles and flying faster in a better direction by learning the characteristics of all the personal-best particles during the search process. In this regard, the cognitive guiding position can be calculated by the following expression.

$$p'_{best,i} = \frac{p_{best,i} + p_{centr} - p_{med}}{2} \quad 12$$

Three position values are included in the cognitive guiding particle, which are the personal best position $p_{best,i}$, the centroid position p_{centr} and the median position (p_{med}). The $p_{best,i}$ and $p_{centr} - p_{med}$ pulls the particle from the local optima. Another new parameter is $a_{cogn,i}$,

That improves cognition terms $a_{cogn,i}$ to fully utilize all the personal-best fitness.

$$a_{cogn,i} = \sum_{a=1}^{n} p'_{best,i}\theta_i - x_i \quad 13$$

Here is the modified velocity equation:

$$v_i^{t+1} = \omega v_i^t + r_1.a_{cogn,i} + c.r_2.(g_t^{best} - x_i^t) \quad 14$$

G. Xu et al [21] applied the idea of martingale theory to analyze the convergence of the SPSO “standard PSO” and QPSO “Quantum PSO”. For this purpose, initially, the swarm state sequence is defined and then its Markov properties are examined according to the theory of SPSO and QPSO. Consequently, the two close sets are determined, one is the optimal swarm state and the other is the optimal particle state set.

D. Tian et al [22] have done a great discussion about adjusting the inertia weight by different researchers, i.e. linear strategies, nonlinear strategies, fuzzy rules and random strategies. They presented an upgraded version of PSO having a chaos-based process of initialization and a robust mechanism of updating the particles. According to the proposed model to develop a good quality of the initial population a logistic map was utilized to uniformly distribute the initial population. To achieve a better compromise between the exploitation and exploration process with the help of nonlinearly reducing and linearly reducing strategies of the inertia weight, a sigmoid function was added to the PSO. The below expression represents the sigmoid-like inertia weight:

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$$\omega(t)=\begin{cases}0.9, & t\le \alpha t_{\max} \\ \dfrac{1}{1+e^{(10t-2t_{\max})/t_{\max}}}+0.4, & otherwise\end{cases} \quad 15$$

Moreover, to increase the diversity of the swarm a wavelet mutation was added and the velocity-position update technique was effectively applied to the global best particle to ensure the convergence of the proposed approach. Shafi et al [23] introduced a new approach to the PSO, by introducing a special mechanism to continuously update the basic parameters by introducing a new formulation for the basic parameters.
For dynamic Inertia weight:

$$w_i = 1-(I-e-\tan(rand()/2))/h \quad 16$$

For C1 and C2:

$$c_1 = rand\times\exp(1) \text{ And } c_2 = \exp(1) - rand() \quad 17$$

The main purpose is to make the swarm more diversified and to keep a good balance between exploitation and exploration searches.
K. K. Mishra et al [24] integrated the basic qualities of human nature such as environmental awareness and the relationship between leader and follower into the basic PSO algorithm. The central idea is to provide the mentioned characteristics of the human's real life to the swarm leader and all the other particles present. Consequently, this technique also improves the status, quality and direction of the particles in the swarm by using a unique model of iterative directional alertness among the swarm particles. Therefore after successive iterations, the particles become more sensitive about their speed and direction of movement by evaluating their performance. The new equations defined are as under:
Velocity equation:

$$v_{new} = v_{old} + rand(p_{best} - g_{best})/1000 \quad 18$$

Position updating equation:

$$x_{worst}^{new\_position} = (p_{best}^{worst} + g_{best})/2 \quad 19$$

The leader particle adds jitter to its velocity v_{Leader} p_{best}^{Leader} . Then the p_{best}^{Leader} is mutated a little near the mean_swarm_position inside the swarm, as the Leader particle attempts to become an ordinary swarm particle. This is done as:

$$v_{Leader} = v_{Leader}/\omega + rand/10 \quad 20$$

$$p_{best}^{Leader} = 0.9 * p_{best}^{Leader} + 0.01 * \text{rand} * (\text{mean\_swarm\_position}) \quad 21$$

The swarm leader also produces a leadership position, which produces a chaos condition among the particles and it is done by the following expression:

$$g_{best} = -g_{best} \quad 22$$

$$v = v - rand * g_{best}/100 \quad 23$$

b. Modification by adding a new operator

N. J. Li et al [25] incorporated a weighted particle in the population to provide more encouraging search regions for all candidates throughout the optimization process. The new weighted particle x^{ω} is used to guide the swarm to a more accurate direction for searching for the best solution. The position of a weighted particle x^{ω} is defined as below:

$$x^{\omega} = \sum_{i=1}^{p} c_i^{-\omega} x_i^{p} \quad 24$$

Where

$$c_i^{-\omega} = (c_i^{\wedge\omega} / \sum_{j=1}^{p} c_i^{\wedge\omega}) \quad 25$$

And

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$$c_i^{-\omega} = \frac{\max_{1\le k\le M}(f(x_k^p)) - f(x_i^p) + \varepsilon}{\max_{1\le k\le M}(f(x_k^p)) - \min_{1\le k\le M}(f(x_k^p)) + \varepsilon}, i = 1,2,3,...M \quad 26$$

and ε is a minor positive value. $c_i^{\wedge\omega}$ is a weighted constant for all the particles. F(x) is the problem's fitness function for the sake of minimization. After introducing this new weighted particle an enhanced PSO with weighted position (EPSOWP) is defined as under:

$$\begin{aligned} & v_i(t+1) = 0, \ \ and \\ & x_i(t+1) = x_i(t) + \Phi_{4i}(x^w(t) - x_i(t), \ \ if \ \text{r and i} \le \text{a} \end{aligned} \quad 27$$

OR

$$\begin{aligned} & v_i(t+1) = \omega_i * v_i(t) + (\phi_{1i} + \phi_{2i} + \phi_{3i})(x_j^p(t) - \text{x}_i(\text{t})) + \\ & \phi_{2i}(x^G(t) - x_j^p(t)) + \phi_{3i}(x^w(t) - x_j^p(t)), \\ & and \\ & x_i(t+1) = x_i(t) + v_i(t+1), \ \text{if rand}_i > a \end{aligned} \quad 28$$

Mahmoodabadi et al [26] developed the High Exploration PSO HEPSO algorithm to increase the exploration ability of the basic PSO optimizer by incorporating two operators from the genetic algorithm and bee colony optimization algorithm. Moreover, the position of the particles is updated due to the inclusion of the bee colony algorithm operator and multi-crossover operator of the GA "genetic algorithm". The Multi-crossover genetic operator is given below:

$$\theta_1^{/} = \theta_1 + r(2\theta_1 - \theta_2 - \theta_3) \quad 29$$

Where, $r \in [0,1]$ is a random number.

The second operator is taken from the ABC "Artificial bee colony" algorithm, and that is the food source finding parameter of the ABC algorithm, which has been used for the particular particles in the PSO strategy. In this way, more positions xi(t + 1) are created by changing the dth dimension of the stochastically selected particle xi(t).

$$x_i^d(t+1) = x_i^d(t) + (2\text{r}-1)(x_i^d(t) + x_j^d(t)) \quad 30$$

Where, $r \in [0,1]$ is a random positive number.

X. Yu et al [27] further modified CLPSO "comprehensive learning PSO" and developed a new name ECLPSO "enhanced comprehensive learning PSO". According to this idea, the perturbation term is included in the equation for updating the velocity of a particle for achieving high-performance exploitation in the EPLPS. The proposed mechanism helps the local search due to dimensional bounds of the personal best positions, while the particle learning probabilities are also updated on the ranking of the personal best fitness values as well as on the particle exploitation and exploration searches to improve convergence.

D. Boudjehem et al [28] proposed a new Improved Heterogeneous Particle Swarm Optimization (IHPSO) capable of a special feature known as an auto-regulating mechanism to improve the system's convergence trends. According to their mechanism, a new term is added to the equation for velocity update in the basic PSO algorithm. The main purpose of the proposed method is to determine the distance between the particle's best position and the neighbour's global best position to increase the exploration of the search space. Also, a weight parameter is added to the novel modification to improve the exploitation searches of the swarm. The new parameter added is L which represents the exploration/ exploitation control parameter. In the IHPSO the velocity will be updated by the given equation.

$$\begin{aligned} & v_i(k+1) = \omega * v_i(k) + c1 * r1 * (p_{lbest} - x_i(k)) + c2 * r2 * (p_{gbest} - x_i(k)) + \\ & c3 * \text{L} * \gamma * (p_{gbest} - p_{lbest})) \end{aligned} \quad 31$$

$$Where, \ \ \gamma = \begin{cases} -1, & \text{if x}_i = p_{lbest} \\ 1, & \text{elsewhere} \end{cases} \ \ \text{and} \ \ L = \frac{f_i - f_{\min}}{f_{\max} - f_{\min}}$$

N. H. A. Rahman et al [29] integrated the mutation function and sigmoid function to the Binary PSO to solve the problem of OPP (Optimal Placement of PMUs) more efficiently. The V-shaped sigmoid function is given below:

$$sig(v_{ij}^{t+1}) = 2 * \left| \frac{1}{1+} e^{-v_{ij}^{t+1}} - 0.5 \right| \quad 32$$

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The expression for updating the position of vector x is given below.

$$x_{ij}^{t+1} = \begin{cases} (x_{ij}^{t})^{-1}, & \text{if rand<sig}(x_{ij}^{t+1}) \\ x_{ij}^{t}, & \text{otherwise} \end{cases} \qquad 33$$

H. Koyuncu et al [30] introduced the “scout bee” phase of the ABC (artificial bee colony) algorithm to the standard PSO technique to improve the convergence capability of the traditional PSO process and to solve the continuous global optimization problems.
Modification by a change in topology

c. <u>Modification by a change in topology</u>

L. Wang et al [31] have proposed the idea of Multi-Layer Particle Swarm Optimization (MLPSO), according to this idea multiple layers are considered in the population instead of only two layers of the basic PSO process, shown in Fig 2.10. The novel method works in such a way that the multiple best particle positions are found simultaneously in several potential optimal areas at each lower layer, while the upper layer guides the next lower layer in intensive searching of the multi-modal regions. During the search process, the individuals jump out from the local optimum via the cross-layered accommodating behavior of the entire swarm. The MLPSO divides the swarm into several swarms layer by layer. A random inertia weight has also been applied in the MLPSO algorithm to restrict and improve the change in velocity and the searching flexibility respectively.

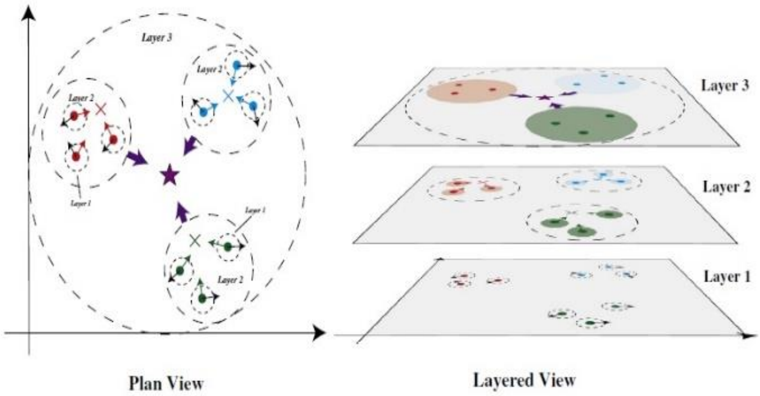

Figure 2 (The layered view of the three layers of global MLPSO, where x represents the best position found so far by the swarmparticle in layer 2 [31]

MLPSO process divides the entire swarm into multiple swarmparticles layer-wise. The distribution of swarmparticles at each layer is defined as:

SP = (SP1, SP2, SP3… SPm) 34

Where SPi denotes the number of swarm particles at layer i-1 and is controlled by a swarmparticle at layer i. The cumulative number of particles can be easily calculated by the following expression:

$$SPN = \prod_{i=1}^{m} SP_i \qquad 35$$

A new expression for velocity update of the ith dimension for local and global MPSO is also devised, which takes information from all the layers and is given by:

For Global MPSO

$$v^i = \omega v^i + \sum_{j=1}^{m} \phi_j r(pbest_j^i - x^i) \quad \phi_j = \frac{\phi}{m} \qquad 36$$

For Local MPSO

$$v^i = \omega v^i + \sum_{j=1}^{m-1} \sum_{k=1}^{n_j} \phi_{jk} r(lbest_{jk}^i - x^i) \quad \phi_{jk} = \frac{\phi}{\sum_{j=1}^{m-1} N_j} \qquad 37$$

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Shiyuan Sun et al [32] presented the idea of the TCPSO "Two-Swarm Cooperative PSO" algorithm according to their approach, the entire population is divided into two swarms, one is the slave swarm and the master swarm. The salve swarm updates from current velocities by learning from the dimension of the neighboring particle, as the novel approach makes the particles of the slave swarm confined toward the local optimum region and in this way, the convergence process is accelerated. The slave swarm particles update the master swarm in static positions, as a result, the diversity of the swarm is maintained with a decent balance between the convergence speed and diversity of the swarm. To make convergence speed faster, a new method has been used to update the particle's velocity:

$$v_{ij}(t+1) = c_1 r_1 (x_{kj}(\mathrm{t}) - x_{ij}(\mathrm{t})) + c_2 r_2 (\mathrm{gbest}_j(\mathrm{t}) - x_{ij}(\mathrm{t})) \quad 38$$

Where $x_k = (x_{k1}, x_{k2}, x_{k3}, ..., x_{kD})$ are chosen at random from the neighbourhood ox xi.

It is noteworthy, that the first term with inertia weight of the traditional PSO has been removed.

Beheshti et al [33] developed an algorithm called the Non-parametric Particle Swarm Optimization (NP-PSO), According to their strategy the balance between the exploration and exploitation searches is maintained without variation in the algorithmic parameter. The central idea of the NPSO is the cooperation among the global and local topologies, in combination with two quadratic interpolation operations to upsurge the searching aptitude.

Yuanxia Shen et al [34] developed the idea of Double Learning Patterns Particle Swarm Optimization (DLP-PSO), According to their new approach two learning patterns are developed, one is a uniform exploration and the second is enhanced exploration. They also introduced the idea of two swarms, master and slave. The purpose of the master swarm is to adopt uniform exploration and to avoid a premature convergence process, while the slave swarm is used for enhanced exploration for quickness of the convergence. Based on this novel idea the new expressions for velocity update of ith particle in both slave and master swarm are given:

For Master Swarm:

$$v_{id}^m(t+1) = \omega v_{id}^m(t) + (\mathrm{c}_1^m + \mathrm{c}_2^m).\mathrm{D}_f^m[L_f^m.pb_{id}^m(t) + (1 - L_f^m)Gb_d^m(t) - \mathrm{x}_{id}^m(t)] \quad 39$$

$$f_{L_f^m}(l_f^m) = \begin{cases} 1, & 0 \le l_f^m < 1 \\ 0, & \text{otherwise} \end{cases} \quad 40$$

$$L_f^m = L_{f,\max}^m - \frac{(L_{f,\max}^m - L_{f,\min}^m)}{iter_{\max}} iter \quad 41$$

Where, ${}^{m}_{f,\max} = 1$ *and* $L_{f,\min}^m = 0$, iter is the maximum number of iterations, Gb_d^m is the global best of the master swarm.

Wang et al [35] proposed a strategy of comparative judgment for the traditional PSO process. The main idea of the strategy is that during the search process, three types of experiences are observed such as individual experience, social experience and a combination of both. According to their strategy, the best experience is selected by the comparative judgment of all three results. All the three equations are given below.

$$V_1 = \omega(t)\,\mathrm{v}_i(t) + \mathrm{c}_1 * r_1 * (pbest_i(t) - \mathrm{x}_i(\mathrm{t})) \quad 42$$

$$V_2 = \omega(t)\,\mathrm{v}_i(t) + \mathrm{c}_2 * r_2 * (\mathrm{g}best_i(t) - \mathrm{x}_i(\mathrm{t})) \quad 43$$

$$V_3 = \omega(t)\,\mathrm{v}_i(t) + \mathrm{c}_1 * r_1(pbest_i(t) - \mathrm{x}_i(\mathrm{t})) + \mathrm{c}_2 * r_2(\mathrm{g}best_i(t) - \mathrm{x}_i(\mathrm{t})) \quad 44$$

$$v_i(t+1) = \left\{ V_k \,\middle|\, f(x_i(t) + V_k) = \min\left\{ f(x_i(t) + V_j), \mathrm{j} = 1, 2, 3 \right\} \right\} \quad 45$$

Where all the parameters are the same as in basic PSO. The eq. 2.44 clearly defines that the comparison between all three experiences will be done and the best experience will be selected for updating the velocity of the particle.

Al-Bahrani et al [36] considering a new learning mechanism known as Orthogonal Particle Swarm Optimization (OPSO) to eliminate the drawbacks of GPSO by applying the orthogonal diagonalization (OD) process. In fact, the matrix diagonalization converts a square matrix, A with size (d×d), into a diagonal matrix, D with size (d×d), as given below:

$$A = Q.D.Q^{-1} \quad 46$$

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According to the newly designed process swarm population is distributed into two categories; one is the active personal best experience particles and the second is the passive collection of the individual experiences of remaining particles (m – d). In the search process, the proposed modification maintains the diversity of the populations. According to their idea, the orthogonal guidance vectors are achieved from the active group and only one guides the particle, the velocity and position vectors of the energetic group of the particles are updated at every iteration while the remaining (m – d) particles remain unchanged. The proposed methods dodge the contradictory situation of the GPSO optimizer and guide the best d particles in the direction of the best solution inside a multi-dimensional search space.

M. S. Nobile et al [37] defined a new version of Fuzzy Logic PSO (FL-PSO), known as Fuzzy Self Tuning PSO (FST-PSO). According to their idea, various learning parameters of PSO are automatically updated and adjusted using a fuzzy logic mechanism. The speciality of FST-PSO is that every particle has its value for inertia weight, social component, cognitive component, lower clamping value and upper clamping value, represented by $\omega_i(t), C_{soc(i)}(t), C_{cog(i)}(t), L_i(t), \mu(t)$ respectively. During different iterations, every particle is dynamically and adaptively tuned and the velocity of each particle is updated according to the following equation:

$$v_i(t) = \omega_i(t-1).v_i(t-1) + C_{social_i}(t-1).R_1 o(x_i(t-1) - g(t-1)) + C_{cog_i}(t-1).R_2 o(x_i(t-1) - b(t-1)) \quad 47$$

Moreover, 15 fuzzy rules have been used for the adjustment of the values of different parameters. D. Tamayo-vera et al [38] introduced two phases in the basic PSO process to advance the exploration and exploitation searches inside the swarm. The first step of the exploitation process is evaluated to ensure that every Pbest position gets a chance to be the Lbest position for some time to increase the chances of fully exploiting the known attraction basin. The novel modification selects the lbest position for a particle by rotating over the usual Pbest positions from which they normally select its lbest position. In the second step, the PSO explore more search space due to reinitiating the particles.

Shanshan Chen et al [39] suggested an innovative 3-D Positioning System based on the PSO along with the Chan algorithm to significantly enhance the particle positioning precision as well as reduce the computational time of the designed mechanism. The proposed system estimates the early position of the target and effectively excludes the "NLOS" error. A modified Chan algorithm accomplishes consecutive calculations more rapidly due to the initial location to obtain the final strict position of the target.

d. Modification by hybridization with other Algorithms

Y. J. Gong et al [40] proposed a new approach introducing genetic evolution into the basic PSO process for encouraging exemplar particles to search space. The central idea behind the method is that the swarm consists of upper and two lower consecutive layers, the upper layer generates the best exemplar, while the remaining layer guides the particles. Exemplars are generated by the genetic operators, and the particles get directions from them as a result, the history of search information of particles delivers guidance for further growth of the exemplars. Consequently, after performing the crossover operation, mutation, and selection based on historical information of particles, the resultant exemplars are not only well expanded but also become highly experienced. As a result of all these processes, both the global search ability and efficiency of the PSO are enhanced. The velocity and position update expressions for the d_{th} dimension $(d = 1, 2, 3, ..., D)$ of particle i are given as:

$$v_{i,d} \leftarrow \omega * v_{i,d} + c1 * r1, d * (p_{i,d} - x_{i,d}) + c2 * r2, d * (g_d - x_{i,d}) \quad 48$$

$$x_{i,d} \leftarrow x_{i,d} + v_{i,d} \quad 49$$

Then for each particle i the crossover operation is performed on pi and g to generate the offspring $O_i(o_i, 1, o_i, 2, ..., o_i, D,)$

$$o_i, D = \begin{cases} r_d \cdot p_{i,d} + (1 - r_d).g_d, & \text{if } f(p_i) < f(p_{k_d}) \\ p_{k_d, d,} & \text{otherwise} \end{cases} \quad 50$$

H. Garg developed the PSO-GA algorithm [41]. According to their proposal, the basic process of exploration and exploitation search capabilities of the particles can be improved by using genetic operators in the PSO algorithm. The importance of this idea is to increase the talent of social thinking in the PSO algorithm by employing the local

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search property of the genetic algorithm. The hybrid PSO-GA approach indicates a more suitable and best optimal algorithm for finding the global optimal solution of the optimization problems.
Based on the psychological model and Weber–Fechner law J. Gou et al [42] introduced individual difference evolution PSO (IDE- PSO). They defined two feelings that can be transferred to the particles, sadness and joy, which resemble the two reactions to judgment. When a particle is in joy condition, it can exploit historical and global experiences and is more dedicated to its current position. But if a particle is in sad condition, it gives more attention to its previous history and alters itself from its normal position. This behavior is like human beings because a particle's emotions change according to the inner elements. The sensitivity of a particle is presented by the following expressions:

$$r_g = -k \ln \frac{S(f(gBest_i) - f(x_i))}{S_0} \quad 51$$

$$r_h = -k \ln \frac{S(f(pBest_i) - f(x_i))}{S_0} \quad 52$$

Where r_g is global perception, r_h is the sense of a particle's history, and k is the constant determined by the experimentation. S represents the stimulus function, while S0 is the stimulus threshold. The novel mechanism splits the overall swarm in three subswarms and chooses the best possible method for each particle as per its emotional status and fitness value. The coefficient value is set dynamically based on the evolutionary performance of every individual particle. Another novelty of this idea is to employ the restarting strategy to restart the corresponding particles to improve the diversity of the whole population. The emotional effect on particle velocity can be determined by the following equations:

$$V_i^{t+1} = \omega.V_i^t + c_1.r_1.r_g.(pBest_i^t - x_i^t) + c_2.r_2.r_h.(gbest^t - x_i^t) \quad 53$$

$$V_i^{t+1} = \omega.V_i^t + c_1.r_1(pBest_i^t - x_i^t) + c_2.r_2.\frac{r_h}{r_g}.r_h.(gbest^t - x_i^t) \quad 54$$

Eq 2.52 presents a joyful particle, while Eq 53 is a sad particle.
N. Singh et al [43] suggested the combination of PSO with Grey Wolf Optimizer (GWO) to improve the convergence capability of the conventional PSO process. Using low-level co-evolutionary diversified hybrid. This proposed hybrid is of low level due to the merger of the functionalities of both variants. The novel hybrid optimizer has the quality of coevolutionary at a low level due to the presence of hybrid algorithmic operators. In light of this, the algorithm variants run in a parallel fashion to achieve the best optimal solution in the search space during the optimization process. To combine PSO and GWO variants, the velocity and updated equation are proposed as follows:

$$v_i^{k+1} = \omega*(v_i^k + c_1r_1(x_1 - x_i^k) + c_2r_2(x_2 - x_i^k) + c_3r_3(x_3 - x_i^k)) \quad 55$$

$$x_i^{k+1} = x_i^k + x_i^{k+1} \quad 56$$

Y. Chen et al [44] presented the idea of using the crossover operator of the differential evolution "DE" with basic PSO (PSOCO) to share the best information among the particles and avoid the premature convergence process. The proposed idea is based on two crossover operators; one is the differential evolution crossover operator and the second is the modified velocity update vector; that updates the PSO process and preserves a proper balance between searches at both global and local levels. Further, a dynamic correction strategy is introduced to increase the exploitation capabilities of the individuals.
Arithmetic crossover:
For each particle 'i' the crossover operation is performed on pi and pj to produce a new individual $V_i = [v_{i,1}, v_{i,2}, ..., v_{i,D}]$ according to the following equation:

$$V_{i,d} = r_{1,d}P_{i,d} + (1 - r_{1,d})P_{j,d} \quad 57$$

Where $r_{i,d}$ is a random number between 0 and 1.
Differential evolution crossover:
The DE crossover operation interchanges some components of the vector Vi with the target vector Pi to make a trial vector Ui. This process can be expressed by the following expression:

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$$U_{i,j} = \begin{cases} V_{i,j,} & \text{if } r_2 \le CR \text{ or } j=j_{rand} \\ P_{i,j,} & \text{otherwise} \end{cases} \quad 58$$

where CR is the user-defined parameter in the range of [0-1],

Askarzadeh et al [45] proposed an improved PSO and applied it to DOCL's “daily optimal chiller loading” problem in a Multi-chiller system to reduce the power consumption of the overall system. The proposed idea is to combine the two algorithms, EPSO “elitism-based PSO” and MA-PSO “multi-agent PSO”. There are predefined numbers of reasonable solutions for every agent and in the DOCL model, every feasible solution comprises the PLR of all the chillers at every hour of the day. Now during different iterations, the solutions are updated by MA-PSO to get the best solution for daily loading. On the other hand, in EPSO, all the particles are graded at each iteration and the first 20 particles are nominated as elite to find the best solution to the “DOCL” problem. Consequently, a competition is executed between two randomly selected particles in the elites and then the superior one is chosen as the remarkable elite particle. That particle, which wants to update its position goes closer to the chosen interesting elite. This process of updating the solutions continues during the different iterations by the EPSO algorithm until and unless the optimal PLR value of all the chillers at every hour of the day is achieved.

K. H. Chao et al [46] designed an improved PSO algorithm and applied it to the shading as well as failure conditions of the MPPT “maximum power point tracking” method in the photovoltaic (PV) module array. As their main approach indicates that the novel formulations have been adopted for the learning and control parameters of the basic PSO to solve the mentioned problem in the photovoltaic module array. The proposed algorithm contains a new formulation for the three basic parameters given below.

$$c_1 = c_{1,\max} - (c_{1,\max} - c_{1,\min}) * \frac{2^j + 1}{2^N + 1} \quad 59$$

$$c_2 = c_{2,\max} - (c_{2,\max} - c_{2,\min}) * \frac{2^j + 1}{2^N + 1} \quad 60$$

$$\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) * \frac{2^j + 1}{2^N + 1} \quad 61$$

Shahira et al [47] applied the modified PSO to data clustering problems for partitioning and also on hard data clusters. The new LPSOC method controlled the drawbacks of the k-means algorithm that were applied previously in data clustering problems. The central idea of the proposal is that the neighboring particles guide other particles towards the global optimal solution of every cluster centroid in the search space. During the evolution process, the entire swarm represents a solution to the clustering problems.

H. Yapjcj et al [48] introduced a new strategy in the traditional PSO, in which the process of optimization is performed in phases, one is the global search randomization phase and the second one is the intensive local search phase. The two phases energize the particles in such a way that exploitation and exploration searches are in proper balance with each other. The novel strategy also supports the algorithm to find the optimum solution and avoid premature convergence.

X. B. Wang [49] suggested a combinational method known as PSO-based VMD (PSO-VMD) to improve the existing method of vibration mode decomposition "VMD" having the self-adapting property related to the LMD and EMD methods. In the proposed method the fault-detection framework is used to separate the detected vibration signals into a sequence of intrinsic modes. During the evolution process, a specific number of the inherent modes are then selected with the help of the “Hilbert transform-based square envelope spectral kurtosis.

J. H. Lee et al [50] suggested an improved PSO algorithm for the optimal design of electric machines. This algorithm operates on the exploration as well as the exploitation capabilities of PSO but efficiently controls several individual particles in the exploitation phase to reduce the total number of calling the function. The proposed algorithm describes its ability to determination that whether the current stage is exploration or exploitation. Consequently, the proposed method was determined by a predefined condition that matches the cost value of g_{best} during the current iteration with the g_{best} in the previous iteration. The algorithm operates in the exploration stage, whenever there is a major difference in the two cost values and it also preserves the number of particles. When the difference between the two cost values is less than the preset threshold value set by the user, then it works in the exploitation stage to reduce the overall number of particles. In this algorithm, the “Euclidean distance”

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equation is used to decide the particles that must be eliminated during the successive iterations. The equation for "Euclidean distance" of the nth particle ln is given below:

$$l_n = \sqrt{\sum_{k=1}^{m} (x_k^{best} - x_k^n)^2} \quad 62$$

C. Wang et al [51] introduced the idea of combining an improved version of PSO with RFID technology to minimize the localization cost and improve the relevant algorithm in Location-based services (LBS). The proposed method is based on the Feed-Forward Neural Network (IMPSO-FNN) to improve indoor localization and control the premature convergence problem of the system. In reality, the proposed approach globally optimizes the structural parameters and network parameters and the novel strategy determines the optimal linkage weights for the system. The new formulation for basic parameters is given here:

$$c_1 = c_{1i} - (c_{1i} - c_{1f}) * (t / T_{max}) \quad 63$$

$$c_2 = c_{2i} - (c_{2f} - c_{2i}) * (t / T_{max}) \quad 64$$

$$\omega(t) = \omega_{max} + (\omega_{max} - \omega_{min}) * \exp(-20 * (t / T_{max})^k) \quad 65$$

Z. Fuxing et al [52] proposed the GCMA-ES technique for the conventional PSO algorithm, which is the abbreviation for "best-guided covariance matrix adaptation evolution strategy". In which the gbest information is applied during the search process to guide the exploitation process to increase the exploitation power of CMA-ES. This technique can effectively use the power of CMA-ES by familiarizing them with the step size and updating the covariance matrix, while simultaneously updating the solution by learning from the most recent best global solution.

Pallero et al [53] applied the PSO optimizer to the uncertainty assessments of the inverse problems, specifically related to the geophysical systems. The use of the PSO optimizer is to scan the equivalence region inside the nonlinear inverse problems. The suggested methodology is general and can be applied to many applications, but in this work, the proposed method addresses the solution of a geophysical problem related to the gravity inversion in sedimentary basins in the application of an uncertainty assessment, which proves the possibility of efficiently performing the sampling-while-optimizing mode in particular.

SUMMARY

Different algorithms have been developed to tackle the challenge of solving complex engineering design problems, the particle swarm optimization is one of them, which is a comparatively simple and easy to implement meta-heuristic algorithm. The problem with PSO is its premature convergence during solving complex real-world problems, i.e. optimization of superconductive magnetic energy storage devices (SMES) and optimization of the amplitude of sidelobes for reducing the interference with adjacent channels in cognitive radio (CR). To eliminate this weakness of PSO researchers have tried to modify the basic PSO in four major directions. The results of different researchers show the key role of all the directions of modifying the basic PSO.

CONCLUSION

This literature review shows that some of the researchers have applied their modified PSO to diverse science and engineering-related applications and got successful results. Now it can be concluded that the PSO algorithm has the strength to be applied to various engineering applications.

POSSIBLE FUTURE DIRECTIONS

The PSO has now become the state-of-the-art technique for solving engineering design problems. To satisfy the ever-increasing demands on numerical methodologies for inverse problems and optimizations, the following challenges should be addressed extensively:

1. The balance of exploiting and exploration searches in existing EAs.
2. Incorporating the preferences of a decision maker in vector EAs.
3. The fast computation methodology and model for robust computing performance of the parameters and constraint treatments.
4. The robust-oriented optimizer for both robust and global optimal solutions.
5. Theoretical issues for existing numerical methodologies and models.

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